ABSTRACT. In The Limits of Science N. Rescher introduces a logical argument known as the Knowability Paradox, according to which, if every true proposition is knowable, then every true proposition is known, i.e. if there are unknown truths, there are unknowable truths. Rescher argues that the Knowability Paradox, giving evidence to a limit of our knowledge (the existence of unknowable truths) could be used for arguing against perfected science. In this article we present two criticisms against Rescher’s argument.

1 Introduction

In The Limits of Science [4], N. Rescher introduces a logical argument known as the Knowability Paradox, according to which, if every true proposition is knowable, then every true proposition is known, i.e. if there are unknown truths, there are unknowable truths. Rescher argues that the paradox, giving evidence to a limit of our knowledge (the existence of unknowable truths) could be used for arguing against perfected science.

In this article we present two criticisms of Rescher’s argument. The first one points out that Rescher is ambiguous on the meaning of “impossibility of a perfected science”: it could be interpreted in at least two different ways, one of which is plainly unproblematic compared with the Knowability Paradox. In the second criticism we argue that the kind of unknowability involved in the paradox is semantic, rather than epistemic. Therefore, it is not a real problem for science. The final conclusion of the paper is, if our criticisms are correct, that the paradox leaves open the possibility of a perfected science.

The paper is divided into three parts. In the first one we give an account of the paradox and our reading of Rescher’s argument. In the second part we argue the first criticism. The third one concerns our second criticism. If our arguments are correct, Rescher’s conclusion, according to which the Knowability Paradox constitutes a problem for perfected science, is mistaken.
2 The Knowability Paradox and Rescher’s argument for the imperfectibility of science

N. Rescher, in The Limits of Science, argues that “perfected science is a mirage; complete knowledge a chimera”.

The above thesis is a consequence of the Knowability Paradox, a logical argument published by F. Fitch in an article entitled A Logical Analysis of Some Value Concepts. Fitch’s argument, starting from the assumption that every true proposition is knowable, reaches the strong conclusion that every true proposition is known or, in different terms: if there are unknown truths, there are unknowable truths. Prima facie, this argument seems to seriously narrow our epistemic possibilities and to constitute a limit for knowledge in general, for scientific knowledge in particular. The argument runs as follows: take the epistemic operator $K$, where $Kp$ stands for “someone knows that $p$” or “it is known that $p$”, and ‘$p$’ is a proposition in a formal language.

Assume the following two properties of knowledge:

1. the distributive property over conjunction (Dist), i.e. if a conjunction is known, then also its conjuncts are, and
2. the factivity of knowledge (Fact), i.e. if a proposition is known, then it is true.

Formally:

\[ K(p \land q) \vdash Kp \land Kq \]  \hspace{1cm} (Dist)
\[ Kp \vdash p \]  \hspace{1cm} (Fact)

Assume the following two unremarkable modal claims, which can be formulated using the usual modal operators $\diamond$ (which is read “it is possible that”) and $\Box$ (which is read “it is necessary that”). The first is the Rule of Necessitation:

\[ if \vdash p, then \Box p \]  \hspace{1cm} (Nec)

The second rule establishes the interdefinability of the modal concepts of necessity and possibility:

\[ \Box \neg p \equiv \neg \diamond p \]  \hspace{1cm} (ER)

Assume also the Knowability Principle according to which every true proposition is knowable, formally:

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1[4], p.150.
2[2]. For an introduction to the literature about the argument see [1].
3$Kp$ is commonly generalized at every subject and time: "someone knows at some time that $p$". For the purposes of our paper the choosed reading of $Kp$ is irrelevant.
∀q(q → ◇Kq) \hspace{1cm} (KP)

Finally, assume that we are not omniscient, i.e. there is at least a truth that is not known:

∃r(r ∧ ¬Kr) \hspace{1cm} (NO)

an instantiation of (NO) is:

(p ∧ ¬Kp) \hspace{1cm} (2)

Consider an example of (KP) resulting by the substitution of q with (2):

((p ∧ ¬Kp) → ◇K(p ∧ ¬Kp)) \hspace{1cm} (3)

By (2) and (3) we obtain:

◇K(p ∧ ¬Kp) \hspace{1cm} (4)

Consider the following argument “per absurdum” (independent from (2)-(4)):

(5) \hspace{1cm} K(p ∧ ¬Kp) \hspace{1cm} [assumption]

(6) \hspace{1cm} Kp ∧ K¬Kp \hspace{1cm} [by (5) and (Dist)]

(7) \hspace{1cm} Kp ∧ ¬Kp \hspace{1cm} [applying (Fact) to (6)]

(8) \hspace{1cm} ¬K(p ∧ ¬Kp) \hspace{1cm} [by (5)-(7), refusing (5) for the inconsistency of (7)]

(9) \hspace{1cm} □¬K(p ∧ ¬Kp) \hspace{1cm} [by (8) and (Nec)]

(10) \hspace{1cm} ¬◇K(p ∧ ¬Kp) \hspace{1cm} [by (9) and (ER)]

(10) is inconsistent with (4)\(^4\). If so, (NO) and (KP) are incompatible. One of the two assumptions must be abandoned. The advocate of the view that all truths are knowable must negate (NO):

¬∃r(r ∧ ¬Kr) \hspace{1cm} (Not-NO)

according (Not-NO) there are not unknown truths, i.e. every truth is known:

\(^4\)Here we have taken the freedom of substituting the argument as it was originally proposed by Rescher with the equivalent, clearer and commonly used formulation. See, for instance, [1].
\[ \forall r(r \rightarrow Kr) \]  \hspace{1cm} \text{(Not-NO*)}

Otherwise, one must negate (KP):
\[ \neg \forall q(q \rightarrow \diamond Kq) \]  \hspace{1cm} \text{(Not-KP)}

obtaining that there are unknowable truths:
\[ \exists q(q \land \neg \diamond Kq) \]  \hspace{1cm} \text{(Not-KP*)}

"This argumentation shows that in the presence of (relatively unproblematic) principles [(Dist)-(Fact)], the thesis that all truths are knowable [(KP)] entails that all truths are known, that is, [(Not-NO*)]. Since the latter thesis is clearly unacceptable, the former must be rejected. We must concede that some truths are unknowable: \[ \exists q(q \land \neg \diamond Kq) \]" [4], p.150.

One of the incredible results obtained using the paradox is that from:
\[ \forall q(q \rightarrow \diamond Kq) \]  \hspace{1cm} \text{(KP)}

and:
\[ \forall r(r \rightarrow Kr) \]  \hspace{1cm} \text{(Not-NO*)}

we obtain the theorem:
\[ \vdash \forall q(q \rightarrow \diamond Kq) \leftrightarrow \forall q(q \rightarrow Kq). \] \hspace{1cm} \text{(T1)}

In fact, it is easy to show that (Not-NO*) entails (KP), a proof needing only the modal principle that what is actual is possible. If we combine the above result with one of the results of Fitch’s paradox – (KP) entails (Not-NO*) – we obtain (T1).

Rescher points out that “No doubt this sort of argumentation for the incompleteness of knowledge is too abstract and ‘general principly’ to carry much conviction in itself. But it does provide some suggestive stage setting for the more concrete rationale of the imperfectibility of science” [4], p.150].

Rescher’s argument for the imperfectibility of science could be analysed in the following way:

if the \textit{Knowability Paradox} holds, then there are unknowable truths (Ass.)  \hspace{1cm} \text{(R1)}

\[^{5}\text{For a formal proof of } \exists q(q \land \neg \diamond Kq), \text{ see [5].}\]
\[^{6}\text{Routley in [5] considers in a more serious way the paradox, as an authentic limitation for knowledge in general. For articles related to Rescher’s one see [6]and [7].}\]
the _Knowability Paradox_ holds (Ass.) \hspace{1cm} (R2)
there are unknowable truths (Conclusion I) \hspace{1cm} (R3)

there are unknowable truths (Conclusion I) \hspace{1cm} (R3)
if there are unknowable truths, then perfected science is impossible (Ass.) \hspace{1cm} (R4)
perfected science is impossible (Conclusion II) \hspace{1cm} (R5)

Rescher’s argument (as it is here reformulated) is based on three different premises, (R1), (R2) and (R4). Here we are not interested in how correct (R1)-(R3) is: we just assume that Fitch’s argument is sound.\(^7\) Is Rescher’s second part of the argument (R3)-(R5) correct?

### 3 First criticism: (R4) is ambiguous

A first problem of Rescher’s argument is that (R4):

If there are unknowable truths, then perfected science is impossible (Ass.) \hspace{1cm} (R4)

is ambiguous. In particular, there are at least two meanings of “imperfectibility of science” – where the expression is here considered as equivalent to “impossibility of a perfected science” – and one of them is plainly unproblematic compared with the paradoxical conclusion.

Consider (R4): if the existence of unknowable truths is a problem for the perfectibility of science, it seems reasonable to think that a perfected science is equivalent or at least implies omniscience. The following is Rescher’s train of thought: if there are unknowable truths, omniscience is impossible, and a perfected science is impossible too.

Here an ambiguity rises. What does it mean that “omniscience is impossible”? We should read it as:

It is impossible that every true proposition is known \hspace{1cm} (IO1)

Formally:

\[ \neg\Diamond\forall q (q \rightarrow Kq) \] \hspace{1cm} (IO1)

But we should also read “omniscience is impossible” as:

\(^7\)There is a long list of criticisms to the paradox. For an introduction to the main literature see [1].
not every true proposition is knowable. \hspace{1cm} (IO2)

Formally:

$$\neg \forall q (q \rightarrow \diamond Kq)$$ \hspace{1cm} (IO2)

Are (IO1) and (IO2) both implied by the paradox? If there are unknown truths, the result of the paradox (according to Rescher) is the negation of the Knowability Principle: (Not-KP) $$\neg \forall q (q \rightarrow \diamond Kq)$$ and (Not-KP) is (IO2). So, (IO2) is the proper conclusion of the paradox.

What about (IO1)? One way of reasoning could be to argue that (IO2) entails (IO1) and – due to the fact that (IO2) is the proper conclusion of the paradox – arguing that the paradox entails (IO1) too. Problem: does (IO2) entail (IO1)? Assume (IO2):

(1) $$\neg \forall q (q \rightarrow \diamond Kq)$$

(2) $$\exists q (q \land \square \neg Kq)$$ \hspace{1cm} [from (1) by (ER)]

(3) $$(r \land \square \neg Kr)$$ \hspace{1cm} [Hypothesis]

(4) $$r$$ \hspace{1cm} [by $$\land$$ Elimination 3]

(5) $$\square \neg Kr$$ \hspace{1cm} [by $$\land$$ Elimination 3]

(6) $$\neg Kr$$ \hspace{1cm} [by $$\square$$ Elimination 5]

(7) $$(r \land \neg Kr)$$ \hspace{1cm} [by $$\land$$ Introduction 4, 7]

(8) $$\exists q (q \land \neg Kq)$$ \hspace{1cm} [by $$\exists$$ introduction 7]

(9) $$\exists q (q \land \neg Kq)$$ \hspace{1cm} [by $$\exists$$ elimination 3]

(10) $$($$ $$\exists q (q \land \square \neg Kq) \rightarrow \exists q (q \land \neg Kq))$$ \hspace{1cm} [by $$\rightarrow$$ introduction 2, 9]

(10) is a theorem. So, by (NEC) we obtain:

$$\square (\exists q (q \land \square \neg Kq) \rightarrow \exists q (q \land \neg Kq)) \hspace{1cm} (11)$$

and by the distribution of the necessity operator ($$\square (A \rightarrow B) \rightarrow (\square A \rightarrow \square B)$$):

$$\square \exists q (q \land \square \neg Kq) \rightarrow \square \exists q (q \land \neg Kq) \hspace{1cm} (12)$$
(12) is equivalent to:
\[ \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \Box \neg \forall q (q \rightarrow Kq). \] 
\( \text{(T1*)} \)

i.e.:
\[ \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \neg \Diamond \forall q (q \rightarrow Kq). \] 
\( \text{(T1*)} \)

The above argument shows that a necessitated version of (IO2) \( \neg \forall q (q \rightarrow \Diamond Kq) \) entails (IO1) \( \neg \Diamond \forall q (q \rightarrow Kq) \). The same result is obtained if we consider that if the Fitch’s paradox holds, then:
\[ \vdash \forall q (q \rightarrow \Diamond Kq) \iff \forall q (q \rightarrow Kq) \] 
\( \text{(T1)} \)

(T1) is logically equivalent to:\(^8\):
\[ \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \iff \neg \Diamond \forall q (q \rightarrow Kq) \] 
\( \text{(T1**)} \)

The left to right side of (T1**) is:
\[ \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \neg \Diamond \forall q (q \rightarrow Kq) \] 
\( \text{(T1*)} \)

again, the result is that a necessitated version of (IO2) implies (IO1).

On the contrary, a contingent version of (IO2) implies that (IO1) is false. In fact, if:
\[ \neg \forall q (q \rightarrow \Diamond Kq) \] 
\( \text{(IO2)} \)

is contingent it is possibly false. Then:
\[ \Diamond \forall q (q \rightarrow \Diamond Kq) \] 
\( \text{(IO2C)} \)

But, by (T1) and the modal rule \( \langle \Diamond A, \Box (A \rightarrow B) \vdash \Diamond B \rangle \):
\[ \Diamond \forall q (q \rightarrow Kq) \] 
\( \text{(Not-IO1)} \)

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\(^8\) In details:
(T1) \( \vdash \forall q (q \rightarrow \Diamond Kq) \rightarrow \forall q (q \rightarrow Kq). \) using the usual rules of propositional logic, is equivalent to:
(T1’) \( \vdash \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \neg \forall q (q \rightarrow Kq). \) By applying the Rule of Necessitation to (T1) we obtain:
(T1’’) \( \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \neg \forall q (q \rightarrow Kq). \) By distribution of the necessity operator \( \langle \Box (A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B) \rangle \):
(T1’’’) \( \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \Box \neg \forall q (q \rightarrow Kq) \) and by (ER):
(T1**) \( \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \rightarrow \neg \Diamond \forall q (q \rightarrow Kq) \)
If (IO2) is contingent, then (IO1) is false. So, given the consistency of the system, (IO2) does not imply (IO1).

Reconsider the starting question: does (IO2) entail (IO1)? If (IO2) is necessary then (IO1); instead, if (IO2) is contingent it does not imply (IO1). Now, the answer to the above question lies in the modal status of (IO2).

If (IO2) is necessary, the starting assumption of the paradox, the non-omniscience thesis (NO) \( \exists q (q \land \neg Kq) \) is necessary too. In fact (T1) is equivalent to:

\[ \vdash \neg \forall q (q \rightarrow \Diamond Kq) \iff \exists q (q \land \neg Kq). \tag{T1} \]

Given the the distribution of the necessity operator (\( \Box (A \iff B) \rightarrow (\Box A \iff \Box B) \)) we obtain:

\[ \vdash \Box \neg \forall q (q \rightarrow \Diamond Kq) \iff \Box \exists q (q \land \neg Kq) \tag{T1***} \]

Therefore, the necessity of (IO2) coimplies the necessity of (NO):

\[ \Box \exists q (q \land \neg Kq) \tag{Nec-NO} \]

and, vice versa, by (T1***) and the usual rules of propositional logic:

\[ \vdash \neg \Box \neg \forall q (q \rightarrow \Diamond Kq) \iff \neg \Box \exists q (q \land \neg Kq) \tag{T1***} \]

(IO2) is contingent iff (NO) is contingent.

Notice that (NO) \( \exists r (r \land \neg Kr) \) - the non-omniscience thesis - is the result of a commonsense observation according to which \textit{de facto} actually there are true propositions that we do not know. It is not a logical principle of the paradox, nor it is introduced through a logical argument.\(^9\)

If it is so, its necessitation does not seem to be plausible. Given the contingency of (NO), as showed, (IO2) is contingent too, and it implies (Not-IO1) \( \Diamond \forall q (q \rightarrow Kq) \). So, accepting the contingency of (NO), (IO2) does not entail (IO1). But, if Fitch’s paradox implies (IO2) and (IO2) does not imply (IO1), Fitch’s paradox does not imply (IO1).

To conclude: given the two readings of “omniscience is impossible”, there are two corresponding reading of “imperfectibility of science”:

Assuming (IO1) \( \neg \Diamond \forall q (q \rightarrow Kq) \) (it is logically impossible that every truth is known), imperfectibility of science is equivalent to the logical impossibility of a perfected science.

Assuming (IO2) \( \neg \forall q (q \rightarrow \Diamond Kq) \) (Not every truth is knowable), and the actual existence of unknown truths, imperfectibility of science is equivalent to the actual unrealizability of a perfected

\(^9\)As C. Wright writes in [8], (NO) says just that p is true and not actually known.


The *Knowability Paradox* is effective only for the second reading (2nd Reading). It is not an argument for the first one (1st Reading). If it is so, Rescher’s premise (R4):

\[(R4) \text{ if there are unknowable truths, then perfected science is impossible}\]

is ambiguous.

Rescher does not seem to be aware of the above specified distinction, and for this reason he falls in the mentioned ambiguity: the *Knowability Paradox* is an argument for (IO2) and the actual unrealizability of a perfected science, but it is not an argument for (IO1) and the logical impossibility of a perfected science.

4 A second criticism: an incorrectness in Rescher’s argument

A second problem of Rescher’s argument is that, given

\[\text{there are unknowable truths (Conclusion I)} \quad (R3)\]

and

\[\text{if there are unknowable truths, then perfected science is impossible (Ass.)} \quad (R4)\]

the conclusion

\[\text{perfected science is impossible (Conclusion II)} \quad (R5)\]

is misleading.

The mistake is due to the fact that Rescher does not take care to the special *status* of the propositions that lead to the paradox. In particular, in the paradox:

\[\forall q (q \rightarrow \Diamond Kq) \quad (KP)\]

is inconsistent with (NO). The unknowable propositions, as the *reductio* (5)-(8) in the paradox showed, are instances of (NO): e.g. (2) \((p \wedge \neg Kp)\). But why is (KP) incompatible with propositions such as (2)?

\footnote{On the other side, if perfected science is possible then omniscience is possible, and it is possible that every truth is known (formally: \(\Diamond \forall q (q \rightarrow Kq)\).}
First of all, let us distinguish between two different kinds of unknowability. A true proposition could be unknowable because of some epistemic limits. Take, for example, Heisenberg’s indetermination principle: according to a certain interpretation of it, the principle seems to give an ineliminable epistemic limit to human knowledge. On the other hand, a different kind of unknowability is just based on semantic considerations: the unknowability of a proposition could result just from its meaning. In the last case there are no effective limits to our (scientific) knowledge.

Consider the proposition:

\[
\text{Perfected science is unrealized} \quad (S)
\]

\(S\) jeopardizes the realization of perfected science: if it is true, then it is false that perfected science is realized. But the reason of such unrealizability is not ascribable to an epistemic limit. It is simply a semantic consequence of the logical law that a proposition is incompatible with its negation (the law of non-contradiction): \((S)\) is incompatible with

\[\text{“perfected science is realized”}. \quad (S^*)\]

Propositions that lead the paradox emerge, and have the logical form (2), are of the same kind, like the conjunction \((S)\) and \((S^*)\). This semantical phenomenon has been studied in the literature and some authors have called this kind of propositions “blindspots”\(^{11}\): \((p \land \neg Kp)\) is unknowable just because it is a conjunction of two propositions, \(p\) and \(\neg Kp\), and the knowledge of the first conjunct implies the falsity of the second one just for semantical reasons: “it is known that \(p\)” is trivially incompatible with “it is not known that \(p\)”.

Notice that the paradox does not concern the knowability of each conjunct in (2). Each of them is independently knowable, whereas their contemporary knowledge is impossible, for the semantic reason showed. If it is so, the problem of the paradox is strictly semantic, not epistemic: it does not concern any specific area of science or, more generally, human epistemic skills.

In light of the above explanation, Rescher’s unproblematic acceptance of (R4):

\[
\text{if there are unknowable truths, then perfected science is impossible} \quad (R4)
\]

is mistaken. The unknowability problematic for science, referred to by Rescher in the antecedent of (R4), is an epistemic one: given our epistemic limits perfected science is impossible. On the other hand, the unknowability resulting in the paradox, assumed in (R3), is a semantical one. So, the conclusion of the paradox is not the intended premise of (R4), and from (R3) and (R4) we can not infer (R5):

\(^{11}\text{See, for instance, [7] and [3].}\)
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Perfected science is impossible (Conclusion II) (R5)

5 Conclusion

To conclude: if our criticisms are satisfactory, Rescher’s argument according to which the Knowability Paradox constitutes a limit for perfected science, is ambiguous and mistaken. Specifically, the Knowability Paradox can not be used - as Rescher has - as an argument for the imperfectibility of science. The final conclusion of our paper is that the paradox leaves open the possibility of a perfected science. 12

BIBLIOGRAPHY


12 An early version of this paper has been read at the “SILFS 2007 Conference”. We are indebted to the participants at the conference for the stimulating discussion. Special thanks to Pierdaniele Giaretta for having improved previous versions of our paper. We also thank Silvia Gaio, Enrico Martino, Vittorio Morato, and Marzia Soavi for their detailed comments.