THREE ESSAYS ON THE NEW KEYNESIAN PHILLIPS CURVE

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ABSTRACT

In the recent years the New Keynesian model has become the main tool to analyse monetary policy and business cycle fluctuations. A fundamental part of this model is the New Keynesian Phillips curve, which suggests a positive relationship between current inflation and the output gap.

This PhD thesis is composed by three essays which tackle recent and controversial aspects of the New Keynesian Phillips curve subject. The first chapter presents a critical survey about the New Keynesian Phillips curve focusing on the so-called “divine coincidence” problem. The second chapter proposes an extension of the Blanchard and Galì New Keynesian Phillips curve and shows that this new formulation helps to explain inflation dynamics. The third chapter examines an extended version of the small-scale New Keynesian model a la Woodford (2003) to assess the role of oil price shocks in influencing inflation and unemployment volatility.

Chapter 1 analyses the genesis and the evolution of the New Keynesian Phillips curve, with particular attention to the solutions proposed by the economic literature to the “divine coincidence” problem, i.e. the absence of a meaningful trade-off faced by the central bank due to the one-to-one mapping between the first and the second best output levels. Our work identifies two main approaches that generate endogenously a trade-off between inflation and the output gap in the presence of a microfounded shock: the first approach, proposed by Blanchard and Gali (2007), focuses on real wage rigidities; the second, elaborated by Ravenna and Walsh (2006), focuses on the cost channel.

Chapter 2 estimates a new version of the New Keynesian Phillips curve (NKPC) in which real wage stickiness implies the existence of unemployment in equilibrium (Blanchard and Gali, 2007, Journal of Money, Credit and Banking). In such model, firms employ a nonproduced input (interpreted as oil) that directly enters the Phillips curve. We fit this model to U.K. and U.S. data, and we compare its empirical performance to that offered by the standard NKPC a la Woodford (2003). The comparison considers the adherence of the results to conventional a priori. Our results point towards the importance of including both the real producer price index and the real effective exchange rate in the New Keynesian
Phillips curve. In particular, the inclusion of the latter makes unemployment coefficient significant and forces it to assume a sign coherent with economic expectation.

Chapter 3 estimates an extended version of the small-scale New Keynesian model a la Woodford (2003) to account for the impact of oil price shocks on U.S. inflation and unemployment. We find four main results. First, in the last two decades, oil price shocks have played an important role in explaining U.S. inflation fluctuations. Second, stronger reactions to oil price swings than the ones historically observed would not have improved the stabilization of inflation and unemployment. Third, the best result in terms of stabilization of inflation and unemployment is obtained by a “progressive central bank” (i.e. a central bank which reacts strongly to unemployment and weakly to inflation) in the presence of oil price shock. Fourth, the more forward-looking the firms are, the more stable the economy is in the presence of oil price shock.
ABSTRACT (IN ITALIAN)

Negli anni recenti il modello neokeynesiano è diventato lo strumento principale per analizzare la politica monetaria e le fluttuazioni del ciclo economico. Una parte fondamentale di questo modello è la curva di Phillips neokeynesiana, la quale suggerisce una relazione positiva tra l’inflazione corrente e l’output gap (cioè il divario esistente tra il prodotto corrente e il prodotto naturale). Questa tesi di dottorato è composta da tre saggi che affrontano aspetti recenti e controversi inerenti il tema della curva di Phillips neokeynesiana. Il primo capitolo presenta una survey critica relativa alla curva di Phillips neokeynesiana mettendo a fuoco il cosiddetto problema della “divina coincidenza””. Il secondo propone un’estensione della curva di Phillips neokeynesiana di Blanchard e Gali e dimostra che questa nuova formulazione aiuta a spiegare le dinamiche dell’inflazione. Il terzo esamina una versione estesa del modello neokeynesiano standard (Woodford (2003)) per valutare quale ruolo rivestono gli shock sul prezzo del petrolio nell’influenzare la volatilità dell’inflazione e della disoccupazione.


Il secondo capitolo stima una nuova versione della curva di Phillips neokeynesiana nella quale la vischiosità del salario reale implica l’esistenza di disoccupazione in equilibrio (Blanchard and Gali (2007), Journal of Money, Credit and Banking). In tale modello le imprese impiegano un fattore di produzione non prodotto (interpretato come petrolio) che entra direttamente nella

Il terzo capitolo stima una versione estesa del modello neokeynesiano standard (Woodford (2003)) per dare conto dell’impatto sull’inflazione e la disoccupazione esercitato dagli shock relativi al prezzo del petrolio. In questo capitolo emergono quattro risultati principali. Primo, nei passati due decenni, gli shock relativi al prezzo del petrolio hanno giocato un ruolo importante nello spiegare le fluttuazioni dell’inflazione negli Stati Uniti. Secondo, reazioni più forti di quelle storicamente osservate a fronte degli shock sul prezzo del petrolio non avrebbero migliorato la stabilizzazione dell’inflazione e della disoccupazione. Terzo, in presenza di uno shock sul prezzo del petrolio il miglior risultato in termini di stabilizzazione dell’inflazione e della disoccupazione è ottenuto da un “banchiere centrale progressista” (cioè da un banchiere centrale che risponde fortemente alla disoccupazione e debolmente all’inflazione). Quarto, tanto più le imprese formano le loro aspettative di prezzo in modo forward-looking (cioè guardando in avanti), tanto più stabile è l’economia in presenza di uno shock sul prezzo del petrolio.
CHAPTER 1


1. Introduction

Since A.W. Phillips (1958) conceived the Phillips curve (a negative relation between unemployment and monetary wages rate of growth), many authors have been interested in this subject. In the last decades the economists’ community has been divided between who believes in the ability of this curve to capture the inflation dynamics and who thinks that this formulation has some lacks. The debate which has taken place allows to formalize a modern formulation of this curve: the New Keynesian Phillips curve.

Today a fundamental framework to analyse monetary policy and business cycle is the New Keynesian model (Woodford 2003). The supply-side of this model is represented by the so-called New Keynesian Phillips curve (NKPC henceforth). It is a relation between current inflation, expected future inflation and the output gap (i.e. the difference between current level of output and the natural output). In the standard form the NKPC reads as follows:

\[ \pi_t = \beta E_{t+1} \pi_{t+1} + (y_t - y_t^*) \]  \hspace{1cm} (1)

where \( \pi_t \) is inflation, \( \beta \) is firm’s discount factor, \( y_t \) represents the log current level of output, \( y_t^* \) is the log natural output, \( E_{t+1} \pi_{t+1} \) is the expectation taken at the time \( t \) of the inflation at the time \( t+1 \). In the standard form the NKPC does not include supply shocks. Moreover the standard formulation of the NKPC does not include elements of inflation inertia. In fact in (1) the inflation lags do not appear.

In this chapter we present a critical survey of the theoretical bases of the NKPC. In particular we emphasize the new approaches proposed by Ravenna and Walsh (2006) and Blanchard and Galì (2007). The objective of this chapter is to lay out critically the works which have contributed to creation of the NKPC. In presenting this survey we pay attention to the limits of the theoretical framework emerged.
The chapter is organised as follows: Section 2 analyses the theoretical bases of the standard NKPC. In Section 3 we discuss the “divine coincidence” problem of the standard formulation of the NKPC, i.e. the absence of a trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap. Section 4 presents the different solutions that many authors have proposed to solve the “divine coincidence” problem. Section 5 proposes some promising avenues to follow to develop the NKPC. Section 6 concludes.

2. Standard New Keynesian Phillips curve

The rational expectations revolution, led by Lucas (1976), imposed a new challenge to the Keynesian economists: the necessity to integrate the business cycle analysis of the short-run, characterized by the presence of nominal rigidities, in a framework in which the economic agents have rational expectations.

This challenge was taken up by important authors. These economists built a new framework which, today, is the most important instrument to analyse the monetary policy. This framework was called New Keynesian model. It is a stochastic and dynamic general equilibrium model. A fundamental block of this model is the NKPC. Three authors have contributed in fundamental way to model the standard NKPC: John Taylor (1979, 1980), Julio Rotemberg (1982, 1982a) and Guillermo Calvo (1983). In the next subsections we explore these three approaches to understand the basic structure of the NKPC.

2.1. Taylor’s approach

A decisive prerogative of all the industrialized economies is that the contract decisions are staggered. According to Taylor this prerogative introduces a type of nominal rigidity which is central to build a macroeconomic tool to inquiry economic fluctuations of the economy.

Taylor supposes that the contracts are drawn up in the way that all contracts last two years. To consider the staggered contract decisions, we assume that half

1 An other important paper to the genesis of the New Keynesian model is the work of Fischer (1977). In fact also this author builds a model which underlines the important role of staggering wages in determining nominal rigidities.
the contracts are drawn up in January and half in July. \( x_t \) is the log of the wage which is set at the time \( t \) for the time \( t \) and \( t+1 \).

In consideration of these hypotheses, in this model the log wage is equal to:

\[
x_t = bx_{t-1} + dE_t x_{t+1} + \gamma (bE_t y_t + dE_t y_{t+1}) + \delta_t
\]

(2)

where \( b \), \( d \) and \( \gamma \) are positive parameters, \( y_t \) is a measure of the excess of demand in the period \( t \) (for instance the output gap, i.e. the difference between current level of output and the natural output), \( \delta_t \) is a random shock, the operator \( E \) expresses the conditional expectation of \( y \) and \( x \) based on information available at the time \( t \) and \( t-1 \).

The equation (2) establishes that contract wage, drawn up in a certain point of the time, depends on: a) the contract wage of the previous period; b) the expected contract wage in the next period; c) a weighted average of expected excess demand in the next two periods.

To explain dynamic behaviour of the Taylor’s contract wage, we have to assume an aggregate demand equation and a policy rule.

The excess demand \( y_t \) is represented by the log deviation of current real output from its natural level. Besides the demand for money is described by this relation:

\[
m_t = y_t + w_t - v_t
\]

(3)

where \( w_t \), \( m_t \), \( v_t \) are respectively the aggregate level of wage, the money supply and a shock (all these variables are expressed as a log deviation from their long-run trends).

The policy rule for money supply is a log linear relation of the aggregate level of wage:

\[
m_t = gw_t
\]

(4)
Rearranging the previous equations, we obtain the aggregate demand equation, which reads as follows:

\[ y_t = -\beta w_t + v_t \]  

(5)

with \( \beta = 1 - g \).

The average wage is equal to:

\[ w_t = \frac{(x_t + x_{t-1})}{2} \]  

(6)

The labour supply curve is:

\[ x_t = \left( \frac{p_t + E_t p_{t+1}}{2} \right) = c_0 - \eta \left( \frac{u_t + E_t u_{t+1}}{2} \right) + \epsilon_t \]  

(7)

where \( p_t \) is log price level, \( u_t \) is the unemployment rate, \( \epsilon_t \) is a white-noise error term which gives accounts for the unobserved factors which influence the wage, \( c_0 \) and \( \eta \) are constant. In particular \( \eta \) is higher than zero. For this reason there is a negative relation between the expected average unemployment rate and the expected average real wage.

Taylor supposes that each firm fixes its price computing a constant markup over the wage. If we normalise the markup to zero, we obtain:

\[ p_t = w_t \]  

(8)

Now the equations (6), (7), (8) can be rearranged and we obtain:

\[ \pi_t = c_0 + E_t \pi_{t+1} - \eta (u_{t-1} + u_t + E_{t-1}u_t + E_t u_{t+1}) + 2(\epsilon_t + \epsilon_{t-1}) + \zeta_t \]  

(9)

where \( \zeta_t \) is an expectational error \( (E_{t-1}p_t - p_t) \), \( \pi_t \) is inflation at the time \( t \) \( (\pi_t = \Delta p_t) \).
In this formulation the inflation at the time $t$ is function of a constant $c_0$, expected future inflation, a moving average of the unemployment rate and an expectational error.

2.2. Rotemberg's approach

To derive the New Keynesian Phillips curve, Rotemberg introduces the quadratic price adjustment cost model. In this framework the firm wants to minimize a function in which are compared two types of costs: a) the cost of modifying the current price; b) the cost of being away from the optimal price for the firm. The problem of the firm can be represented as:

$$
\min_{(p)} \Omega_t = E_t \left[ \sum_{\tau=t}^{\infty} \theta^{\tau-t} \left( (p_{\tau} - p^*_\tau)^2 + c (p_{\tau} - p_{\tau-1})^2 \right) \right] 
$$

where $\Omega_t$ represents the total cost, $p$ is the log price of the firm at the time $t$, $p^*_\tau$ is the log price that the firm would set, if the adjustment costs did not exist, $\theta$ is a constant discount factor, $c$ is a parameter which gives account for the ratio between the cost of modifying the price and the cost of being away from the price desired by the firm.

Solving this minimization problem, we obtain this first order condition:

$$
E_t \left\{ (p_t - p^*_t) + c \left[ (p_t - p_{t-1}) - \theta (p_{t+1} - p_t) \right] \right\} = 0 
$$

(11)

We assume that $\theta$ is equal to one, then (11) becomes:

$$
\pi_t = E_t \pi_{t+1} - \left( \frac{1}{c} \right) (p_t - p^*_t) 
$$

(12)

where $\pi_t = \Delta p_t$

Rotemberg assumes that the desired log price for the firm follows this rule:

$$
p^*_t = p^*_{tj} + \beta y_t + \epsilon_t
$$

(13)
where $y_t$ is the log deviation between current output and the natural output (i.e. the output gap), $p_t^{of}$ is the price which the other firms set at the time $t$, $\beta$ is a positive parameter and $\epsilon_t$ is a i.i.d. random error. The equation (13) describes a positive relation between the optimal price of the firm and the aggregate output.

Because we assume that all firms are identical, we can replace $p_t^{of}$ with $p$ in the equation (13) and rearranging we obtain:

$$\pi_t = E_t \pi_{t+1} + \left( \frac{\beta}{c} \right) y_t + \frac{\epsilon_t}{c}$$  \quad (14)

The equation (14) expresses inflation at the time $t$ as a function of expected future inflation, the output gap and the error term. In particular, when the current output gap rises, current inflation increases.

2.3. Calvo’s approach

In the Calvo’s model at any time only a fraction of firms can reset the price. This fraction of firm is expressed by $1 - \theta$. The remaining firms, represented by the fraction $\theta$, can not modify their prices.

The firms, which can change their prices, fix them in log to minimize a loss function. The loss function takes this form:

$$L(z_t) = \sum_{k=0}^{\infty} (\theta \beta)^k E_t \left( z_t - p_{t+k}^* \right)^2$$  \quad (15)

where $z_t$ is the log price that the firm would fix at the time $t$, if it had this opportunity, $p_{t+k}^*$ is the optimal price for the firm at the time $t+k$, $\beta$ is a discount factor included between zero and one.

The solution of the minimization problem (15) is given by:

---

This solution suggests that, when the firm can reset its price, it fixes a price \( z_i \) which is a weighted average of the prices that it would have fixed in the presence of perfectly flexible prices.

The optimal price for the firm is given by:

\[
p^*_i = \mu + mc_i
\]  

(17)

where \( mc_i \) is the log nominal marginal cost of the firm and \( \mu \) is a constant markup.

Replacing (17) in (16), we obtain:

\[
z_i = (1 - \theta \beta) \sum_{k=0}^{\infty} (\theta \beta)^k E_t p^*_i(k)
\]  

(18)

In the Calvo’s model the aggregate level of log price takes this form:

\[
p_i = \theta p_{i-1} + (1 - \theta) z_i
\]  

(19)

The aggregate log price level at the time \( t \) is a weighted average between the log price fixed by the fraction \( (1 - \theta) \) of firms at the same time and the log price of the previous time weighted for \( \theta \).

By some rearrangements we obtain the equation which describes the NKPC:

\[
\pi_i = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \theta \beta)}{\theta} (\mu + mc'_i)
\]  

(20)

where \( \pi_i \) is the inflation rate \( (\pi_i = p_i - p_{i-1}) \) and \( mc'_i \) is the real marginal cost. According to (20), inflation at the time \( t \) is function of expected future inflation and real marginal cost.
Finally, if we suppose a positive relation between real marginal cost and the output gap \( mc_t^\mu + \mu = \lambda y_t \), we can reformulate the NKPC as follows:

\[
\pi_t = \beta E_t \pi_{t+1} + \gamma y_t
\]  

(21)

where \( \gamma = \frac{\lambda (1-\theta)(1-\theta\beta)}{\theta} \). The equation (21) expresses inflation at the time \( t \) as function of expected future inflation and the output gap.

2.4. Comments on the standard NKPC

The three different approaches, which we have examined in this section, have some characteristics in common.

In the first place they involve some form of nominal rigidities. The approaches of Rotemberg (1982, 1982a) and Calvo (1983) incorporate nominal price rigidities. Instead the Taylor’s model is built on the nominal wage rigidities.

In spite of this difference, all approaches analysed can be represented by the equation (1):  

\[
\pi_t = \beta E_t \pi_{t+1} + (y_t - y_t^*)
\]

In the second place it is interesting to note that expectations involved in the NKPC are only forward-looking. For this reason in this standard model the inflation inertia does not have any role in explaining inflation dynamics.

3. "Divine coincidence"

In the NKPC there is a shortcoming. This shortcoming is the absence of trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap (i.e. the deviation between current output and the first best

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3 The equation (1) is also a correct derivation of the Taylor’s approach. In fact, as Roberts (1995) explains, the unemployment rate is strongly serially correlated, therefore current unemployment rate is a valid proxy for future, current and lagged unemployment rate. Finally, it is possible to replace the unemployment rate with the output gap, using Okun’s law.
output\textsuperscript{4}, when a shock (for example a change in the price of oil) happens. Blanchard and Gali (2007) call this prerogative of the NKPC “divine coincidence”.

The source of “divine coincidence” is the constant relation between the first best output and the second best output. Because of this constant relation, when policy makers stabilize the output gap (i.e. the gap which arises between current output and the second best output), they automatically stabilize the welfare-relevant output gap (i.e. the gap which arises between current output and the first best output). From (1) we see that the stabilization of inflation is compatible with the stabilization of the output gap. Then, because the relation between the first best and the second best output is constant, it follows that, when policy maker stabilizes the output gap in response to shock, in the same time she stabilizes also the welfare-relevant output gap.

4. Alternative approaches to solve the “divine coincidence” problem

To solve the “divine coincidence” problem, different solutions are proposed in the economic literature. In this section we analyse the four different paths which economists have followed to tackle this question. The four different approaches are:

a) distortion shocks;

b) alternative structures of wage and price fixing;

c) cost channel;

d) real wage rigidities.

4.1. Distortion shocks

The more immediate solution to the “divine coincidence” puzzle is to add a “cost-push” shock to the standard form of the NKPC. This solution is practised for the first time by Clarida, Gali and Gertler (1999, CCG henceforth). These authors consider a theoretical framework, in which there are two main blocks: a) an expectational IS curve, which establishes an inverse relation between the output gap and the real interest rate; b) a NKPC, which fixes a positive relation between

\textsuperscript{4} The first best output is the one that would realize under fully flexible prices and perfect competition in all markets. By contrast, the second best output is the one that would prevail under flexible prices in the presence of real distortions such as monopolistic competition.
inflation and the output gap and in which there is a “cost-push” shock. The form of the NKPC considered by CGG is:

$$\pi_t = \beta E\pi_{t+1} + \lambda x_t + u_t$$  \hspace{1cm} (22)$$

where $u_t$ is a “cost-push” shock which is a random variable which follows an autoregressive process ($u_t = \rho u_{t-1} + \hat{u}_t$ with $u_t \geq 0$ and $\hat{u}_t$ which is a i.i.d. random variable with mean and variance respectively equal to zero and $\sigma_u^2$), and $x_t$ is the output gap.

This approach is able to create a trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap, but it seems to us unsatisfactory. The reason is that there is not a theoretical microfoundation which justifies the presence of the “cost-push” shock. In fact the term $u_t$ appears a foreign body inside equation (22). To make acceptable its presence it would be necessary to derive it by a microfoundation process.

Along the same line of reasoning we find other economists who have followed the CCG (1999) approach, but in more complex way. Among these authors there are Steinsson (2003), Smets and Wouters (2003), Clarida, Gali and Gertler (2001).

Steinsson (2003) analyses optimal monetary policy using a hybrid version of the NKPC, i.e. a formulation which includes not only a forward-looking term of inflation, but also a backward-looking term. This author introduces the “cost-push” shock considering a time-varying income tax and assuming that the elasticity of substitution among goods is stochastic. The author uses a standard stochastic general equilibrium model\(^5\). The households, which live infinitely, are represented by a continuum of measure 1. The utility function of the representative household/producer is:

$$E_t \sum_{s=t}^{\infty} \beta^s \left[ u\left(C_s^0;\xi_s\right) - u\left(y_s(z);\xi_s\right) \right]$$  \hspace{1cm} (23)$$

\(^5\) The literature of reference for this subject is Yun (1996), Obstfeld and Rogoff (1996), Rotemberg and Woodford (1997, 1999).
Where $\beta$ is a discount factor, $\xi$ is a vector of shocks to households’ tastes and production aptitude, $C_i$ is household $i$’s consumption of a composite consumption good, $y_i$ is the differentiated good that the household $i$ produces.

The flow budget constraint of the representative household takes this form:

$$P_tC_t + E_t\left[R_{t,t+1}B_{t+1}\right] \leq B_t + (1 - \tau_t) p_t(z) y_t(z) + T_t$$

(24)

where $P_t$ is the price level in the period $t$, $B_t$ is the nominal value of the household’s portfolio of financial assets held in the period $t$, $\tau_t$ is the time-varying income tax rate levied by government, $T_t$ is a lump sum transfer paid by the government, $R_{t,t+1}$ is the stochastic discount factor.

Following Calvo’s rule, Steinsson (2003) supposes that only a fraction of producers/households, $1 - \alpha$, can reset the price in any time. The other fraction of producers/households, $\alpha$, does not reoptimize the price, but can rise it accordingly the steady state inflation rate ($\bar{\pi}$). Moreover only a fraction of producers/households, $(1 - \omega)$, modifies the price following a forward-looking manner (i.e. forward-looking households). The remaining fraction of producers/households which changes the price, $\omega$, sets it using a backward-looking rule-of-thumb (i.e. backward-looking households).

According to these assumptions, the aggregate price level is equal to:

---

6 This composite consumption good has the familiar Dixit-Stiglitz form:

$$C_i = \int c_i'(z) \frac{\theta_i}{\bar{\theta}_i} \frac{d\theta_i}{\bar{\theta}_i-z}$$

where $c_i'(z)$ is the consumption of the good $z$ for the household $i$ at the time $t$. Because $\theta_i$ is stochastic, the markup over the marginal cost is constantly varying.

7 For each producer/household the probability of changing its price is independent on the time elapsed since the last change of it.

\[ P_t = \left[ \alpha (\pi P_{t-1})^{1-\theta} + (1-\alpha)(1-\omega)(p_f)^{1-\theta} + (1-\alpha)\omega (p_b)^{1-\theta} \right]^{1-\theta} \]  

(25)

where \( p_f \) and \( p_b \) represent the prices set by the forward-looking and backward-looking households respectively.

Log-linearizing the supply block, we find a hybrid version of the NKPC which reads as follows:

\[ \pi_t = \chi_f \beta E_t \pi_{t+1} + k_1 x_t + k_2 x_{t-1} + \chi_b \pi_{t-1} + \eta_t \]  

(26)

where \( x_t \) is the output gap, \( \eta_t \) is a distortion exogenous shock generated by variation in the income tax rate, \( \tau_t \), and variation in the elasticity of substitution among goods, \( \theta_t \), and \( \chi_f \), \( \chi_b \), \( k_1 \), \( k_2 \) are parameters.

Some analyses very close to the Steinsson’s model are done by Smets and Wouters (2003) and Clarida, Galì and Gertler (2001).

In particular Smets and Wouters (2003) consider three “cost-push” shocks (a shock to the markup in the goods market, a shock to the markup in the labour market and a shock to the risk premium on capital). Their model generates a NKPC which takes this form:

\[ \hat{\pi}_t = \frac{\beta}{1+\beta \gamma_p} E_t \hat{\pi}_{t+1} + \frac{1}{1+\beta \gamma_p} \left[ (1-\beta \gamma_p)x_t \left( 1-\frac{1}{\beta \gamma_p} \right) \right] \left[ \alpha \hat{p}_{t+1}^k + (1-\alpha)\hat{w}_t - \epsilon_t + \eta_t \right] + \frac{\gamma_p}{1+\beta \gamma_p} \hat{\pi}_{t-1} \]  

(27)

where the variables with hat indicate the log deviation from their steady state values, \( \beta \) is the discount factor, \( \epsilon_t \) is a productivity shock, \( \gamma_p \) is the degree of price indexation, \( w_t \) is the real wage, \( r_t^k \) is the rental price of capital service, and \( \alpha \) is the parameter of the Cobb-Douglas production function of the intermediate good’s...

---

9 Steinsson shows that equation (26), assuming four values for \( \omega \), gives values of the parameters which are compatible with values found in different empirical researches [Gali and Gertler (1999), Fuhrer and Moore(1995)].
producers, \((1 - \xi_p)\) is the constant probability that in any period a producer can change his price, \(\eta^p\) is the “cost-push” shock to the NKPC\(^{10}\).

According to (27), current inflation is related to expected future inflation, past inflation and marginal cost \([\alpha \hat{\pi}^k + (1 - \alpha) \hat{\pi}^u - \hat{\pi} + \eta^p]\) which depends on the rental price of capital service, the real wage, a productivity shock and a “cost-push” shock.

Clarida, Gali and Gertler (2001) add to the standard NKPC a “cost-push” shock \((\eta^u)\). This “cost-push” shock is related to the wage markup \((\omega^u)\). This ad hoc “cost-push” shock gives account for factors which influence the marginal cost and do not change in proportional way with the output gap.

Altogether the “cost-push” shock’s approach seems unsatisfactory to solve the “divine coincidence” problem. In particular Blanchard and Gali (2007) affirm that this kind of shock introduces a trade-off between inflation and the welfare-relevant output gap. But it does not eliminate the “divine coincidence” problem with respect to the supply shocks such as change in the oil price or technological shocks.

### 4.2. Alternative structures of wage and price fixing

Erceg, Henderson and Levin (2000, HEL henceforth) introduce an optimizing-agent dynamic general equilibrium model in which nominal rigidities are originated from both staggered price and wage decisions\(^{11}\). The result of this approach is the apparent elimination of the “divine coincidence”.

The structure of model involves, on production side, that firm competes monopolistically and choices the price following Calvo’s rule. On the other side household, which has monopolistically power, sets staggered nominal wage.

---

\(^{10}\) In particular \(\eta^p\) is a shock which hits a stochastic parameter determining the time-varying markup in the goods market.

\(^{11}\) This innovative approach stands out to other works which analyse the relation between inflation and the output gap involving only staggered price decisions. Among these works there are Goodfriend and King (1997), King and Wolman (1999), Ireland (1997), Rotemberg and Woodford (1997, 1999).
Firms
In detail each firm produces a differentiated good competing monopolistically. The aggregate demand for each differentiated good reads as follows:

\[ Y_f(t) = \left[ \frac{P_t(f)}{P_t} \right]^{1+\theta_p} Y_t \]  \hspace{1cm} (28)

where \( Y_f(t) \) is the aggregate demand for good \( f \), \( P_t(f) \) is the price of the good \( f \), \( P_t \) is the aggregate price index, \( Y_t \) is the aggregate output index and \( \theta_p > 0 \).

Given an identical Cobb-Douglas production function for each firm, the marginal cost is given by:

\[ MC_t = \frac{W_t L_t^\alpha}{(1-\alpha)\bar{K} X_t} = \frac{W_t}{MPL_t} \]  \hspace{1cm} (29)

where \( L \) is labour, \( \bar{K} \) is fixed capital stock, \( X_t \) is total factor productivity, \( W \) is wage index, \( MPL \) is marginal product of labour.

Following Calvo’s rule, in any period only a fraction of firms can reset the price optimally. This fraction is \( 1-\xi_p \). The firm chooses \( P_t(f) \) to maximize this profit function:

\[ E_t \sum_{j=0}^\infty \xi_p^j \psi \left( (1+\tau_p) \Pi^j P_t(f) Y_{t+j}(f) - MC_{t+j} Y_{t+j}(f) \right) \]  \hspace{1cm} (30)

---

12 The aggregate price index is:

\[ P_t = \left[ \int P_t(f)^{\frac{1}{\theta_p}} df \right]^{-\theta_p} \]

13 The aggregate output index is:

\[ Y_t = \left[ \int Y_t(f)^{\frac{1}{\theta_p}} df \right]^{-\theta_p} \]
where $\tau_p$ is fixed subsidy rate of firm’s output, $\psi_{i+1}$ is a discount factor and $\Pi$ is unconditional mean rate of gross inflation.

The maximization of equation (30) gives this result:

$$E_i \sum_{j=0}^{\infty} \xi_j \psi_{i+1} \left( \frac{1+\tau_p}{1+\theta_p} \right)^j \Pi^j P_i (f) - MC_{i+1} Y_{i+1} (f) = 0 \quad (31)$$

The price fixed by the firm makes equal the expected value of discounted real marginal revenue and the expected value of discounted real marginal cost.

**Households**

Each household supplies a differentiated labour service to the firms. All households compete monopolistically in the labour market.

The aggregate demand for the household $h$’s labour, $N_i (h)$, is given by:

$$N_i (h) = \left[ \frac{W_i (h)}{W_i} \right]^{\sigma} L_i \quad (32)$$

where $L_i$ is the aggregate labour index$^{14}$, $W_i$ is the aggregate wage index, $W_i (h)$ is household $h$’s wage rate and $\theta_w > 0$.

The utility function of household $h$ reads as follows:

$$E_i \sum_{j=0}^{\infty} \beta^j \left( U \left( C_{i+1} (h), Q_{i+1} \right) + V \left( N_{i+1} (h), Z_{i+1} \right) + \frac{\mu_0}{1-\mu} \left( \frac{M_{i+1} (h)}{P_{i+1}} \right)^{1-\mu} \right) \quad (33)$$

$$U \left( C_i (h), Q_i \right) = \frac{1}{1-\sigma} \left( C_i (h) - Q_i \right)^{1-\sigma}$$

$^{14}$ The labour index is $L_i = \left[ \int N_i (h)^{1+\theta_w} dh \right]^{1+\theta_w}$; the wage index is $W_i = \left[ \int W_i (h)^{\frac{1}{\theta_w}} dh \right]^{-\theta_w}$.
\[ V(N_r(h), Z_r) = \frac{1}{1-\chi} (1-N_r(h) - Z_r)^{\chi} \]

where \( \beta \) is a discount factor with \( 0 < \beta < 1 \), \( C(h) \) is household \( h \)’s consumption, \( Q \) is consumption shock, \( Z \) is leisure shock, \( M \) is nominal money balances.

Following Calvo’s rule, households reset nominal wages in staggered way. In any period only a fraction of households, \( (1-\xi_w) \), resets the nominal wages. When a household \( h \) can reset its nominal wage, it sets \( W_r(h) \) to maximize (33). The result of this maximization problem is:

\[
E \sum_{j=0}^{\infty} \beta^j \xi_w \left( \frac{1 + \tau_w}{1 + \theta_w} \right) \left( W_r(h) \Pi^w(h) \frac{U_{C,j,t+j}}{P_{t+j}} + V_{N(h),t+j} \right) N_{t+j}(h) = 0 \tag{34}
\]

where \( \tau_w \) is fixed subsidy rate of labour income. The nominal wage, fixed by the household, makes equal the expected value of discounted marginal utility of income and the expected value of discounted marginal disutility.

In this model the NKPC takes this form:

\[
\pi_t = \beta E_{t+1} \pi_{t+1} + k_p (\zeta_t - mpl_t) \tag{35}
\]

\[
k_p = \frac{(1-\zeta_p \beta)(1-\zeta_p)}{\xi_p}
\]

where \( \zeta_t \) is real wage rate, \( mpl_t \) is log deviation between marginal product of labour and its value of steady state, \( \pi_t \) is inflation rate.

In EHL (2000) current price inflation is function of expected future price inflation and the difference between real wage rate and marginal productivity rate of labour. In this model, because wage and price are set in staggered way, there is a trade-off among the stabilization of the output gap, price inflation and wage
inflation. Besides HEL (2000) show that it is possible only for one variable among the output gap, price inflation and wage inflation to have zero variance.

Using Blanchard and Gali terminology (presented in subsection 4.4), we explain that EHL approach does not eliminate the “divine coincidence” at all\(^{15}\).

EHL show that, if wages and prices are staggered following Calvo’s rule, a trade-off between the stabilization of price inflation and the stabilization of the output gap arises. In EHL model wage inflation is:

\[
\pi^w = \beta E\pi^w (1) - \lambda_w \left( w - mrs - \mu^w \right)
\]

\[
\pi^w = \beta E\pi^w (1) - \lambda_w \left( w - \alpha m - (1 - \alpha + \phi) n - \xi - \mu^w \right)
\]

\[
\pi^w = \beta E\pi^w (1) - \lambda_w \left( w - w_2 \right) + \lambda_w \left( 1 + \frac{\phi}{1 - \alpha} \right) (y - y_2)
\]  

(36)

where \( w \) is current level of real wage, \( w_2 \) is the second best level of real wage, \( y \) is current level of output, \( y_2 \) is the second best level of output\(^{16}\), \( \mu^w \) is constant desired wage markup, \( \lambda_w \) is a coefficient, \( m \) represents oil, \( \alpha \) is the share of oil used in production, \( \phi \) is the slope of the labour supply curve, \( n \) represents labour, \( mrs \) is the household’s marginal rate of substitution, \( \xi \) is a preference parameter.

Price inflation is described by this equation:

\[
\pi^p = \beta E\pi^p (1) + \lambda_p \left( mc + \mu^p \right)
\]

\[
\pi^p = \beta E\pi^p (1) + \lambda_p \left( w - \alpha m + \alpha n - \log(1 - \alpha) + \mu^p \right)
\]

\[
\pi^p = \beta E\pi^p (1) + \lambda_p \left( w - w_2 \right) + \lambda_p \frac{\alpha}{1 - \alpha} (y - y_2)
\]  

(37)

where \( \lambda_p \) is a coefficient and \( \mu^p \) is constant desired price markup.

\(^{15}\) The following reasoning is developed by Blanchard and Gali (2007).

\(^{16}\) In this context \( y_2 \) is the output that would be realized, if there was perfectly flexible prices and wages but market distortions (i.e. the second best output). By contrast \( y_1 \) is the output that would be produced in the presence of perfectly flexible prices and wages and perfect competition in all markets (i.e. the first best output).
Wage inflation and price inflation are influenced not only by the output gap, but also by the difference between current real wage and the second best level of real wage. Consequence of this fact is that the “divine coincidence” disappears, either in price inflation or in wage inflation.

However the solution proposed by EHL (2000) does not eliminate the “divine coincidence” at all. In fact, if we consider a composite inflation rate \( \pi = \left( \lambda_w \pi^w + \lambda_p \pi^p \right) / \left( \lambda_w + \lambda_p \right), \) it is possible to represent the NKPC in this way:

\[
\pi = \beta E \pi(1) + k(y - y_2) \quad (38)
\]

with \( k \equiv \frac{\lambda_w \lambda_p (1 + \phi)}{(\lambda_w + \lambda_p)(1 - \alpha)} \)

But the difference between the first best level of output and the second best level of output is constant:

\[
y_1 - y_2 = \frac{\mu(1 - \alpha)}{1 + \phi} = \delta \quad (39)
\]

where \( \mu \equiv \mu^w + \mu^p \) and \( \delta \) is a constant.

Therefore the “divine coincidence” is still present. In fact we obtain:

\[
\pi = \beta E \pi(1) + k(y - y_1 + \delta) \quad (40)
\]

where \( y_1 \) is the first best level of output.

Equation (40) shows that the stabilization of a weighted average of wage and price inflation is compatible with the stabilization of the welfare-relevant output gap.

**4.3. Cost channel**

Ravenna and Walsh (2006), RW henceforth) solve the “divine coincidence” problem introducing in the standard New Keynesian model the concept of the cost
channel. In the presence of cost channel the marginal cost of the firms is directly influenced by nominal interest rate. The cost channel fades the scenario suggested by the standard New Keynesian model: in fact in the cost channel’s situation a trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap arises.

We present the elements of the RW’s model.

**Households**

Each household aims to maximize the expected value of its utility:

\[
E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{\xi_j C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right] \tag{41}
\]

where \( \beta \) is a discount factor, \( C_t \) is a composite consumption good, \( \xi_j \) is a taste shock, \( N_t \) is labour.

The composite consumption good is made up by differentiated goods. These goods are produced in a monopolistically competitive final market. This market is populated by a continuum of firms which has measure 1. We can formalize the composite consumption good in this way:

\[
C_t = \left[ \int c_{j,t}^{(\theta-1)/\theta} df \right]^{-\theta/(\theta-1)} \tag{42}
\]

where \( c_{j,t} \) is the amount of consumption of good \( j \), which is produced by the firm \( j \), and \( \theta > 1 \).

The demand for good \( j \) on the part of household is:

\[
c_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} C_t \tag{43}
\]

where \( p_{j,t} \) is the good \( j \)’s price and \( P_t \) is the aggregate price index given by:
The dynamic budget constraint of household is:

\[ M_{t+1} = M_t + W_t N_t - D_t - P_t C_t + R_t D_t + \Pi_t - T_t \]  \hspace{1cm} (45)

where \( M_t \) is money, \( W_t N_t \) is wage income, \( R_t \) is the gross nominal interest rate, \( D_t \) represents deposits, \( P_t C_t \) is purchase of goods, \( \Pi_t \) is aggregate profits from firms and intermediaries, \( T_t \) represents taxes.

In household’s equilibrium, characterized by a positive nominal interest rate, these first order conditions must be respected:

\[ \xi_t C_t^{-\sigma} = \beta E_t \left( \frac{R_t P_t}{P_{t+1}} \right) \xi_{t+1} C_{t+1}^{-\sigma} \]  \hspace{1cm} (46)

\[ \frac{X^\gamma \eta}{\xi_t C_t^{-\sigma}} = \frac{W_t}{P_t} \]  \hspace{1cm} (47)

\[ P_t C_t = M_t + W_t N_t - D_t \]  \hspace{1cm} (48)

To obtain goods market equilibrium, we have to have \( Y_t = C_t + G_t \), where \( G_t \) represents government consumption. We suppose that government consumption is proportional to output (i.e. \( G_t = (1 - \gamma_t) Y_t \) where \( \gamma_t \) has stochastic nature and is constrained between zero and one).

Then the aggregate resource constraint reads as follows:

\[ C_t = \gamma_t Y_t \]  \hspace{1cm} (49)
**Firms**

Each firm competes monopolistically, following Calvo’s rule. In any time only a fraction of firms can reset the price. This fraction is equal to $1 - \omega$. The firms which can not reset their prices restrict themselves to revise the previous prices updating them considering the steady state inflation rate.

All firms have the same real marginal cost, which is given by:

\[ \varphi_i \equiv \frac{R_i w_i}{A_i} = R_i S_i \tag{50} \]

where $w_i$ is real wage, $A_i$ is marginal product of labour, $S_i$ is labour’s share of output.

**The flexible-price equilibrium (second best)**

In the second best equilibrium each firm makes equal the real wage plus interest costs and the ratio between the marginal product of labour and the markup:

\[ R_i w_i^f = \frac{A_i}{\Phi} \tag{51} \]

where $\Phi$ is the constant markup.

Each household makes equal the real wage and the marginal rate of substitution between consumption and leisure:

\[ \frac{\chi N_i^p}{\xi C_i^{\sigma}} = w_i^f \tag{52} \]

Considering an aggregate production function, represented by $Y_t = A_t N_t$, and the resource constraint (49), when the labour market is in equilibrium, the output equilibrium value in a flexible-price context is:

\[ ^{17} \text{We distinguish the variables concerning flexible-price equilibrium situation by a superscript } f. \]
By equation (53) we can calculate the steady state value of flexible-price output:

\[
\bar{Y} = \left[ \frac{\xi_0 \gamma_{-\sigma} A_t^{1+\eta}}{\chi \Phi R_t^{-1}} \right]^{-1/(\sigma+\eta)}
\]

(54)

The output of flexible-price (second best) equilibrium, measured in log deviation from its steady state value, is given by:

\[
\hat{\gamma}_t' = \left( \frac{1}{\sigma+\eta} \right) \left( (1+\eta) \hat{A}_t - \sigma \hat{\gamma}_t + \hat{\xi}_t - \hat{R}_t' \right)
\]

(55)

The equation (55) shows that the second best output is influenced by productivity shock \( A_t \), fiscal shocks \( \gamma_t \), taste shocks \( \xi_t \) and the nominal interest rate\(^{18}\). The existence of cost channel determines that the flexible-price output is influenced by nominal interest rate. When nominal interest rate increases, labour demand decreases and flexible–price output equilibrium value reduces.

**Sticky-price equilibrium**

If \( \omega > 0 \), then there is the sticky-price case. In this scenario real marginal cost is influenced by nominal interest rate coherently with (50):

\[
\hat{\varphi}_t = \hat{R}_t + \hat{s}_t
\]

(56)

\(^{18}\)The hat expresses a log deviation of a variable with respect to its steady state value.
where \( \hat{s}_t \) is the log deviation of labour’s share of output around its steady state value (\( \hat{s}_t = \hat{w}_t + \hat{n}_t - \hat{y}_t \)).

Finally we obtain the NKPC. It has this form:

\[
\pi_t = \beta E_t \pi_{t+1} + k \left( \hat{R}_t + \hat{s}_t \right) \tag{57}
\]

where \( k \) is a parameter given by \( \frac{(1-\omega)(1-\omega\beta)}{\omega} \).

Inflation at the time \( t \) is function of a) expected future inflation, b) nominal interest rate plus labour’s share of output, i.e. real marginal cost.

The gap between sticky-price and flexible-price output levels is represented by:

\[
\hat{Y}_t - \hat{Y}'_t = E_t \left[ \hat{Y}_{t+1} - \hat{Y}'_{t+1} \right] - \left( \frac{1}{\sigma} \right) \left[ \left( \hat{R}_t - E_t \pi_{t+1} \right) - \hat{r}'_t \right] \tag{58}
\]

where \( \hat{r}'_t \) is flexible-price real interest rate.

It is possible to reformulate the NKPC as:

\[
\pi_t = \beta E_t \pi_{t+1} + k \left( \sigma + \eta \right) \left( \hat{Y}_t - \hat{Y}'_t \right) + k \left( \hat{R}_t - \hat{R}'_t \right) \tag{59}
\]

According to equation (59) current inflation is function of expected future inflation, the current gap between sticky-price and flexible-price output levels and the current gap between sticky-price and flexible-price nominal interest rates. With respect to the standard NKPC, the RW’s NKPC includes nominal interest rate among the determinants of inflation.

RW estimate their version of the NKPC for the U.S. economy. They use quarterly data from 1960:1 until 2001:1. The estimates are done by GMM. They find empirical evidence of the nominal interest rate’s influence on inflation behaviour.
Optimal monetary policy

The present discounted value of household’s utility is approximately given by:

$$\sum_{t=0}^{\infty} \beta^t U_t = \bar{U} - \Omega \sum_{t=0}^{\infty} \beta^t L_t$$ (60)

with

$$L_t = \pi_t^2 + \lambda \left( \hat{Y}_t - \hat{Y}_t' - z^* \right)^2$$ (61)

$$\hat{Y}_t' = \frac{(1+\eta) \hat{A}_t + \hat{\xi}_t + (1-\sigma) \hat{\gamma}_t}{\sigma + \eta}$$ (62)

where $z^*$ is the gap between flexible-price steady state output and the first best steady state output, $\hat{Y}_t'$ is the first best level of output taken in log deviation around its steady state value.

RW show that the relation between the welfare-relevant output gap and the output gap is given by:

$$\hat{Y}_t, \hat{Y}_t' - z^* = \left( \hat{Y}_t - \hat{Y}_t' \right) - \left( \frac{1}{\sigma + \eta} \right) \left( \hat{R}_t' + \hat{\gamma}_t \right) - z^*$$ (63)

The welfare-relevant output gap is influenced by a) the out gap (as in the standard New Keynesian model), b) flexible-price nominal interest rate, c) fiscal shocks.$^{19}$

---

$^{19}$ We assume that $z^* = \frac{\bar{\gamma} \Phi \bar{R} - 1}{\bar{\gamma} \Phi \bar{R} (\sigma + \eta)}$, which gives account for efficiency distortions, is equal to zero. The upper bar expresses a steady state value.
With \( z^* = 0 \) and in the presence of the policy rule given by \( \hat{R}' = 0 \), the equilibrium level of flexible-price output, the welfare-relevant output gap and real marginal cost are described by (64), (65) and (66) respectively:

\[
\hat{Y}_t = \frac{(1+\eta)\hat{A}_t - \sigma \hat{\gamma}_t + \hat{\xi}_t}{\sigma + \eta} \quad (64)
\]

\[
\hat{Y}_t - \hat{Y}_t^c = \hat{Y}_t - \hat{Y}_t^* - \left( \frac{1}{\sigma + \eta} \right) \hat{\gamma}_t \quad (65)
\]

\[
\hat{\varphi}_t = (\sigma + \eta) \left( \hat{Y}_t - \hat{Y}_t^* \right) + \hat{R}_t \quad (66)
\]

The monetary policy problem takes this form:

\[
\max -\frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \lambda \left[ x_t - \left( \frac{1}{\sigma + \eta} \right) \hat{\gamma}_t \right]^2 \right\} \quad (67)
\]

subject to

\[
x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) \left( \hat{R}_t - E_t \pi_{t+1} \right) + u_t \quad (68)
\]

and

\[
\pi_t = \beta E_t \pi_{t+1} + k (\sigma + \eta) x_t + k \hat{R}_t \quad (69)
\]

where \( x_t = \hat{Y}_t - \hat{Y}_t^* \), \( u_t \) is an exogenous demand shock, which is influenced by productivity shocks, taste shocks and fiscal shocks and

\[
\lambda = \left[ (1 - \omega)(1 - \omega \beta) / \omega \right] \left[ (\sigma + \eta) / \theta \right].
\]

The solution of the monetary policy problem under discretion takes this form:
\[ \pi_t = -\left( \frac{\lambda}{kn} \right)x_t - \left( \frac{1}{\sigma + \eta} \right)^\gamma_t \]  

(70)

The equation (70) suggests that, in the presence of cost channel, there is a greater inflation volatility for a given output gap volatility\(^{20}\). The consequence is that stabilizing inflation requires a higher cost in term of output.

In the presence of cost channel the policy maker can not use nominal interest rate to neutralize productivity or taste shocks’ effects on the output gap without generating volatility of inflation. The cost channel creates a trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap.

4.4. Real wage rigidities

This approach is proposed by Blanchard and Galì (2007, BG henceforth). In the BG’s model the innovation consists on the introduction of the friction represented by real wage rigidities.

The BG’s framework reproduces the standard New Keynesian model, but there is an important difference. In the production function is included a nonproduced input, which is offered exogenously. This fact permits us to consider shocks on this nonproduced input as supply shocks. In this framework technological shocks are not expressly included, but we consider shocks on nonproduced input equivalent to technological shocks. This procedure gives us the advantage that supply shock is directly measurable.

**Firms**

We assume that there is a continuum of monopolistically competitive firms. Each firm has an iselastic demand and produces a differentiated product.

The production function of the representative firm is:

\[ Y = M^\alpha N^{1-\alpha} \]  

(71)

\(^{20}\) In fact in the standard New Keynesian model the solution under discretion shows that the coefficient on \( x_t \) is equal to \( \left( \frac{\lambda}{k(\sigma + \eta)} \right) < \left( \frac{\lambda}{k\eta} \right) \).
where \( Y \) is output, \( M \) is nonproduced input (for example oil) and \( N \) is labour.

We describe with lower case natural logarithm of variables. Real marginal cost, expressed in natural logarithm, is:

\[
mc = w - mpn = w - (y - n) - \log(1 - \alpha)
\]

where \( w \) is natural logarithm of real wage which is exogenously given for firms.

\( \text{Households} \)

There is a large number of households. All households are identical. The utility function for representative household is:

\[
U(C, N) = \log(C) - \exp\{\xi\} \frac{N^{1+\phi}}{1 + \phi}
\]

\( N \) is level of employment, \( C \) is composite consumption, \( \xi \) is a preference parameter.

Given this utility function, the marginal rate of substitution is:

\[
mrs = c + \phi n + \xi
\]

\( \text{First best allocation} \)

If we assume perfect competition in all markets, for firms it results that:

\[
w = mpn = (y - n) + \log(1 - \alpha)
\]

and for consumers, it results that:

\[
w = mrs = y + \phi n + \xi
\]
In this equation we assume that \( c = y \), because we imagine that in each period all output is consumed.

Using expression (75) and (76), it is possible to obtain the first best allocation’s levels of employment and output. They result:

\[
(1 + \phi) n_1 = \log(1 - \alpha) - \xi \\
y_1 = \alpha m + (1 - \alpha) n_1
\]

\( (77) \)

\( (78) \)

**Second best allocation**

In this situation each firm has a monopoly power in the goods market. For this reason optimal price setting requires that \( mc + \mu^p = 0 \). \( \mu^p \) is the markup of price over cost. Combining this expression with (72), we obtain:

\[
w = y - n + \log(1 - \alpha) - \mu^p
\]

\( (79) \)

Now it is possible to obtain the second best allocation’s levels of employment and output. They result:

\[
(1 + \phi) n_2 = \log(1 - \alpha) - \mu^p - \xi \\
y_2 = \alpha m + (1 - \alpha) n_2
\]

\( (80) \)

\( (81) \)

The source of “divine coincidence” emerges clearly considering jointly the first best output and the second best output:

\[
y_1 - y_2 = \frac{\mu^p (1 - \alpha)}{1 + \phi} \equiv \delta
\]

\( (82) \)

where \( \delta \) is a constant.
Even though the levels of the first best and the second best output change over time, the gap between them results always constant.

Real wage rigidities

We assume that prices are staggered in accordance with Calvo’s rule. Following this rule, the equation which describes inflation around zero-inflation steady state is:

\[
\pi = \beta E\pi(+1) + \lambda (mc + \mu^p)
\]  

(83)

where \(mc + \mu^p\) represents log deviation of real marginal cost from its value in a zero-inflation steady state, \(\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\) and \(\theta\) is the fraction of firms which does not adjust the price in any period.

Using Calvo’s rule, the New Keynesian Phillips curve takes this form:

\[
\pi = \beta E\pi(+1) + k(y - y_2)
\]  

(84)

with \(k = \frac{\lambda (1+\phi)}{1-\alpha}\)

This traditional formulation of the NKPC has a lack, which Blanchard and Gali (2007) call “divine coincidence”. In fact in the presence of a supply shock (for example a change in oil price) there is not a trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap. The stabilization of inflation is consistent with the stabilization of the welfare-relevant output gap.

We consider a rule of wages adjustment, which involves real rigidities:

\[
w = \gamma w(-1) + (1 - \gamma) mrs
\]  

(85)

where \(\gamma\) is a parameter of real wage rigidities.
Considering this assumption, the first best output and the first best employment remain unchanged, but now the second best output and the second best employment change.

In particular in the new situation, it results that:

\[
w = \gamma w(-1) + (1 - \gamma)\left( y + \phi m + \xi \right)
\]

\[
w = \gamma w(-1) + (1 - \gamma)\left[ \alpha (m - n) + (1 + \phi) n + \xi \right]
\]

\[
w = m p n - \mu^p
\]

\[
w = \alpha (m - n) + \log (1 - \alpha) - \mu^p
\]

\[
[y_2 - y_1 + \delta] = \Theta[y_2(-1) - y_1(-1) + \delta] + \Theta (1 - \alpha) \left[ \Delta m + \left( \frac{1}{1 + \phi} \right) \Delta \xi \right]
\]

where \( \Theta = \frac{\gamma \alpha}{\gamma \alpha + (1 - \gamma)(1 + \phi)} \in [0,1] \)

The introduction of real wage rigidities breaks down the constant relation between the first and the second best output. The consequence is that, if a supply shock happens, there is a trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap (i.e. the difference between current output and the first best output). In this case the stabilization of inflation is still compatible with the simultaneous stabilization of the output gap (i.e. the distance between current output and the second best output) but, because the difference between the first best output and the second best output is no longer constant, it does not guarantee the simultaneous stabilization of the welfare-relevant output gap.

Using Calvo’s rule and by some rearrangement, we find:

\[
(mc + \mu^p) = \gamma (mc + \mu^p)(-1) + x_z
\]
where
\[ x_2 = \frac{(1-\gamma)(1+\phi)(y-y_2) + \gamma \alpha (\Delta y - \Delta y_2)}{1-\alpha} \]

Combining (89) with (83), we obtain the relation between inflation and the output gap described by Blanchard and Gali’s model:

\[ \pi = \beta \bar{E} \pi(1) + \frac{\lambda}{1-\gamma L} x_2 \]  \hspace{1cm} (90)

This relation implies still the absence of trade-off between the stabilization of inflation and the stabilization of the output gap. According to equation (90), if the output gap remains constant, inflation results constant. But in this situation the “divine coincidence” disappears. The reason is that now the gap between the first best output and the second best output is no longer constant. The relation between inflation and the welfare-relevant output gap is:

\[ \pi = \beta \bar{E} \pi(1) + \frac{\lambda}{1-\gamma L} x_1 - \frac{\lambda \gamma \alpha}{1-\gamma L} \left[ \Delta m + \left( \frac{1}{1+\phi} \right) \Delta \xi \right] \]  \hspace{1cm} (91)

where
\[ x_1 \equiv \frac{(1-\gamma)(1+\phi)(y-y_1 + \delta) + \gamma \alpha (\Delta y - \Delta y_1)}{1-\alpha} \]

In this situation it is impossible to stabilize simultaneously inflation and the welfare-relevant output gap in response to supply shocks or preference shocks: a trade-off arises for policy maker.

Let \( v \) be the real price of nonproduced input (for instance oil). It is possible to rewrite inflation in function of \( \Delta v \) and the welfare-relevant output gap:

\[ \pi = \beta \bar{E} \pi(1) + \frac{\lambda}{1-\Gamma L} \left[ (1-\Gamma)(1+\phi)(y-y_1 + \delta) + \Gamma \alpha \Delta v \right] \]  \hspace{1cm} (92)

where \( \Gamma \equiv \frac{\gamma}{1-\alpha(1-\gamma)} \in [0,1] \)
If a supply shock causes an increase of oil price, to keep constant inflation, it is necessary to accept a decrease in current output with respect to the first best output.

Relation between inflation and unemployment

In the first place we define $n_s$ as quantity of labour which households desire to offer at the level of current wage and given marginal utility of income. We obtain:

$$w = y + \phi n_s + \xi$$  \hspace{1cm} (93)

In the second place we define involuntary rate of unemployment, $u$, as log deviation between the desired supply of labour and current employment:

$$u = n_s - n$$  \hspace{1cm} (94)

when $\gamma > 0$, it results that:

$$\Delta w = \frac{(1-\gamma)\phi u}{\gamma}$$  \hspace{1cm} (95)

In the presence of real wage rigidities there is a negative relation between unemployment rate and real wage.

Finally we can represent the BG’s NKPC as follows:

$$\pi = \frac{\beta}{1+\beta} E\pi(-1) - \frac{\lambda(1-\alpha)(1-\gamma)\phi u}{\gamma(1+\beta)} + \frac{1}{1+\beta} \pi(-1) + \frac{\alpha\lambda}{1+\beta} \Delta v + \zeta$$  \hspace{1cm} (96)

Inflation is expressed in term of expected future inflation, unemployment rate, past inflation and percent change in the real price of nonproduced input. Table 1 synthesizes the qualitative results of the different approaches to the NKPC discussed in this chapter.
<table>
<thead>
<tr>
<th>Model</th>
<th>Drivers of inflation</th>
<th>Source of rigidities</th>
<th>Is “cost-push” shock endogenous?</th>
<th>Our judgement about effectiveness of the “divine coincidence” solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor (1979, 1980)</td>
<td>a)E.F.I.; b)A moving average of the U.R.</td>
<td>Nominal wage</td>
<td>“Cost-push” shock is not present</td>
<td>“Divine coincidence” is not solved</td>
</tr>
<tr>
<td>Rotemberg (1982, 1982a)</td>
<td>a)E.F.I b)The O.G.</td>
<td>Nominal price</td>
<td>“Cost-push” shock is not present</td>
<td>“Divine coincidence” is not solved</td>
</tr>
<tr>
<td>Calvo (1983)</td>
<td>a)E.F.I.; b)Real marginal cost</td>
<td>Nominal price</td>
<td>“Cost-push” shock is not present</td>
<td>“Divine coincidence” is not solved</td>
</tr>
<tr>
<td>Clarida, Gali, Gertler (1999)</td>
<td>a)E.F.I.; b)The O.G.; c)Exogenous “cost-push” shock</td>
<td>Nominal price</td>
<td>No</td>
<td>*</td>
</tr>
<tr>
<td>Smets, Wouters (2003)</td>
<td>a)E.F.I.; b)Rental price of capital service; c)Real wage; d)Productivity shock; e)”Cost-push” shock; f)P.I.</td>
<td>a)Nominal price; b)Nominal wage</td>
<td>No</td>
<td>**</td>
</tr>
<tr>
<td>Erceg, Henderson, Levin (2000)</td>
<td>a)E.F.I.; b)Real wage rate; c)log deviation between marginal product of labour and its value of steady state</td>
<td>a)Nominal price; b)Nominal wage</td>
<td>No</td>
<td>***</td>
</tr>
<tr>
<td>Blanchard, Galì (2007)</td>
<td>a)E.F.I.; b)U.R.; c)P.I.; d)Percent change in the real price of nonproduced input</td>
<td>Real wage</td>
<td>Yes</td>
<td>****</td>
</tr>
</tbody>
</table>

**Legend:** E.F.I: expected future inflation; P.I.: past inflation; The O.G.: the output gap; U.R.: unemployment rate; N.I.R.: nominal interest rate taken in log deviation around its value of steady state; L.S.O.: labour’s share of output taken in log deviation around its value of steady state. ****: very good; ***: good; **: quite good; *: sufficient.
Estimates

Blanchard and Gali (2007) estimate equation (96), considering the parameters in reduced form, for the U.S. economy. They consider annual data from 1960 until 2004. The estimates are done using instrumental variables. Inflation is measured by the percent change in the GDP deflator. Unemployment is measured by the civilian unemployment rate. The effect on inflation of the real price of nonproduced input is measured by the percent change in the PPI relative to the GDP deflator. The instrument set was composed by four lags of the previous three variables.

Blanchard and Gali (2007) find this result (standard errors in brackets):

\[ \pi = 0.48 E\pi (+1) - 0.08 u + 0.52 \pi (-1) + 0.014 \Delta v + \zeta \]

All coefficients have the expected sign and are statistically significant.

Boldrin (2008) estimates an extension of BG’s NKPC for open economy which reads as follows:

\[ \pi = \psi_1 E\pi (+1) + \psi_2 u + \psi_3 \pi (-1) + \psi_4 \Delta v + \psi_5 \Delta e + \zeta \]

(97)

where \( \Delta e \) is the percent change in the real effective exchange rate. The estimates refer to the U.K. and U.S. economies\(^{21}\). The results are summarized in the table 2:

<table>
<thead>
<tr>
<th>Country</th>
<th>( \psi_1 )</th>
<th>( \psi_2 )</th>
<th>( \psi_3 )</th>
<th>( \psi_4 )</th>
<th>( \psi_5 )</th>
<th>J-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>0.62( (0.02) )**</td>
<td>-0.21( (0.02) )**</td>
<td>0.56( (0.02) )**</td>
<td>0.26( (0.05) )**</td>
<td>0.02( (0.008) )**</td>
<td>5.86</td>
<td>0.88</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.57( (0.01) )**</td>
<td>-0.02( (0.009) )*</td>
<td>0.59( (0.006) )**</td>
<td>0.08( (0.005) )**</td>
<td>0.03( (0.001) )**</td>
<td>5.11</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Estimated curve: \( \pi = \psi_1 E\pi (+1) + \psi_2 u + \psi_3 \pi (-1) + \psi_4 \Delta v + \psi_5 \Delta e + \zeta \). All the coefficients are assumed to take a positive value except for \( \psi_2 \), which is assumed to be negative. Note: GMM point estimates reported in the Table (Newey-West robust, standard errors in brackets). **/* identify 1/5% significance level. Instruments: constant, four lags of \( \pi \), four lags of \( u \), four lags of \( \Delta v \), four lags of \( \Delta e \).

\(^{21}\) The period of the analysis is 1980-2004. The estimates are done by GMM using annual data.
These estimates show that the BG’s NKPC fit the data well. In particular all coefficients have the expected sign and are statistically significant.

5. Future developments and new challenges

In the recent years the economic literature involving the New Keynesian model in general and the NKPC in particular is growing in a massive way. In fact today it is possible to identify the New Keynesian model as the main tool to analyse the short-run fluctuations of the economy. The quick growth of the New Keynesian paradigm has been generated by the continuous progresses realized in the past two decades. Although these improvements, to reach a more satisfactory level of the reality’s comprehension, it is essential to tackle some problematic aspects which have not had still a response.

In particular this survey has analysed a specific problem afflicting the NKPC, known in the economic literature as “divine coincidence”, and the solutions proposed to it currently. The specifications of the NKPC, which are arisen by the solutions proposed, will have to be checked in the future analyses to understand their degree of validity. Nevertheless it seems obvious that the complexity of the problem, because of its links with a multiplicity of variables, will require further energies to develop the research along different straight.

1) The specifications of the NKPC, addressed to solve the “divine coincidence”, are hybrid in the sense that inflation is explained by both past inflation and expected future inflation. Many economists have tried to quantify the relative importance of these variables in influencing inflation dynamics. Currently there is not a large consensus about what is predominant between these two variables. It is possible to highlight two different positions in this subject. The first is represented by Galì and Gertler (1999) and Gali, Gertler and López-Salido (2005). According to these authors the expected forward-looking component is preponderant in explaining inflation. The second is expressed by Rudd and Whelan (2006, 2007). These authors doubt that existing empirical researches show a prevalence of expected future inflation in explaining inflation behaviour.

22 The points 3), 4) and 5) have been clearly expressed by Galì and Gertler (2007) and Gali (2008).
2) In the empirical analyses of the NKPC in general and with particular attention to the specifications used to solve the “divine coincidence”, a relevant problem is the choice of the variable which expresses the real marginal cost (which in general is proxied by the output gap). The failure of the New Keynesian Phillips curve in fitting the data when the excess of demand is represented by the log of the detrended real GDP induces many authors to criticize this variable because of its inadequacy as proxy to real marginal cost (i.e. the variable which theoretically drives inflation in the NKPC). For this reason Gali and Gertler (1999) and Sbordone (2002) propose to use labour’s share of income as proxy of real marginal cost. This choice is brought into question by Rudd and Whelan (2006, 2007). These economists elaborate an empirical estimate using labour’s share of income as proxy of real marginal cost, and find a weakness of the NKPC in fitting the data when it is estimated in this way.

3) Rigidities can be introduced in the New Keynesian model following two methods\textsuperscript{23}: i) time-dependent models; ii) state-dependent models\textsuperscript{24}. The solution more used is i). All the models analysed in this survey belong to the time-dependent approach. In this approach (for example the Calvo’s model) firms adjust their prices according to a fixed frequency. Differently in the state-dependent approach firms adjust their prices when certain variables (for examples costs) achieve some values. In this way the rules of price adjustment become endogenous. Recent analyses inspired by state-dependent approach are led by Dotsey, King and Wolman (1999), Gertler and Leahy (2006), Midrigan (2006) and Golosov and Lucas (2007).

4) The standard New Keynesian model and the implied NKPC suppose a neoclassical labour market in which the workers modify their hours worked but there is not involuntary unemployment. Many researchers try to introduce in the New Keynesian model some frictions of the labour market traditionally analysed in the matching and search literature. This

\textsuperscript{23} This classification is proposed by Ball, Mankiw and Romer (1988).

\textsuperscript{24} Probably the first formalization of the state-dependent approach is developed by Caplin and Spulber (1987).
procedure can improve the comprehension of the reality in order to a fundamental aspect as labour market’s dynamics. Some attempts along this line are proposed by Walsh (2005), Trigari (2005), Blanchard and Gali (2006) and Gertler, Sala and Trigari (2007).

5) A fundamental hypothesis of the New Keynesian model is that the financial markets are perfectly competitive markets. Today a growing number of economists tries to remove this assumption to create a model which considers the imperfections which characterize the real financial markets. This procedure offers the opportunity to inquiry more deeply the role played by the monetary policy on the short-run fluctuations. This kind of analysis can improve the comprehension of inflation dynamics involved by the NKPC. This line of research has been followed by Bernanke, Gertler and Gilchrist (1999), Gilchrist and Leahy (2002), Christiano, Motto and Rostagno (2006), Faia and Monacelli (2006), Monacelli (2006), Iacoviello (2006).

6. Conclusions

This chapter discusses a fundamental part of the New Keynesian model, the so-called NKPC. We analyse the theoretical bases of the NKPC and emphasize that the standard explanation of inflation behaviour in the New Keynesian framework has a shortcoming. This lack is called by Blanchard and Gali (2007) “divine coincidence”, i.e. the absence of a trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap, when a shock hits the economy.

We identify in the economic literature four solutions to this problem. The solutions which are proposed have in common the attempt of representing some form of “cost-push” shock to generate a trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap.

In particular two solutions among the others seem to us particularly efficacious in tackling the “divine coincidence” problem. The two theoretical analyses are proposed by Blanchard and Gali (2007) and Ravenna and Walsh (2006). Both are able to generate a “cost-push” shock endogenously. But the difference between them consists of the source which nourishes this “cost-push” shock. In the Blanchard and Gali’s work the source of the “cost-push” shock is
represented by real wage rigidities, in the Ravenna and Walsh’s model by the cost channel.
References


CHAPTER 2

“Assessing the Relevance of Oil and Exchange Rates in the New Keynesian Phillips Curve”

1. Introduction

In the modern macroeconomic literature the most popular framework for analysing the monetary policy and business cycle is certainly the New Keynesian model (Woodford 2003). An important ingredient of this model is the New Keynesian Phillips curve, which may be represented as follows: \( \pi_t = \beta E_t \pi_{t-1} + k(y_t - y^*_t) \). In this formulation \( \pi \) represents inflation, \( \beta \) represents firms’ discount factor, \( E_t \) identifies expectations taken at time \( t \) (i.e. with an information set updated up to \( t \)), \( y \) is a measure of current GDP taken in logs, \( y^*_t \) is the log of the natural level of output, and \( (y_t - y^*_t) \) is the “output gap”, i.e. the measure of economic slack identifying the inflationary pressures coming due to aggregate demand.

Many authors underline that the New Keynesian Phillips curve has a lack (e.g. Mankiw (2001)). This lack consists in the absence of a trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap (say, after a supply shock has hit the economy). The reason is that in this model the wedge between the first and the second best output level is constant, then when stabilizing output around the second best output level, monetary policy makers also stabilize the first best output gap.\(^1\)

Blanchard and Gali (2007) name this peculiarity of the standard New Keynesian model “divine coincidence”. They also offer a rationale for the “divine coincidence”: in the standard New Keynesian framework, no role is acknowledged to non trivial real imperfections. Interestingly, they show that once real wage rigidities are allowed to enter the model, the relationship between the first and the second best output levels is not constant anymore. This implies the disappearance of the “divine coincidence”, i.e. a meaningful inflation-output gap stabilization trade-off arises. Notably, the Blanchard and Gali New Keynesian Phillips curve (BGNKPC henceforth) also features the presence of the relative price of the

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\(^1\) The second best output level is the one that would prevail under flexible prices in the presence of real distortions such as monopolistic competition. By contrast, the first best output level is the one that would realize under fully flexible prices and perfect competition in all markets.
nonproduced input (e.g. oil) as a regressor, so explicitly accounting for one of the main sources of inflationary pressures in OECD countries.

This chapter estimates three different versions of the New Keynesian Phillips curve for the United Kingdom and the United States of America in the sample 1980-2004. Our empirical results point towards the likely misspecification of the standard New Keynesian Phillips curve a la Woodford (2003). In fact, when adding the producer price index (relative to the GDP deflator) and the growth rate of the real effective exchange rate, these two regressors turn out to be largely significant in both countries considered. Interestingly, the real effective exchange rate helps corroborating the BGNKPC by i) capturing an otherwise missed source of persistence (movements in the terms-of-trade) and ii) making unemployment regressor’s sign significant and coherent with economic expectation.

This paper develops as follows. Section 2 offers a brief discussion of the “divine coincidence” and presents the Blanchard-Gali proposal. Section 3 proposes the three different versions of the supply curve we take to the data. In Section 4 we present our empirical findings, we compare the different abilities of our competing models to fit the data and line up with an economist’s expectations in terms of significance and sign of the included regressors. Section 5 concludes.

2. NKPC: standard model and Blanchard-Gali proposal

In the economic literature different solutions have been proposed to solve the “divine coincidence”. A first approach is put forward by Clarida, Gali and Gertler (1999). They add a “cost-push shock” in the New Keynesian Phillips curve. In this way they create a trade-off between the stabilization of inflation and the stabilization of the output gap. However, the so-called “cost-push” shock is a non-micro-founded disturbance. Ideally, a model should endogenously generate a time-varying wedge between the first and the second best level of output. In an attempt to consider this criticism, Smets and Wouters (2003) and Clarida, Gali and Gertler (2001) model “distortionary shocks” by allowing for variations in the desired firms’ mark-up. Steinsson (2003) embeds variations in tax changes in an otherwise standard model. In both scenarios the second best output is influenced by distortionary shocks. Due to this reason, the relationship between the first and the second best levels of output is not constant anymore, then monetary authorities
actually face the inflation-output volatility trade-off. Once more, the solution relies on the manipulation of an exogenous autoregressive process, and it can not be considered as being theoretically satisfactory.

A different approach to eliminate the “divine coincidence” is the one proposed by Erceg, Henderson and Levin (2000). They show that, if both wages and prices are staggered following Calvo’s rule, a meaningful gap arises. In Erceg, Henderson and Levin’s model wage inflation and price inflation are influenced not only by the output gap, but also by difference between current wage and the second best level of wage. Consequence of this fact is that the “divine coincidence” disappears either in price inflation or in wage inflation. However, it is easy to show that in Erceg, Henderson and Levin’s model the stabilization of a weighted average of wage and price inflation is compatible with the stabilization of the welfare-relevant output gap. In fact, if we consider a composite inflation rate \( \pi = \frac{\pi^w + \lambda_p \pi^p}{(\lambda_w + \lambda_p)} \), it is possible to rewrite the New Keynesian Phillips curve in this way: \( \pi = \beta E\pi(+1) + k(y - y^*) \), where \( \pi^w \) is price inflation, \( \pi^w \) is wage inflation, \( k = \frac{\lambda_w \lambda_p (1+\phi)}{((\lambda_w + \lambda_p)(1-\alpha))} \). \( \alpha \) is share of the nonproduced input in production, \( \phi \) represents the slope of the labour supply, \( \lambda_w \) and \( \lambda_p \) are coefficients expressed in term of structural parameters. Then, the difference between the first best level of output and the second best level of output is still constant, i.e. \( y_1 - y_2 = \delta^2 \). Consequently the “divine coincidence” reappears.

A more theoretically appealing strategy to solve the problem of the “divine coincidence” is proposed by Blanchard and Galì (2007). This approach is based on the introduction of non trivial real imperfections in a New Keynesian model featuring a CRS production function displaying labour and a nonproduced input (a natural resource, e.g. oil). Following the lead by other authors (e.g. Hall (2005)), Blanchard and Galì (2007) model real wage rigidities in order to create a gap between desired labour supply and actual employment. Turning to the supply side, the production function of the representative firm, which operates in monopolistic competition, embeds an exogenously offered nonproduced input. They interpret shocks to this nonproduced input as supply shocks, i.e. technological shocks. This

\[ y_1 \] represents the output that would realize under fully flexible prices and wages with perfect competition in all markets; \( y_2 \) stands for the natural output, i.e. the output that would obtain under both flexible prices and wages in the presence of real distortion (for example monopolistic competition); \( \delta \) is a constant.
assumption is very convenient, because it gives the econometrician the advantage of directly measuring the supply shock.

In the first place we define \( n_s \) as quantity of labour which households desire to offer at the level of current wage and given marginal utility of income. In the second place we define involuntary rate of unemployment, \( u \), as log deviation between desired supply of labour and actual employment: \( u = n_s - n \). Blanchard and Gali (2007) show that when the index of real wage rigidities \( \gamma > 0 \), \( \Delta w = -\{(1-\gamma)\varphi/\gamma\}u \), where \( w \) stands for real wage and \( \varphi \) represents the slope of the labour supply. Notice that, in the presence of real wage rigidities, there is a negative relationship between unemployment rate (\( u \)) and real wage (\( w \)).

The Phillips curve as proposed by Blanchard and Gali (2007) reads as follows:

\[
\pi = \beta/(1+\beta)E\pi(+1) - \{[\lambda(1-\alpha)(1-\gamma)\varphi]/[\gamma(1+\beta)]\}u + [1/(1+\beta)]\pi(-1) + [\alpha\lambda/(1+\beta)]\Delta v + \zeta
\]

where \( \alpha \) is share of nonproduced input in production, \( \beta \) represents firm’s discount factor, \( \gamma \) stands for real wage rigidities, \( \theta \) represents the fraction of firms not changing the price in any period, and \( \lambda = 0^{-1}(1-0)(1-\beta0) \).

In this equation inflation is expressed in term of expected future inflation, \( E\pi(+1) \), unemployment rate, \( u \), past inflation, \( \pi(-1) \), and percent change in the real price of nonproduced input \( \Delta v \). Blanchard and Gali (2007) estimate a version of equation (1) with the parameters in reduced form for the U.S. economy sample 1960-2004 and find empirical support for the role of unemployment and relative producer prices in the inflation schedule.

3. Empirical investigation

The aim of this chapter is to understand if the BGNKPC finds empirical support and is somewhat superior with respect to the standard NKPC. To do so, we empirically investigate inflation dynamics in two OECD countries. The countries we concentrate on are the United Kingdom and the United States of America. The sample we consider spans the period 1980-2004. We consider three different formulations of the Phillips curve: 1) standard NKPC; 2) BGNKPC; 3) an “augmented” BGNKPC, extended in order to account for the role exerted by
real effective exchange rate fluctuations in shaping inflation. In fact, several recent contributions point towards an increasing importance of external pressures in explaining countries’ inflation (e.g. Rogoff (2007)). We present the three schedules below.

Standard New Keynesian Phillips curve (NKPC)

The standard New Keynesian Phillips curve we focus on features the following form (with the parameters in reduced form):

\[ \pi = \psi_1 E\pi(+1) + \psi_2 u + \zeta \]  \hspace{1cm} (2)

where – here and in the subsequent equations - all the parameters are assumed to take a positive value except for \( \psi_2 \), which is assumed to be negative. In order to increase the degree of comparability between the standard NKPC and the BGNKPC, we substitute the output gap with the unemployment rate. Notice that these two macroeconomic indicators are very correlated, as also postulated by Okun’s law\(^3\). Moreover, Roberts (1995, 2006) and Gordon (1997) show that the standard NKPC estimated with the unemployment rate as the proxy of the business cycle displays good empirical fit.

Blanchard and Gali New Keynesian Phillips curve (BGNKPC)

We also estimate the Blanchard and Gali New Keynesian Phillips curve with the parameters in reduced form, which takes this form:

\[ \pi = \psi_1 E\pi(+1) + \psi_2 u + \psi_3 \pi(-1) + \psi_4 \Delta v + \zeta \]  \hspace{1cm} (3)

With respect to equation (2), it is immediate to see that, given the presence of lagged inflation in equation (3), the latter is more suited for capturing the persistence typically displayed by the inflation rate. Moreover equation (3) offers us the possibility of assessing the role played by an indicator like the producer price index (considered in relative terms with respect to the GDP deflator, and

\(^3\) The correlation between unemployment rate and the output gap in the two countries under investigation reads as follows. U.K.: -0.50; U.S.: -0.77.
taken in growth-rates) in shaping a country’s inflation. This is one of the main insights coming from Blanchard and Gali (2007).

“Augmented” BGNKPC

This version of the BGNKPC is tested in order to assess the role played by foreign pressures (proxied by the percent change in the real effective exchange rate) in the determination of a country’s inflation. Admittedly, the real effective exchange rate is added in an ad hoc fashion. However, as discussed below, its presence is very important for capturing otherwise unmodeled inflationary pressures. This version of the New Keynesian Phillips curve (with the parameters in reduced form) reads as follows:

\[
\pi = \psi_1 E\pi(+1) + \psi_2 u + \psi_3 \pi(-1) + \psi_4 \Delta v + \psi_5 \Delta e + \zeta
\]  

(4)

where \(\Delta e\) is the percent change in the real effective exchange rate. Using this ad hoc formulation, we show that it is possible to increase the explanatory power of the Blanchard and Gali New Keynesian Phillips curve about inflation dynamics.

As already pointed out, equations (2)-(4) are inflation curves with the parameters in reduced form. The choice of these inflation schedules mimics the one by Blanchard and Gali (2007), and it is due to the identification problems affecting these models\(^4\).

4. Estimates of the three versions of the New Keynesian Phillips curve

Our objective is to estimate inflation dynamics for the United Kingdom and the United States of America, using inflation equations (2), (3), (4). We estimate these equations using annual data from 1980 until 2004. The source of these data is the OECD database. We estimate the models (2), (3), (4) using GMM.

\(^4\) In fact, we have to estimate five parameters, \(\psi_1\) and \(\psi_3\) give us information limited to the discount factor \(\beta\). The remaining regressors do not contain enough information for jointly identifying \(\alpha, \theta, \gamma, \phi\). A possible choice would be that of calibrating some of the structural parameters. For a discussion on the pitfalls related to “calibration-estimation” mixed strategies, see Canova and Sala (2006).
**Standard New Keynesian Phillips curve**

Table 1 summarizes our results. It is immediate to see that these estimates point towards model misspecification. Indeed, the estimated parameter for the unemployment rate is insignificant and takes the wrong sign. According to these estimates, the monetary policy transmission via demand channel would work neither in the U.K. nor in the U.S., a conclusion that goes against the conventional wisdom. Notice that, as highlighted by the J-test, this result does not appear to be driven by badly selected instruments, at least as far as the orthogonality condition is concerned. It is then interesting to move to alternative, empirically more successful models of inflation.

### Table 1. Standard New Keynesian Phillips Curve

<table>
<thead>
<tr>
<th>Country</th>
<th>$\psi_1$</th>
<th>$\psi_2$</th>
<th>J-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>0.64(0.16)**</td>
<td>0.07(0.08)</td>
<td>2.65</td>
<td>0.85</td>
</tr>
<tr>
<td>U.S.</td>
<td>1.50(0.45)**</td>
<td>0.02(0.14)</td>
<td>2.71</td>
<td>0.84</td>
</tr>
</tbody>
</table>

*Estimated curve: $\pi = \psi_1 \pi (+1) + \psi_2 u + \zeta$. Note: GMM point estimates reported in the Table (Newey-West robust, standard errors in brackets). **/* identify 1/5% significance level. Instruments: constant, four lags of $\pi$, four lags of $u$.*

**Blanchard and Gali New Keynesian Phillips curve**

Table 2 displays our estimates of the hybrid Phillips curve a la Blanchard-Gali (2007). Interestingly, the presence of oil prices is not rejected by the data, a finding supporting the novel Phillips curve formulation by Blanchard and Gali (2007). In general, our econometric exercise leads to a sensible description of the forces driving inflation in the U.K., with all the estimated parameters being significant and taking the correct sign. Differently, the model is not successful in tracking U.S. inflation, at least as regards the sensibility of the estimated parameters. As before, the estimated unemployment parameter is wrongly signed and insignificant. By contrast, a better picture is the one regarding the U.K., whose Phillips curve estimated parameters are all significant and with the correct sign. However, one may wonder if the U.K. GDP deflator inflation, which is computed also by considering imported inputs and intermediate goods employed for the production of domestic final goods, is affected by external inflationary pressures. This is the reason why we move to the third inflation model, i.e. a
version of the Blanchard and Gali Phillips curve embedding the real effective exchange rate.

### Table 2. Blanchard and Gali New Keynesian Phillips Curve

<table>
<thead>
<tr>
<th>Country</th>
<th>$\psi_1$</th>
<th>$\psi_2$</th>
<th>$\psi_3$</th>
<th>$\psi_4$</th>
<th>J-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>0.52</td>
<td>−0.13</td>
<td>0.59</td>
<td>0.29</td>
<td>5.00</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.07)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.25</td>
<td>0.05</td>
<td>0.73</td>
<td>0.20</td>
<td>5.33</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimated curve: $\pi = \psi_1 \pi (t+1) + \psi_2 u + \psi_3 \pi (t-1) + \psi_4 \Delta v + \zeta$.  

Note: GMM point estimates reported in the Table (Newey-West robust, standard errors in brackets). **/* identify 1/5% significance level. Instruments: constant, four lags of $\pi$, four lags of $u$, four lags of $\Delta v$.

"Augmented” BGNKPC

This subsection presents our estimates of the open economy version of the Blanchard-Gali model. As already pointed out, external inflationary pressures are captured by “appending” the real effective exchange rate to the microfounded version of the Blanchard-Gali supply curve. Admittedly, this renders the curve non-structural. However, this strategy i) allows us to understand at least the correlation between the U.S. and U.K. GDP deflator inflation and exchange rates, and ii) is not subject to estimation biases potentially coming from misspecification of the structural economic model from which the Phillips curve is derived.

Table 3 collects our estimates of the model (4). Some interesting results stand out. First, unemployment becomes significant and takes the correct sign in the estimated Phillips curve for the United States, so offering support to the demand channel typically seen as being the main monetary policy transmission channel. Second, the presence of the real exchange rate is corroborated by the data. This finding suggests that both economies are affected by external pressures, an evidence indicated by some authors as one of the consequences of the increasing openness featuring most of the industrialized countries at a world-wide level (Rogoff (2007)). Moreover, the presence of the real exchange rate in the Phillips curve influences the inflation-output volatility trade-off, so affecting optimal monetary policies (Walsh (1999)). Third, the presence of the exchange rate augments the estimated parameter of unemployment in the U.K. Phillips curve, so suggesting a stronger systematic effect of monetary policy moves on inflation and, possibly, a more precise estimation of the sacrifice ratio in the U.K.. Fourth, the
introduction of the real exchange rate does not sweep away the statistical relevance of oil, i.e. the model proposed by Blanchard and Gali (2007) is supported by the data.

Table 3. “Augmented” Blanchard and Gali New Keynesian Phillips Curve

<table>
<thead>
<tr>
<th>Country</th>
<th>$\psi_1$</th>
<th>$\psi_2$</th>
<th>$\psi_3$</th>
<th>$\psi_4$</th>
<th>$\psi_5$</th>
<th>J-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>0.62(0.02)**</td>
<td>-0.21(0.02)**</td>
<td>0.56(0.02)**</td>
<td>0.26(0.05)**</td>
<td>0.02(0.008)**</td>
<td>5.86</td>
<td>0.88</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.57(0.01)**</td>
<td>0.02(0.009)*</td>
<td>0.59(0.006)**</td>
<td>0.08(0.005)**</td>
<td>0.03(0.001)**</td>
<td>5.11</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Estimated curve: $\pi = \psi_1 \pi (+1) + \psi_2 u + \psi_3 \pi (-1) + \psi_4 v + \psi_5 \Delta e + \zeta$. Note: GMM point estimates reported in the Table (Newey-West robust, standard errors in brackets). **/*/ identify 1/5% significance level. Instruments: constant, four lags of $\pi$, four lags of $u$, four lags of $\Delta v$, four lags of $\Delta e$.

Table 4 offers a summary of this chapter’s findings.

Table 4. Qualitative empirical results

<table>
<thead>
<tr>
<th>Country</th>
<th>SNKPC</th>
<th>BGNKPC</th>
<th>“Augmented” BGNKPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>misspecified</td>
<td>coherent with theoretical indications</td>
<td>coherent with theoretical indications; stronger evidence supporting the demand channel</td>
</tr>
<tr>
<td>U.S.</td>
<td>misspecified</td>
<td>u: wrong sign and not significant</td>
<td>coherent with theoretical indications</td>
</tr>
</tbody>
</table>

5. Conclusions

This chapter estimates three different versions of the New Keynesian Phillips curve (standard New Keynesian Phillips curve, Blanchard and Gali New Keynesian Phillips curve, “augmented” Blanchard and Gali New Keynesian Phillips curve) for the United Kingdom and the United States of America. We find that the standard New Keynesian Phillips curve specified in terms of unemployment is severely misspecified. In fact, the presence of oil – in particular, the producer price index computed in relative terms with respect to the GDP deflator – turns out to be significant in both economies. However, we also find evidence in favour of the presence of the real exchange rate in the estimated
supply curves. Importantly, such a regressor helps identifying the value of other estimated parameters in the two schedules under investigation.

It seems to us that the Blanchard and Gali New Keynesian Phillips curve, above all when interpreted in an open economy fashion, may very well be a credible alternative to the standard, output gap-driven supply curve. This calls for further analysis concerning optimal monetary policy in the presence of explicitly modelled supply shocks on the one hand, and the role played by such supply shocks in triggering inflation and business cycle fluctuations on the other hand. These efforts belong to our research agenda.
References


Walsh C. (1999), “Monetary Policy Trade-Offs in the Open-Economy,” University of California at Santa Cruz, manuscript.

Data appendix

The data used come from: a) OECD Economic Outlook for GDP deflator; b) OECD Main Economic Indicator for PPI and real effective exchange rate; c) OECD Labour Force Survey for standardised unemployment rate.

We define the variables of the equations (2)-(4) in the following way:

**Inflation \([\pi(-1) \text{ and } E\pi(+1)]\):** for inflation we use the annual percent change in the GDP price deflator.

\[
\pi = \left\{ \frac{\text{GDP} - \text{GDP}(-1)}{\text{GDP}(-1)} \right\} \cdot 100
\]

where GDP is gross domestic product price deflator.

For inflation at the time minus one, \(\pi(-1)\), we consider the realized inflation at the previous year.

For future expected inflation, \(E\pi(+1)\), we consider the realized inflation at the next year.

**Unemployment \((u)\):** we consider unemployment using the percent standardised unemployment rate.

**Percent change in the price of nonproduced input \((\Delta v)\):** we define the percent change in the price of nonproduced input as the annual percent change in the PPI relative to the GDP price deflator.

\[
\Delta v = \left\{ \frac{\text{PPI}/\text{GDP} - \text{PPI}(-1)/\text{GDP}(-1))}{\text{PPI}(-1)/\text{GDP}(-1))} \right\} \cdot 100
\]

\(\Delta v\): annual percent change in the price of nonproduced input.

PPI: producer price index.

GDP: gross domestic product price deflator.

**Percent change in the real effective exchange rate \((\Delta e)\):** we define the percent change in the real effective exchange rate as the annual percent change in the real effective exchange rate.

\[
\Delta e = \left\{ \frac{\varepsilon - \varepsilon(-1)}{\varepsilon(-1)} \right\} \cdot 100
\]

\(\Delta e\): annual percent change in the real effective exchange rate.
CHAPTER 3

“Assessing the Role of Oil Price Shocks in a New Keynesian Model”

1. Introduction

The New Keynesian model (for a textbook presentation see Woodford (2003)) is the core analytical tool to study monetary policy and the business cycle nowadays. A key schedule of this model is the New Keynesian Phillips curve (NKPC henceforth). In the standard form the NKPC reads $\pi_t = \beta E_t \pi_{t+1} + k(y_t - y_t^*)$, where $\pi$ is inflation, $\beta$ identifies firms’ discount factor, $E_t$ expresses expectations taken at time $t$ (i.e. with an information set updated up to $t$), $y$ represents current GDP taken in logs, $y^*$ is the log of the natural level of output (i.e. the second best output), and $(y - y^*)$ is the output gap.

Many economists have highlighted an important lack afflicting the NKPC (e.g. Mankiw (2001)). This lack is the absence of a trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap (i.e. the gap between current and the first best level of output), when a supply shock strikes the economy. The reason is the following. In the standard New Keynesian model the difference between the first and the second best output remains always constant. Therefore, if a central bank stabilizes simultaneously inflation and the output gap, in the same time it stabilizes the welfare-relevant output gap. Blanchard and Galì (2007) call this feature of the standard New Keynesian model “divine coincidence”. Interestingly, they show that, if non trivial real imperfections are endogenized in the New Keynesian model, the “divine coincidence” does not work anymore. In particular they include real wage rigidities in the model determining a break of the constant relation between the first and the second best output. The effect is that the policy maker can not stabilize simultaneously inflation and the welfare-relevant output gap. Because of real wage rigidities, a trade-off between inflation and the welfare-relevant output gap arises in front of central banker in the presence of a supply shock. From their assumptions, Blanchard and Gali (2007) obtain a new version of the New Keynesian model.

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1 The second best output level is the one that would prevail under flexible prices in the presence of real distortions such as monopolistic competition. By contrast, the first best output level is the one that would realize under fully flexible prices and perfect competition in all markets.
Keynesian Phillips curve (BGNKPC henceforth), characterized by the presence of the real price of a nonproduced input (e.g. oil) among the independent variables. Interestingly, this feature allows a researcher to investigate the role played by oil shocks in shaping inflation and the business cycle, a role object of an intense debate in the literature [(Blanchard and Galì (2007); Nakov and Pescatori (2007)].

This chapter estimates and simulates a small-scale DSGE New Keynesian model including the BGNKPC for the United States in the quarterly data sample 1984:1–2007:4. We then perform factual and counterfactual simulations to gauge the role played by different shocks on inflation, unemployment and policy rate. Our main findings are the followings. First, oil price shocks have played an important role in explaining inflation fluctuations in the U.S. economy in the last two decades. In particular, the explained variance of the U.S. inflation due to such shocks amounts to 9.27 percent. This is due both to the direct impact that oil price shocks exert on GDP inflation and the indirect impact working throughout their influence on the U.S. systematic monetary policy, but it is largely due to the former. By contrast, the contribution of oil price shocks in determining unemployment fluctuations has been very modest, i.e. 0.05 percent. Moreover the driving shocks for inflation and unemployment are respectively non-oil supply shock (81.08%) and demand non-policy shock (98.52%). Second, stronger reactions to oil price swings than the ones historically observed would not have improved the stabilization of inflation and unemployment. Third, the best result in terms of stabilization of inflation and unemployment is conditional to the type of shock hitting the economy. In particular the best result is obtained by a “progressive central bank” (i.e. a central bank which reacts strongly to unemployment and weakly to inflation) in the presence of oil price shock, and by an “interventionist central bank” (i.e. a central bank which reacts strongly both to inflation and to unemployment) in the presence of monetary policy shock. Fourth, the more forward-looking the firms are, the more stable the economy is in the presence of oil price shock.

This chapter develops as follows. Section 2 underlines the economic literature about oil related to our work. Section 3 presents the version of small-scale DSGE New Keynesian model including the BGNKPC. Section 4 proposes the estimates of this model. In the section 5 we perform different counterfactual
simulations to gauge the role of different monetary policy rules and different inflation expectations hypotheses in explaining inflation, unemployment and short term interest rate fluctuations. Section 6 presents variance decomposition data to identify key-shocks. Section 7 compares our model with that of Nakov and Pescatori (2007). Section 8 concludes.

2. Literature about oil related to our work

Our work is an empirical work which aims to focus the role of oil price shock in explaining inflation and unemployment fluctuations. To conduct this analysis and to assess the found results we identify some links with other economic papers which tackle the oil subject in macroeconomic key. On the empirical hand useful references for us are Darby (1982), Hamilton (1983), Burbidge and Harrison (1984). These authors find that increases of oil price are followed by decreases of U.S. output growth (Hamilton (1983)), by decreases of growth rate of the real GNP for the U.S. and other countries (Darby (1982)), by rises of inflation and decreases of industrial production for the U.S. and other countries (Burbidge and Harrison (1984)). This authors signal an important role of oil price shocks in explaining inflation and output fluctuations. The conclusions of Hamilton (1983) are called into question by Bernanke, Gertler and Watson (1997). According to these authors the main cause of U.S. recessions after World War II was a restrictive monetary policy more than oil price rises. In support to the Bernanke, Gertler and Watson’s thesis there are two papers: Barsky and Kilian (2001) and Kilian (2005). These authors claim that the stagflation phenomenon of the seventies was generated essentially by monetary dynamics, while the role of oil price increases was limited. Our analysis allows us to affirm that in the period from 1984 to 2007 the oil price shocks have played a role which appears irrelevant in determining unemployment fluctuations, but is important in determining inflation fluctuations. On the theoretical hand relevant papers which incorporate oil in a macroeconomic framework are written by Rotemberg and Woodford (1996), Finn (1995, 2000), Leduc and Sill (2004), Carlstrom and Fuerst (2005). All these papers have in common that they suppose an exogenous path for oil price. This approach appears unsatisfactory, because in a model conceived in this way for policy maker there is not a trade-off between stabilizing inflation and
stabilizing the welfare-relevant output gap in the presence of oil price shock. A solution for this lack is proposed by Nakov and Pescatori (2007), who make endogenous the optimal oil price markup of the dominant supplier of oil (e.g. OPEC). This solution allows two authors to create a trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap in the presence of oil price shock. As we argue in the introduction, Blanchard an Gali (2007) reach the same result introducing in their model real wage rigidities. The model which we analyse tries to take in both i) Nakov and Pescatori’s proposal (in fact in our framework we model the percent change in the real price of oil as a function of past change in unemployment rate, past short term interest rate and past percent change in the real price of oil) and ii) Blanchard and Gali solution using the BGNKPC, which incorporates the real price of oil.

3. A small-scale DSGE New Keynesian model

The small-scale DSGE New Keynesian model we consider is composed by four equations.

The first equation is the BGNKPC\(^2\). This equation reads as follows:

\[
\pi = \varphi_1 E\pi(+1) + \varphi_2 \Delta u + \varphi_3 \pi(-1) + \varphi_4 \Delta v + \zeta
\]  

(1)

where \(\pi\) represents current inflation, \(E\pi(+1)\) is expected future inflation, \(\Delta u\) is the change in unemployment rate, \(\pi(-1)\) is past inflation, \(\Delta v\) is the percent change in the real price of oil and \(\zeta\) is the inflation shock\(^3\).

The second equation is the Euler equation with unemployment rate in place of output gap\(^4\):

\[
\Delta u = \varphi_5 \Delta u(-1) + \varphi_6 [i - E\pi(+1)] + \rho
\]  

(2)

---

\(^2\) With respect to the original version of the BGNKPC, we replace unemployment in levels with its first difference to capture the transmission mechanism going from unemployment to inflation in a more satisfactory fashion from an empirical standpoint.

\(^3\) In the following part of the chapter we refer to the change in unemployment rate and the percent change in the real price of oil respectively as unemployment and real price of oil.

\(^4\) In order to augment the degree of comparability among different equations of the DSGE New Keynesian model, we substitute the output gap with the change in unemployment rate. As postulated by the Okun’s law, these two macroeconomic variables are very correlated. Gordon (1997) and Roberts (1995, 2006) propose a similar solution with respect to the New Keynesian Phillips curve.
where $i$ is short term interest rate and $\rho$ is the unemployment shock.

The third equation is the Taylor rule [Taylor (1993)]:

$$i = \varphi_7 \pi + \varphi_8 \Delta u + \varphi_9 i(-1) + \psi \tag{3}$$

where $\psi$ is the monetary policy shock.

The fourth equation refers to the real price of oil:

$$\Delta v = \varphi_{10} \Delta v(-1) + \varphi_{11} \Delta u(-1) + \varphi_{12} i(-1) + \varepsilon \tag{4}$$

where $\varepsilon$ is the oil price shock.

All coefficients are expected to take a positive value except for $\varphi_2$, $\varphi_6$, $\varphi_{11}$, $\varphi_{12}$ which are assumed to be negative.

4. Estimates

We estimate the small-scale DSGE New Keynesian model expressed by the equations (1)–(4) for the U.S. economy using quarterly data which span the sample 1984:1–2007:4. The choice of 1984:1 as the first quarter of our analysis is justified by our willingness to study the U.S. economy in a stable policy regime [Clarida, Gali, Gertler (2000)]. We assume that the four kinds of shock are uncorrelated. Following this assumption we estimate the equations (1)–(4) separately. In particular we estimate the BGNKPC, the Euler equation and the Taylor rule by GMM; while we estimate the equation (4) by OLS.

The table 1 displays the estimates for the U.S.. The estimated coefficients exhibit signs in line with to the economic wisdom and are significant at 5% (but $\varphi_6$ and $\varphi_{11}$ are significant at 10%). The only exception is represented by the coefficient on unemployment in the BGNKPC which takes a positive, but not significant, sign.

According to our estimates in the BGNKPC expected future inflation is predominant in explaining inflation dynamics with respect to past inflation. The real price of oil exerts a positive effect on inflation, with an estimated coefficient equal to 0.05.
The estimate of equation (2) (i.e. the Euler equation) shows that unemployment is highly persistent. The expected future interest rate influences positively current unemployment.

The estimate of equation (3) (i.e. the Taylor rule) gives results coherent with our expectations. The coefficient on inflation is positive, signalling a positive impact on the short term interest rate by the side of inflation. Importantly, the Taylor principle is satisfied. This principle requires that in the long-run the short term interest rate reacts more than one to one to a change in inflation. The long-run coefficient on inflation in the Taylor rule is equal to \([\phi/(1-\phi)]\), and it has to be higher than one to guarantee that the move of the policy rate in a certain direction is followed by the move of the real interest rate in the same direction. According to our estimates, \([\phi/(1-\phi)]=3.28\). Still focusing on the Taylor rule, unemployment exerts a very strong negative impact on the short term interest rate in the U.S., highlighting a very active role played by the FED in contrasting negative shocks on the business cycle. Moreover the short term interest rate is characterized by a high level of persistence.

Finally, the estimate of equation (4) suggests an important level of persistence in the path of the real price of oil. Besides the effects played by past short term interest rate and past unemployment on the real price of oil are both negative. This is due to the fact that, when past short term interest rate or past unemployment increases, the oil demand falls determining a reduction of the real price of oil. In particular the negative impact of past unemployment is more intensive with respect to that of past short term interest rate.

5. Impulse response functions

5.1. Standard case:

We estimate impulse response functions of the small-scale DSGE New Keynesian model (1)–(4) using the estimated coefficients presented in the previous section (we label this scenario as “standard case”)\(^5\). We assume that four kinds of shock can hit the economy: a) inflation shock, which hits the BGNKPC;

\[^5\text{Here and in the subsequent simulations we calibrate } \phi_2=-0.20, \text{ following Roberts (2006). Moreover, although the estimated value of coefficient } \phi_8 \text{ is } -2.17, \text{ we calibrate this coefficient equal to } -1.245, \text{ i.e. the average between } -1.49 \text{ (i.e. } \phi_8 + 2\sigma_{\phi_8}, \text{ where } \sigma_{\phi_8} \text{ is the estimated standard deviation for } \phi_8 ) \text{ and } -1 \text{ (i.e. the value attributed to } \phi_8 \text{ by Orphanides and Williams (2006)).}\]
b) unemployment shock, which works via Euler equation; c) monetary policy shock, which acts on Taylor rule equation (3); d) oil price shock, which influences equation (4). We assume that estimated residuals may be interpreted as structural shocks. Here and in the remaining part of the chapter we assume that, in estimating the impulse response functions, each innovation is equal to a standard error as estimated in this chapter. The period of simulation is fixed in forty quarters.

We are interested in understanding the paths of inflation, unemployment and short term interest rate in the presence of each of these shocks. The figures 1–4 display the estimated impulse response functions.

Oil price shock:

The response of U.S. inflation, when an oil price shock hits the U.S. economy, is positive.

This variable jumps to about 0.31 percent above its steady state value and then it begins to decline. After one year and one quarter after this shock it is equal to −0.02 percent with respect to the steady state value. U.S. inflation returns to its steady state value after six years and one quarter after this shock.

Differently U.S. unemployment responds negatively in reaction to this kind of shock. Initially unemployment goes to about −0.0003 percent below its steady state value. Then it increases and after one year and one quarter is above the steady state value of about 0.0016 percent. Subsequently it declines and returns to its steady state value after seven years after this shock.

Initially U.S. short term interest rate responds in positive way to oil price shock. It jumps to 0.072 percent above its steady state value. It continues to increase until 0.09 percent after three quarters after the shock and then it declines and comes back to its steady state value after eight years and three quarters after this shock.

The oil price shock determines an initial increase of the real price of oil to about 4.5 percent with respect to its steady state value. Afterwards this variable decreases to −0.1 percent with respect to its steady state value after one year and one quarter after the shock. Then it goes newly to its steady state value after three years after this shock.
Among the variables analysed we see that, in reaction to oil price shock, the real price of oil is characterized by more variance, while U.S. short term interest rate is settled by more persistence.

**Inflation shock:**

The initial effect of inflation shock on inflation is higher than that on short term interest rate or unemployment or real price of oil. After this shock U.S. inflation jumps above its steady state value. Then it decreases quickly in the way that after four quarters is about −0.05 percent with respect to steady state. Inflation returns to steady state after about six years after this shock.

The inflation shock exerts a very low effect on U.S. unemployment. In fact the maximum deviation of this variable from steady state is equal to about 0.0041 percent after one year after this shock. Unemployment returns to its steady state value after about nine years after this shock.

U.S. short term interest rate augments to about 0.225 percent with respect to its steady state value in consequence of inflation shock; afterwards it decreases and reassumes its steady state value after eight years and half after this shock.

After three quarters after the inflation shock, the real price of oil decreases to −0.14 percent with respect to steady state. Then it increases and reassumes its steady state value after eight years and one quarter after this shock.

Among the variables analysed we see that, in reaction to inflation shock, U.S. inflation is characterized by more variance, while U.S. unemployment is settled by more persistence.

**Unemployment shock:**

Inflation goes to about −0.058 percent below its steady state value after the unemployment shock. Then, after one year and half, it increases to about 0.076 percent with respect to its steady state value and then it returns to steady state value after eight years and half after this shock.

While the initial response of U.S. inflation to unemployment shock is negative, U.S. unemployment follows a positive path. U.S. unemployment reaches a value of about 0.14 percent above its steady state value and then declines to
about −0.002 percent below the steady state after two years. U.S. unemployment comes back to its steady state value after five years and half after this shock.

Initially U.S. short term interest rate reacts in negative manner to unemployment shock. In fact it goes to about −0.19 percent with respect to its steady state value. Then it still declines to about −0.29 percent after three quarters. From that point it increases and reassumes the steady state value after eight years and three quarters after this shock.

The real price of oil goes to −0.55 percent below its steady state value after two quarters after this shock. Then it increases to 0.12 percent above steady state after about one year and three quarters. Afterwards it decreases and comes back to its steady state value after nine years after this shock.

Among the variables considered we see that, in reaction to unemployment shock, the real price of oil is characterized by more variance and more persistence.

Monetary policy shock:

The initial response of U.S. inflation to monetary policy shock is negative. U.S. inflation goes to about −0.115 percent with respect to its steady state value. Then it declines further on to about −0.132 percent after two quarters and subsequently increases until returning newly to its steady state value after eight years and three quarters after this shock.

Initially U.S. unemployment responds in positive way to monetary policy shock. It jumps to about 0.0049 percent above its steady state value. Then it still increases to about 0.0072 percent above the steady state after one year after this shock. From that point U.S. unemployment declines and returns to steady state value after nine years after this shock.

After the monetary policy shock the initial response of U.S. short term interest rate is positive. In fact this variable augments to about 0.42 percent above its steady state value. Subsequently it decreases continually and returns to steady state after eight years and one quarter after this shock.

In response to monetary policy shock the real price of oil decreases to −0.235 percent below its steady state value after three quarters. Then it increases until returning to steady state after eight years and half.
Among the variables examined we see that, in reaction to monetary policy shock, U.S. short term interest rate is characterized by more variance, while U.S. unemployment is settled by more persistence.

The table 2 synthesizes all these results.

5.2. Alternative Taylor rules

We simulate the equations (1)–(4) performing different monetary policy rules or assuming different weights for backward–looking and forward–looking expectations in the BGNKPC. We consider five hypothetical scenarios: 1) different responses of short term interest rate to inflation in the Taylor rule (case called “Taylor rule I”); 2) different responses of short term interest rate to unemployment in the Taylor rule (case called “Taylor rule II”); 3) different responses of short term interest rate to inflation and unemployment in the Taylor rule (case called “mixed Taylor rules I and II”); 4) different responses of short term interest rate to real price of oil in the Taylor rule (case named “Taylor rule III”); 5) different coefficients on past inflation and expected future inflation in the BGNKPC (case called “Different inflation expectations”).

Taylor rule I:

This counterfactual analysis deviates from the standard case analysed in the section 5.1 for the reason that we suppose that the monetary authority reacts to inflation differently with respect to the value estimated for \( \varphi_7 \). We assume two hypothetical situations. The first situation consists in an aggressive response of the central bank to inflation. In this case we assume \( \varphi_7 = 0.35 \). The second situation hypothesizes that the monetary authority reacts weaker than reality to inflation. In this case \( \varphi_7 = 0.11 \). We compare these two hypothetical scenarios and that represented by the standard case (which is the benchmark). In particular, we try to understand the different effects of oil price shock and monetary policy shock on inflation, unemployment and short term interest rate under the two hypothetical cases and the standard case. The results are represented in the figure 5.

\[ \varphi_7 = \varphi_7 + 2\sigma_\varphi, \]

\[ \varphi_7 = \varphi_7 - 2\sigma_\varphi. \]

\( \sigma_\varphi \) is the estimated standard deviation for \( \varphi_7 \) in the Taylor rule.
For the U.S. economy a more aggressive behaviour of the central bank against inflation does not give a general advantage in terms of inflation variance’s reduction. In fact, if an oil price shock hits the U.S. economy, inflation exhibits lower variance in the case of weak response of central bank to inflation (while in the presence of this shock all the three scenarios analysed produce the same result in terms of inflation persistence’s reduction). Following this shock, also the standard case realizes a better performance in terms of inflation variance’s reduction with respect to the case involving strong reaction to inflation. Instead, in the presence of monetary policy shock, the appropriate behaviour to stabilize U.S. inflation quickly and with lower variance is the strong response of short term interest rate to inflation.

As for U.S. inflation, also for U.S. unemployment there is not only one best central bank’s response to inflation to reduce variance, whatever is the shock involved. In fact, if an oil price shock hits the U.S. economy, the best strategy for the central bank to minimize the variance of unemployment is to react in weak way to inflation. On the contrary, if a monetary policy shock realizes, the best strategy for unemployment variance’s reduction is to react in strong way to inflation. In terms of unemployment persistence’s reduction, the best strategy is to respond strongly to inflation in the presence of whatever shock.

The best strategy for the central bank to reduce the variance of U.S. short term interest rate is: i) the weak response to inflation in the presence of oil price shock; ii) the strong response to inflation in the presence of monetary policy shock. The best strategy for the monetary authority to minimize the persistence of U.S. short term interest rate is the strong response to inflation in the presence of whatever shock.

The results which we have found are displayed in the table 3.

*Taylor rule II:*

This counterfactual analysis differs from the standard case examined in the section 5.1. because now we suppose that the monetary authority can react to unemployment differently with respect to the value estimated for \( \phi_8 \). We assume two hypothetical cases. The first case consists in a strong reaction of the central
bank to unemployment. In this case we assume $\varphi_8 = -1.49^8$. The second case hypothesizes that the monetary authority reacts more weakly to unemployment. In this case $\varphi_8 = -1^9$. We compare these two hypothetical cases and the standard case in terms of inflation, unemployment and short term interest rate’s adjustment paths in the presence of oil price shock and monetary policy shock.

The results are represented in the figure 6.

If an oil price shock comes about, the strong response to unemployment is the best strategy to reduce the variance of unemployment and short term interest rate. Instead, in the presence of oil price shock, the three strategies under investigation produce the same result in terms of inflation variance’s reduction.

In the presence of monetary policy shock the best strategy for the central bank to reduce variance of inflation, unemployment and short term interest rate is the strong response to unemployment.

Moreover, in the presence of whatever shock, the strong response to unemployment, the weak response to unemployment and the standard case realize the same results in terms of persistence’s reduction for inflation, unemployment and short term interest rate.

The table 4 summarizes the results which we have found.

Mixed Taylor rules I and II:

In this scenario we assume that the central bank reacts to inflation and unemployment using mixed strategies arisen from the jointly consideration of the Taylor rule I and Taylor rule II cases. In particular we identify four situations characterized by different values assigned to parameters $\varphi_7$ and $\varphi_8$:

1) $\varphi_7 = 0.35$ and $\varphi_8 = -1$. In this case the central bank fights aggressively inflation and contrasts weakly unemployment (case called “conservative central bank”, CCB henceforth);

2) $\varphi_7 = 0.11$ and $\varphi_8 = -1.49$. In this case the central bank fights weakly inflation and contrasts strongly unemployment (case called “progressive central bank”, PCB henceforth);

3) $\varphi_7 = 0.35$ and $\varphi_8 = -1.49$. In this case the central bank fights weakly inflation and contrasts weakly unemployment (case called “conservative central bank”);

4) $\varphi_7 = 0.11$ and $\varphi_8 = -1$. In this case the central bank fights strongly inflation and contrasts strongly unemployment (case called “progressive central bank”);

---

$^8 \varphi_8 = \varphi_7 + 2\sigma_\varphi$, where $\sigma_\varphi$ is the estimated standard deviation for $\varphi_8$ in the Taylor rule.

$^9$ Orphanides and Williams (2006) impose this value in calibrating an equation very similar to our.
3) $\varphi_7=0.11$ and $\varphi_8=-1$. In this situation the central bank fights weakly both inflation and unemployment (case called “laissez-faire central bank”, LFCB henceforth);

4) $\varphi_7=0.35$ and $\varphi_8=-1.49$. In this situation the central bank contrasts strongly both inflation and unemployment (case called “interventionist central bank”, ICB henceforth).

The results are depicted in the figures 7–8.

In the presence of oil price shock, PCB is the best response of the central bank to stabilize inflation, unemployment and short term interest rate.

In the presence of monetary policy shock, ICB represents the best reaction of the central bank to stabilize inflation, unemployment and short term interest rate.

The best responses identified in this section are more efficacious in terms of stabilization with respect to the responses generated by the standard case.

The table 5 summarizes the results which we have found.

*Taylor rule III:*

We analyse the adjustment paths towards the steady state values of U.S. inflation, unemployment and short term interest rate assuming a different structure of the Taylor rule. Opposite to the equation (3), we include the real price of oil among the independent variables of the Taylor rule. Now we assume that the central bank considers also this variable in moving short term interest rate. For this reason the new Taylor rule takes this form:

$$i = \varphi_7 \pi + \varphi_8 \Delta u + \varphi_o i(-1) + \varphi_{oil} \Delta v + \psi$$  (5)

The standard case is characterized by $\varphi_{oil}$ equal to zero. Now we calibrate $\varphi_{oil}$ to represent two alternative hypothetical strategies in response to real price of oil. We call the situations with $\varphi_{oil}^1=0.10$ and $\varphi_{oil}^2=0.20$ respectively “weak response to real price of oil” and “strong response to real price of oil”\(^{10}\). We compare U.S. inflation, unemployment and short term interest rate’s adjustment paths in the

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\(^{10}\) $\varphi_{oil}^1$ and $\varphi_{oil}^2$ are calibrated following Bernanke, Gertler and Watson (2004) and Carlstrom and Fuerst (2005).
presence of oil price shock and monetary policy shock under the standard, weak response to real price of oil, strong response to real price of oil cases.

The results are represented in the figure 9.

If an oil price shock hits the economy, the more variance’s reduction for inflation is obtained by interest rate’s no response in reaction to real price of oil (i.e. the standard case). Moreover, when an oil price shock strikes the economy, also U.S. unemployment and short term interest rate require no response of short term interest rate to real price of oil to minimize their variance. Furthermore, in the presence of oil price shock, the strong response of short term interest rate in reaction to real price of oil produces always the worst result in terms of inflation, unemployment, short term interest rate variance’s reduction. Instead, in the presence of oil price shock, the strong response to real price of oil, the weak response to real price of oil and the standard case realize the same results in terms of inflation, unemployment and short term interest rate persistence’s reduction.

In the presence of monetary policy shock, the strong response to real price of oil realizes the more variance and persistence’s reduction for inflation, unemployment and short term interest rate.

These results are displayed in the table 6.

5.3. Different inflation expectations:

We simulate the equations (1)–(4) assuming different values for coefficients on expected future inflation and past inflation, $\phi_1$ and $\phi_3$, in the BGNKPC. The calibration of these two parameters allows us to reproduce three cases: i) standard case, in which $\phi_1$ and $\phi_3$ take the estimated values; ii) more forward-looking case (with respect to the standard case, MFL henceforth), in which $\phi_1=0.925$ and $\phi_3=0.075$; iii) purely forward-looking case (PFL henceforth), in which $\phi_1=1$ and $\phi_3=0$.

This exercise tries to clarify the role played by inflation expectations on inflation, unemployment and short term interest rate paths in the presence of oil price shock.

The results are displayed in the figure 10.

Under the PFL case U.S. inflation returns to its steady state value with lower variance with respect to the MFL and standard cases, in the presence of oil price shock.
shock. Inflation expectations performed in purely forward-looking manner reduce the cost of disinflation. This fact is coherent with the idea that the more credible the commitment offered by the monetary authority to economic agents is, the more forward-looking firms’ inflation expectations are. When the credibility of commitment is very high, firms’ inflation expectations are only forward-looking. Instead, in the presence of oil price shock, the PFL, MFL and standard cases assure substantially the same performance in terms of inflation persistence’s reduction.

Analogous tendencies concern U.S. unemployment and short term interest rate. When an oil price shock hits the U.S. economy, the PFL case ensures unemployment and short term interest rate paths with the minimum variance. In regard to unemployment and short term interest rate, the PFL, MFL and standard cases offer identical results in terms of persistence’s reduction.

The table 7 synthesizes the results which we have described.

6. Variance decomposition

We compute the variance decomposition of U.S. inflation, unemployment and short term interest rate to deliver the key-shocks which generate fluctuations of these variables. We show that in the standard case the role played by oil price shock in determining fluctuations is important for U.S. inflation. Instead it appears to be irrelevant for U.S. unemployment and short term interest rate. U.S. inflation fluctuations are explained by oil price shock for 9.27 percent. Besides oil price shock is responsible of only 0.05 percent of U.S. unemployment fluctuations and of 2.87 percent of short term interest rate fluctuations. Our results are different in terms of oil price shock’s impact in explaining inflation and short term interest rate variance with respect to those found by Nakov and Pescatori (2007). In particular, according to Nakov and Pescatori (2007), oil price shocks are altogether responsible of 33.54 percent of inflation variance and 13.96 percent of short term interest rate variance.

On the other hand, monetary policy shock explains an important fraction of U.S. inflation variance and a little fraction of U.S. unemployment variance. In fact monetary policy shock influences U.S. inflation and unemployment fluctuations respectively by 6.48 and 1.07 percent. Notably, also Christiano, Eichenbaum and
Evans (2005) estimate an impact of monetary policy shock in terms of inflation variance decomposition similar to that we have found (7% in a twenty quarters period ahead the exercise).

The difference between our results and those of Nakov and Pescatori (2007) is smaller, if we recalculate the variance decomposition of variables considering \( \Phi_4 = \Phi_4 + 2 \sigma_\omega \). In this situation, with \( \Phi_4 = 0.09 \), oil price shock is responsible of 22.86 of inflation fluctuations, of 7.87 percent of interest rate fluctuations and of 0.15 percent of unemployment fluctuations. The results of variance decomposition are reported in the tables A and B.

### Table A. Standard case; variance decomposition (in percent)

<table>
<thead>
<tr>
<th>U.S. variable</th>
<th>Oil price shock</th>
<th>Inflation shock</th>
<th>Unemployment shock</th>
<th>Monetary policy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFLATION</td>
<td>9.27</td>
<td>81.08</td>
<td>3.17</td>
<td>6.48</td>
</tr>
<tr>
<td>UNEMPLOYMENT</td>
<td>0.05</td>
<td>0.36</td>
<td>98.52</td>
<td>1.07</td>
</tr>
<tr>
<td>SHORT TERM INTEREST</td>
<td>2.87</td>
<td>16.85</td>
<td>38.49</td>
<td>41.79</td>
</tr>
</tbody>
</table>

### Table B. Variance decomposition (in percent) with \( \Phi_4 = \Phi_4 + 2 \sigma_\omega \)

<table>
<thead>
<tr>
<th>U.S. variable</th>
<th>Oil price shock</th>
<th>Inflation shock</th>
<th>Unemployment shock</th>
<th>Monetary policy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFLATION</td>
<td>22.86</td>
<td>63.37</td>
<td>4.61</td>
<td>9.16</td>
</tr>
<tr>
<td>UNEMPLOYMENT</td>
<td>0.15</td>
<td>0.33</td>
<td>98.50</td>
<td>1.02</td>
</tr>
<tr>
<td>SHORT TERM INTEREST</td>
<td>7.87</td>
<td>14.66</td>
<td>40.24</td>
<td>37.23</td>
</tr>
</tbody>
</table>

7. **Comparison between our model and that of Nakov and Pescatori (2007)**

In the economic literature the paper which is closest to ours is that of Nakov and Pescatori (2007, NP henceforth). We share with this work different choices. First, we employ a model in which the real price of oil is not exogenous, but is influenced by demand conditions. For this reason our approach represents the real price of oil as function of past unemployment, past short term interest rate and the
past real price of oil. In this way we make endogenous the real price of oil as in NP (2007). Second, we estimate the coefficients of our model. The estimators employed are GMM for the BGNKPC, the Euler equation and the Taylor rule and OLS for the equation of the real price of oil. NP (2007) estimate their model with Bayesian techniques. This fact give us the possibility of assessing the impact of oil price shock on inflation and business cycle volatility in more precise way with respect to case based on calibration of the model. Third, in our and Nakov and Pescatori’s model the monetary authority follows a type of Taylor rule offering a commitment to private sector. The discretionary monetary policy is not admitted. Our variance decomposition results underline a relevant impact of oil price shock in explaining inflation fluctuations, but less than in NP (2007). In particular according to us oil price shocks are responsible of 9.27 percent of inflation fluctuations, while according to NP (2007) the impact of oil price shocks on inflation variance amounts to 33.54 percent. But, if we recalculate the variance decomposition assuming that the coefficient on the real price of oil in the BGNKPC is equal to its estimated value plus two times its estimated standard deviation, we find a result more in line with NP’s estimates. In fact in this situation oil price shock explains 22.86 percent of inflation volatility. The value added offered by our work with respect to NP (2007) is represented by a large set of counterfactual analyses. By these we are able to understand what strategy of monetary policy could minimize the volatility of inflation and unemployment in the presence of different shocks.

8. Conclusions

This chapter estimates a small-scale DSGE New Keynesian model a la Woodford (2003) for the U.S. in the quarterly data sample 1984:1–2007:4 and performs factual and counterfactual analyses to make clear the role played by oil price shocks, different monetary policy rules and different inflation expectations in explaining inflation and unemployment fluctuations.

Our main findings are four. First, oil price shocks have played an important role in explaining inflation fluctuations in the U.S. economy in the last two decades. In particular, the explained variance of U.S. inflation due to such shocks amounts to 9.27 percent. This is due both to the direct impact that oil price shocks
exert on GDP inflation and the indirect impact working throughout their influence on the U.S. systematic monetary policy, but it is largely due to the former. By contrast, the contribution of oil price shocks in determining unemployment fluctuations has been very modest, i.e. 0.05 percent. Moreover the driving shocks for inflation and unemployment are respectively non-oil supply shock (81.08%) and demand non-policy shock (98.52%). Second, stronger reactions to oil price swings than the ones historically observed would not have improved the stabilization of inflation and unemployment. Third, the best result in terms of stabilization of inflation and unemployment is conditional to the type of shock hitting the economy. In particular the best result is obtained by a “progressive central bank” (i.e. a central bank which reacts strongly to unemployment and weakly to inflation) in the presence of oil price shock, and by an “interventionist central bank” (i.e. a central bank which reacts strongly both to inflation and to unemployment) in the presence of monetary policy shock. Fourth, the more forward-looking the firms are, the more stable the economy is in the presence of oil price shock.

Overall, our results corroborate the search for a better understanding of the relationship among oil price, unemployment and inflation.
References


Table 1. Estimates of DSGE New Keynesian model for the standard case.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_1$</td>
<td>0.85(0.09)***</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>0.31(0.66)</td>
</tr>
<tr>
<td>$\varphi_3$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\varphi_4$</td>
<td>0.05(0.02)**</td>
</tr>
<tr>
<td>$\varphi_5$</td>
<td>0.54(0.08)***</td>
</tr>
<tr>
<td>$\varphi_6$</td>
<td>0.009(0.005)*</td>
</tr>
<tr>
<td>$\varphi_7$</td>
<td>0.23(0.06)***</td>
</tr>
<tr>
<td>$\varphi_8$</td>
<td>$-2.17(0.34)$***</td>
</tr>
<tr>
<td>$\varphi_9$</td>
<td>0.93(0.02)***</td>
</tr>
<tr>
<td>$\varphi_{10}$</td>
<td>0.23(0.10)**</td>
</tr>
<tr>
<td>$\varphi_{11}$</td>
<td>$-4.57(2.48)$*</td>
</tr>
<tr>
<td>$\varphi_{12}$</td>
<td>$-0.45(0.17)$***</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>0.89</td>
</tr>
<tr>
<td>$\sigma_\rho$</td>
<td>0.16</td>
</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>4.49</td>
</tr>
</tbody>
</table>

Estimated curves:

$$\pi = \varphi_1 E\pi(+1) + \varphi_2 d\pi + \varphi_3 d(-1) + \varphi_4 d\pi + \xi$$

$$\Delta u = \varphi_5 d\pi(-1) + \varphi_6 [i - E\pi(+1)] + \psi$$

$$i = \varphi_7 \pi + \varphi_8 d\pi + \varphi_9 d(-1) + \psi$$

$$\Delta v = \varphi_{10} d\pi(-1) + \varphi_{11} d\pi(-1) + \varphi_{12} d(-1) + \varepsilon$$

Note: GMM point estimates reported in the table for coefficients from $\varphi_1$ to $\varphi_9$ (Newey-West robust, standard errors in brackets). OLS point estimates reported in the table for coefficients from $\varphi_{10}$ to $\varphi_{12}$ (Newey-West robust, standard errors in brackets). The value of coefficient $\varphi_3$ is restricted to $1 - \varphi_1$. ***/**/* identify 1/5/10% significance level.

Instruments: constant, two lags of $\pi$, four lags of $\Delta u$, four lags of $\Delta v$ in the BGNKPC; constant, two lags of $\Delta u$, two lags of $[i - E\pi(+1)]$, four lags of $i$, three lags of $\Delta v$ in the Euler equation; constant, three lags of $\pi$, two lags of $\Delta u$, two lags of $i$ in the Taylor rule.

The BGNKPC (J-test: 6.40; p-value: 0.49); the Euler equation (J-test: 7.13; p-value: 0.62); the Taylor rule (J-test: 3.70; p-value: 0.44).
<table>
<thead>
<tr>
<th>U.S. variable</th>
<th>Oil price shock</th>
<th>Inflation shock</th>
<th>Unemployment shock</th>
<th>Monetary policy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFLATION</td>
<td>Initially positive</td>
<td>Initially positive</td>
<td>Initially negative</td>
<td>Initially negative</td>
</tr>
<tr>
<td></td>
<td>**;+++</td>
<td>*;++++</td>
<td>**;+</td>
<td>***;++</td>
</tr>
<tr>
<td>UNEMPLOYMENT</td>
<td>Initially negative</td>
<td>Initially positive</td>
<td>Initially positive</td>
<td>Initially positive</td>
</tr>
<tr>
<td></td>
<td>***;+</td>
<td>****;+</td>
<td>***;+,</td>
<td>*;++++</td>
</tr>
<tr>
<td>SHORT TERM INTEREST RATE</td>
<td>Initially positive</td>
<td>Initially positive</td>
<td>Initially negative</td>
<td>Initially positive</td>
</tr>
<tr>
<td></td>
<td>****;+</td>
<td>***;+++</td>
<td>***;+++</td>
<td>*;++++</td>
</tr>
<tr>
<td>REAL PRICE OF OIL</td>
<td>Initially positive</td>
<td>Initially negative</td>
<td>Initially negative</td>
<td>Initially negative</td>
</tr>
<tr>
<td></td>
<td>*;+++</td>
<td>**;+</td>
<td>****;+++</td>
<td>*;+++</td>
</tr>
</tbody>
</table>

**Persistence:** ****=the higher persistence; ***=the upper medium-size persistence; **=the medium-sized persistence; *=the lower persistence.

**Variance:** ++++=the higher variance; +++=the upper medium-size variance; ++=the medium-sized variance; +=the lower variance.
Table 3. Taylor rule I

<table>
<thead>
<tr>
<th>U.S. variable</th>
<th>Kind of response to inflation</th>
<th>Oil price shock</th>
<th>Monetary policy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strong</td>
<td><em>;</em>**</td>
<td>*;+</td>
</tr>
<tr>
<td>INFLATION</td>
<td>Weak</td>
<td>*;+</td>
<td><em><strong>;</strong></em></td>
</tr>
<tr>
<td>Standard case</td>
<td><em>;</em>**</td>
<td>**;+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td><em>;</em>**</td>
<td>*;+</td>
</tr>
<tr>
<td>UNEMPLOYMENT</td>
<td>Weak</td>
<td>***;+</td>
<td><em><strong>;</strong></em></td>
</tr>
<tr>
<td>Standard case</td>
<td>**;+</td>
<td>**;+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td><em>;</em>**</td>
<td>*;+</td>
</tr>
<tr>
<td>SHORT TERM INTEREST RATE</td>
<td>Weak</td>
<td>***;+</td>
<td><em><strong>;</strong></em></td>
</tr>
<tr>
<td>Standard case</td>
<td>**;+</td>
<td>**;+</td>
<td></td>
</tr>
</tbody>
</table>

Persistence: ***=the higher persistence; **=the medium-sized persistence; *=the lower persistence.

Variance: +++=the higher variance; ++=the medium-sized variance; +=the lower variance
<table>
<thead>
<tr>
<th>U.S. variable</th>
<th>Kind of response to unemplmy.</th>
<th>Oil price shock</th>
<th>Monetary policy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strong</td>
<td>*;+</td>
<td>*;+</td>
</tr>
<tr>
<td></td>
<td>Weak</td>
<td>*;+</td>
<td>*;+++</td>
</tr>
<tr>
<td>Standard case</td>
<td>*;+</td>
<td>*;+</td>
<td>*;++</td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td>*;+</td>
<td>*;+</td>
</tr>
<tr>
<td></td>
<td>Weak</td>
<td>*;+++</td>
<td>*;+++</td>
</tr>
<tr>
<td>Standard case</td>
<td>*;++</td>
<td>*;+</td>
<td>*;+</td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td>*;+</td>
<td>*;+</td>
</tr>
<tr>
<td>Standard case</td>
<td>*;+++</td>
<td>*;+</td>
<td>*;+</td>
</tr>
</tbody>
</table>

Persistence: ***=the higher persistence; **=the medium-sized persistence; *=the lower persistence.
Variance: +++=the higher variance; ++=the medium-sized variance; +=the lower variance.
Table 5. Mixed Taylor rules I and II

<table>
<thead>
<tr>
<th>U.S. variable</th>
<th>Kind of shock</th>
<th>The best strategy for the central bank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oil price shock</td>
<td>PCB</td>
</tr>
<tr>
<td>INFLATION</td>
<td>Monetary policy shock</td>
<td>ICB</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNEMPLOYMENT</td>
<td>Oil price shock</td>
<td>PCB</td>
</tr>
<tr>
<td></td>
<td>Monetary policy shock</td>
<td>ICB</td>
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<th>U.S. variable</th>
<th>Kind of response to real price of oil</th>
<th>Oil price shock</th>
<th>Monetary policy shock</th>
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<td>*;+</td>
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<td>Weak</td>
<td>*;++</td>
<td>**;++</td>
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<td>*;+</td>
<td><strong>;++;</strong>*+++</td>
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<tr>
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<td>*;+++</td>
<td>*;+</td>
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<tr>
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<td>Weak</td>
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<td>**;++</td>
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<td><strong>;++;</strong>*+++</td>
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<td>**;++</td>
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<tr>
<td></td>
<td>Standard case</td>
<td>*;+</td>
<td><strong>;++;</strong>*+++</td>
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Persistence: **=*the higher persistence; **=*the medium-sized persistence; *=the lower persistence.

Variance: +++=*the higher variance; ++=*the medium-sized variance; +=the lower variance
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<thead>
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<tr>
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<td>ST</td>
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</tbody>
</table>

*PFL: purely forward-looking expectations; MFL: more forward-looking expectations (with respect to the standard case); ST: the standard case.

Persistence: ***=the higher persistence; **=the medium-sized persistence; *=the lower persistence.

Variance: +++=the higher variance; ++=the medium-sized variance; +=the lower variance.
Data appendix

The data used come from: a)OECD Economic Outlook for GDP deflator; b)OECD Main Economic Indicator for PPI, short term interest rate and real effective exchange rate; c)OECD Labour Force Survey for standardised unemployment rate.

We define the variables of the equations (1)-(4) in the following way:

Inflation \(\pi(-1)\) and \(E\pi(+1)\): for inflation we use the annualized percent change in the GDP price deflator.

\[
\pi = \left\{ \frac{\text{GDP} - \text{GDP}(-1)}{\text{GDP}(-1)} \right\} \cdot 4 \cdot 100
\]

where GDP is gross domestic product price deflator.

For inflation at the time minus one, \(\pi(-1)\), we consider the realized inflation at the previous quarter.

For expected future inflation, \(E\pi(+1)\), we consider the realized inflation at the next quarter.

Change in unemployment (\(\Delta u\)): we define the change in unemployment using the percent standardised unemployment rate.

\[
\Delta u = u - u(-1)
\]

Percent change in the real price of oil (\(\Delta v\)): we define the percent change in the real price of oil as the annualized percent change in the PPI relative to the GDP price deflator.

\[
\Delta v = \left\{ \frac{\left\{ \frac{\text{PPI}}{\text{GDP}} - \left\{ \frac{\text{PPI}(-1)}{\text{GDP}(-1)} \right\} \right\}}{\left\{ \frac{\text{PPI}(-1)}{\text{GDP}(-1)} \right\}} \right\} \cdot 4 \cdot 100
\]

\(\Delta v\): annualized percent change in the real price of oil.

PPI: producer price index.

GDP: gross domestic product price deflator.

Short term interest rate (\(i\)): annualized short term interest rate.
Figure 1
Standard case
Figure 2

Standard case
Figure 3

Standard case

0 quarters after the unemployment shock

percent deviation

UNEMPLOYMENT SHOCK

inflation

unemployment

short term interest rate

real price of oil
Figure 4

Standard case
Figure 5

Taylor rule I

[Graphs showing the effects of oil price and monetary policy shocks on inflation, unemployment, and short-term interest rates in the standard case, and in the presence of strong or weak response to inflation.]
Figure 6
Taylor rule II
Figure 7

Mixed Taylor rules I and II

[Graphs showing responses to oil price shock and monetary policy shock]
Figure 8

Mixed Taylor rules I and II (comparison with the standard case)
Figure 9
Taylor rule III

OIL PRICE SHOCK

- Inflation in the standard case
- Inflation in the presence of strong response to real price of oil
- Inflation in the presence of weak response to real price of oil

- Unemployment in the standard case
- Unemployment in the presence of strong response to real price of oil
- Unemployment in the presence of weak response to real price of oil

- Short term interest rate in the standard case
- Short term interest rate in the presence of strong response to real price of oil
- Short term interest rate in the presence of weak response to real price of oil

MONETARY POLICY SHOCK

- Inflation in the standard case
- Inflation in the presence of strong response to real price of oil
- Inflation in the presence of weak response to real price of oil

- Unemployment in the standard case
- Unemployment in the presence of strong response to real price of oil
- Unemployment in the presence of weak response to real price of oil

- Short term interest rate in the standard case
- Short term interest rate in the presence of strong response to real price of oil
- Short term interest rate in the presence of weak response to real price of oil
Figure 10

Different inflation expectations

- Inflation expectations
  - OIL PRICE SHOCK
  - Inflation in the standard case
  - Inflation in the MFL case
  - Inflation in the PFL case

- Unemployment
  - OIL PRICE SHOCK
  - Unemployment in the standard case
  - Unemployment in the MFL case
  - Unemployment in the PFL case

- Short-term interest rate
  - OIL PRICE SHOCK
  - Short-term interest rate in the standard case
  - Short-term interest rate in the MFL case
  - Short-term interest rate in the PFL case

0
5
10
15
20
25
30
35
40

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0
0.5
1
0
5
10
15
20
25
30
35
40

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x 10^-4

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