The Knowability Paradox, Perfectibility of Science and Reductionism

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Abstract

A logical argument known as *Fitch's Paradox of Knowability* seems to prove that there are unknowable truths. This claim seems incompatible with a certain view in epistemology and philosophy of science according to which, at least in principle, a perfected science – considered as an ideal of omniscience and of completeness for scientific knowledge – is possible. In the present paper we give a possible way out from the paradoxical conclusion. Our proposal is based on an emended version of the type-solution suggested by Linsky (2008): it solves his problem of *ad-hoc*ness linking types to kinds of justifications, giving in this way a reason for the introduction of types in the case of knowledge. Noteworthy, our proposal strictly depends on the degree of reductionism adopted in science: in particular, a complete reductionist stance in science is not compatible with the suggested solution.

Introduction

A logical argument known as *Fitch's Paradox of Knowability*,[1] starting from the assumption that every truth is knowable, leads to the consequence that every truth is also actually known. Then, given the ordinary fact that some true propositions are not actually known, it concludes, by *modus tollens*, that there are unknowable truths. The main literature on the topic has been focusing on the threat the argument poses to the so called semantic anti-realist theories, which aim to epistemically characterize the notion of truth; according to those theories, every true proposition must be knowable. But the paradox seems to be a problem also for epistemology and philosophy of science: the conclusion of the paradox – the claim that there are unknowable truths – seems to seriously narrow our epistemic possibilities and to constitute a limit for knowledge. This fact contrasts with certain views in philosophy of science according to which every scientific truth is in principle knowable and, at least at an ideal level, a perfected, "all-embracing", omniscient science is possible.

The main strategies proposed in order to avoid the paradoxical conclusion, given their effectiveness, are able to address only semantic problems, not epistemological ones. However, recently Bernard Linsky (2008) proposed a solution to the paradox that seems to be effective also for the epistemological problems. In particular, he suggested a possible way to block the argument employing a type-distinction of knowledge.

In the present paper, firstly, we introduce the paradox and the threat it represents for a certain views in epistemology and philosophy of science;

[1] *Fitch's Paradox of Knowability* has been called also *anonymous' argument*, *Fitch's theorem 5*, *Church-Fitch's Paradox* or simply, *Knowability Paradox*. For a detailed historical account see Salerno (2008).

secondly, we show Linsky's solution; thirdly, we argue that this solution, in order to be effective, needs a certain kind of justification, and we suggest a way of justifying it in the scientific field; fourthly, we show that the effectiveness of our proposal depends on the degree of reductionism adopted in science: it is available only if we do not adopt a complete reductionism.

I. The Knowability Paradox and the Limits of Science

The *Knowability Paradox* is a logical argument published by F. Fitch in an article entitled *A Logical Analysis of Some Value Concepts* in 1963. The argument has the following structure. Take the epistemic operator K, where "Kp" stands for "someone knows that p" or "it is known that p", and "p" is a proposition in a formal language.

Assume the following two properties of knowledge:

1. the distributivity over conjunction: if a conjunction is known, then its conjuncts are also known. Formally:

Dist) $K(p \wedge q) \vdash Kp \wedge Kq$

2. the factivity of knowledge: if a proposition is known, then it is true. Formally:

Fact) $Kp \vdash p$

Assume the following two uncontroversial modal claims, formulated using the modal operators $\Diamond$ (read "it is possible that") and $\Box$ (read "it is necessary that"). The first is the *Rule of Necessitation*:

Nec) if $\vdash p$, then $\Box p$

The second rule establishes the interdefinability of the modal concepts of necessity and possibility:

ER) $\Box\neg p \vdash \neg\Diamond p$

Assume also the Knowability Principle according to which every true proposition is knowable, formally:

KP) $\forall q\ (q \rightarrow \Diamond Kq)$

Finally, assume that no one is omniscient, i.e. there is at least a truth that is not known (by anyone):

NO) $\exists r\ (r \wedge \neg Kr)$

An instantiation of (NO) is:

2) $(p \wedge \neg Kp)$

Consider an example of (KP) resulting from the substitution of q in (KP) with (2):

3) $(p \wedge \neg Kp) \rightarrow \Diamond K(p \wedge \neg Kp)$

By (2) and (3) we obtain:

4) $\Diamond K(p \wedge \neg Kp)$

Consider the following argument "per absurdum" (independent from (2)-(4)):

5) $K(p \wedge \neg Kp)$ assumption

6) $Kp \wedge K\neg Kp$ by (5) and (Dist)

7) $Kp \wedge \neg Kp$ applying (Fact) to (6)

8) $\neg K(p \wedge \neg Kp)$ by (5)-(7), from the inconsistency of (7)

9) $\Box\neg K(p \wedge \neg Kp)$ by (8) and (Nec)

10) $\neg\Diamond K(p \wedge \neg Kp)$ by (9) and (ER)

(10) is inconsistent with (4).[2] If so, (NO) and (KP) are incompatible. One of them must be abandoned.

The advocate of the view that all truths are knowable must deny (NO):

Not-NO) $\neg\exists r\ (r \wedge \neg Kr)$

According to (Not-NO) there are not unknown truths, i.e. every truth is known:

Not-NO*) $\forall q\ (q \rightarrow Kq)$

Otherwise, one must deny (KP):

Not-KP) $\neg\forall q\ (q \rightarrow \Diamond Kq)$

[2] There are different formulations of the paradox. Here we follow Broogard and Salerno (2002).

obtaining that there are unknowable truths:

Not-KP*) $\exists q (q \wedge \neg\Diamond Kq)$

Putting Fitch's paradox conclusion in an informal way, if there are unknown truths (NO), there are unknowable truths (Not-KP*). It seems an evident and ordinary fact that we are not omniscient beings and that some (probably many) truths are not actually known. Given this assumption, the paradox forces us to conclude that there are unknowable truths.

As noted in the introduction, the conclusion of the *Knowability Paradox* has been considered a problem at least for two philosophical stances: 1) the semantic anti-realist theories. Those theories aim to epistemically characterize the notion of truth, and the minimal requirement for such characterization is that every truth must be knowable. 2) Certain perspectives in philosophy of science, according to which every scientific proposition is, at least in principle, knowable. Consider, for example, the view that Shapiro (1993) called *Gödelian Optimism*, that refers to some positions expressed by D. Hilbert and K. Gödel: Hilbert argues for the solvability of every mathematical problem on the basis of methodological considerations,[3] Gödel argues for a rationalistic optimism on the basis of aesthetic considerations.[4] Another well-known position in philosophy of science threatened by Fitch's Paradox argues for the possibility of reaching perfected science, considered as an ideal of omniscience and of

[3] «This convinction of the solvability of every mathematical problem is a powerful incentive for the worker. We hear the perpetual call: There is a problem. Seek its solution. You can find it… for in mathematics there is no ignorabimus». From Hilbert's "Mathematical Problems" lecture, 1999.

[4] «Those parts of mathematics which have been systematically and completely developed… show an amazing degree of beauty and perfection. In those fields, by entirely unsuspected laws and procedures …means are provided… for solving all relevant problems». In Wang (1974), pp. 324-325.

completeness of scientific knowledge:[5] the paradox shows the existence of unknowable truths, and this is incompatible with the realization of such an ideal. Nicholas Rescher, in *The Limits of Science* (1984), pointed out explicitly the issue, arguing that, in consequence of Fitch's paradox:

> «Perfected science is a mirage; complete knowledge a chimera. [...] No doubt this sort of argumentation for the incompleteness of knowledge is too abstract [...] to carry much convinction in itself. But it does provide some suggestive stage setting for the more concrete rationale of the imperfectibility of science»[6].

The semantic and the epistemological problems raise different sorts of issues, which should be approached in different ways. In particular, two main strategies of solution have been proposed against the semantic problem, in defence of the anti-realist point of view. The first strategy put a restriction on the required *principle of knowability* (KP) bounding it to a range of unproblematic propositions (propositions without the form of (2) ($p \wedge \neg Kp$), responsible of the paradox) (*restriction strategy*).[7] The second strategy challenges the logic underlying the argument (classical logic) arguing that there are no problems using alternative logics to classical logic (intuitionistic or paraconsistent logics) (*revision strategy*).[8] Unfortunately, both the strategies are useless against the epistemological problem. In fact, 1) the adoption of a different restricted principle of knowablity cannot cancel the epistemologically problematic claim that there are unknowable truths resulting from the paradox. 2) In epistemology and in science

[5] See, for example, Peirce 1935, pag. 139.
[6] Rescher (1984), p. 150. An anologous position is that of Routley (1981). He considers the paradox a strong proof of the limits of knowledge. For articles related to Rescher's one see also Schlesinger (1986) and Zemach (1987).
[7] Well known restrictions are those of N. Tennant (1997), M. Dummett (2001) and D. Edgington (1985).
[8] See, for example, T. Williamson (1982).

classical logic is generally admitted and there are no particular reasons for a logical revision. Therefore, also if the solutions would be effective for the semantic problem, the epistemological one would remain open. To solve it, a different strategy is required.

II. A type theory-based solution of the paradox and the scientific knowledge case

A proposal of solution for the paradox, able to address also the epistemological problem, has been recently developed by Bernard Linsky (2008).[9] He offers a solution based on a type-distinction. Though the *Knowability Paradox* makes no use of self-referential sentences, it is nevertheless invalid on a typed account of knowledge.[10] The account rests on the following two basic rules:

(1) If φ has no occurrences of K, φ is of type 0 (φ_0);

(2) If φ is of type n, then Kφ is of type $n + 1$ ($K_{n+1}\varphi_n$).

A further rule must be added to the basic ones for the typing of complex propositions:

(3) If ψ is a complex proposition, and the atomic proposition included in ψ with the maximum type is of type n, then ψ is of type n.[11]

[9] The type solution of Fitch's argument was already suggested by A. Church in 1945. A. Paseau has independently suggested it in a forthcoming article (2008).
[10] For a logical account of the type-theory see Russell (1908).
[11] Linsky has forgot this further rule, but it is necessary in the case of the knowability paradox, where, at step (5), there is knowledge of a conjunction, (2) ($p \wedge \neg Kp$).

The intuitive idea is that types reflect the number of occurrences of K within each formula. On this view the *Knowability Paradox* can be blocked. In fact, consider the following example. Let p be of type 0. Then, steps (5) – (7) run as follows:

5*) $K_2(p_0 \wedge \neg K_1 p_0)$ assumption

6*) $K_2 p_0 \wedge K_2 \neg K_1 p_0$ by (5*) and (Dist)

7*) $K_2 p_0 \wedge \neg K_1 p_0$ applying (Fact) to (6*)

Unless higher types collapses in lower ones (i.e. in steps (5*) – (7*), K_2 implies K_1), line (7*) is not a contradiction. There is no incoherence in not knowing p at the lower level and knowing it at the higher level. So, if (7*) is consistent, (5*) can be maintained, and (10) $\neg\Diamond K(p \wedge \neg Kp)$, the contradictory of (4), cannot be derived. Hence, both the assumptions (KP) and (NO) can be (consistently) maintained.

The proposed solution, if effective, seems to adequately address the epistemological problem. In fact, unlike the solutions based on a logical revision, the type-solution works also with classical logic, and unlike the restriction strategy (that avoids the problems bounding the *Knowability Principle* to a range of unproblematic propositions by the exclusion of propositions with the form of (2) $(p \wedge \neg Kp)$) the type-solution gives an account of the knowledge of every proposition (also if at different type-levels), included the problematic ones, as the steps (5*) – (7*) show.

But, at this stage, two questions arise:

1) Is the introduction of types suitably justified in the case of knowledge?

2) What prevents the collapse of higher order types into lower order ones?

In the next section (III) we are going to answer the first question. In section IV we will consider the second one.

III. How to justify the introduction of types?

Why should we distinguish different types of knowledge? Without a good justification it seems that the indiscriminate introduction of a type-distinction of knowledge would not avoid some standard criticisms to other solutions of the *Knowability Paradox* e.g. of being *ad hoc*.[12] In response to the above question one could argue that the type-solution of Fitch's Paradox is just the application of a formal tool (a type-distinction) to a logical argument and that, in similar cases, the introduction of types does not require further justifications. However, in our opinion, the introduction of types alone – without further justifications – is artificial and rather counterintuitive: for example, if we indiscriminately apply the type-distinction to knowledge, the meaning of this term seems to change from type to type and to become ambiguous. Take the Socratic saying: «The only true wisdom is in knowing you know nothing». Adopting the type-distinction, the two occurrences of the verb "to know" in the sentence, having different types, should be used in different contexts and so they would have different meanings. This fact contrasts with our commonsensical intuitions pushing for a single, non-ambiguous, reading of the meaning of "to know" in the above sentence.

[12] *Ad-Hoc*ness is the common criticism to restriction strategies. See, for instance, Tennant's restriction (Tennant 1997) and the criticism of *ad-hoc*ness in (Hand & Kvanvig 1999).

One could reply that the type-solution is not only effective for this paradox, but for a broad range of logical paradoxes, including many epistemic ones like, for example, the *Knower Paradox* and the *Paradox of the Preface*.[13] The train of thought is the following one: if all these paradoxes are threatened by inconsistency, all can be cured by the same antidote. The answer to this reply (let's call it the *analogical argument*) is that even if all these paradoxes can be solved with the same logical tool, it does not mean that the use of the tool is principled. As Linsky himself observed: «determining whether certain arguments are valid in intensional logic with logical types is distinct from the general issue of the appropriateness of using type-theory to resolve paradoxes in philosophical logic» [Linsky 2008, p. 8]. Moreover, it is important to distinguish the application of type theory to paradoxes involving self-reference from the application of type theory to paradoxes that do not involve self-reference. The type-solution is usually adopted to solve problems involving self-reference, but the *Knowability Paradox* does not rely on any sort of self-reference. Therefore the *analogical argument* seems to be weakened by some important differences between this paradox and the others.

Given that, in order to avoid the criticism of ad-hocness, a justification of type-introduction is needed one could try to justify it on the basis of what is known – the content of knowledge. In particular, if type-levels reflect the number of occurrences of the K-operator within each formula, it seems plausible to distinguish types following the distinction between epistemic and non-epistemic contents of the known propositions.

But this is not a good idea. In fact, (i) such a move faces the same problems shown before: the distinction carries an ambiguity on the otherwise unambiguous notion of knowledge; (ii) the distinction of type-levels on the ground of the epistemic (or non-epistemic) content alone of the

[13] On this see Linsky (2008) and Halbach (2008), pp. 114-117.

known proposition is not *per se* a good reason for justifying a type-solution: the distinction between epistemic and non-epistemic propositions is affected by some problems. For example, the obtaining of some non-epistemic state of affairs concerning agent's properties could have effects also on his epistemic state. "John is lying down in the bed" is not an epistemic proposition, it does not involve any occurrence of the verb *to know*, but it prevents John from knowing what is happening in the kitchen now. Although the proposition "John knows what happens in the kitchen now" possesses an epistemic content and therefore it is epistemic, its truth-value depends on the truth or falsity of the non-epistemic proposition "John is lying down in the bed".[14] Conversely, every agent's change of epistemic conditions affects also his psychophysical state and has consequences on the agent's body and on the environment. Furthermore, it is also a debated matter if propositions about ignorance – as that in the Socratic saying or proposition (2) ($p \wedge \neg Kp$) in the paradox – can be considered epistemic.[15] For those reasons, a distinction between epistemic and non epistemic propositions seems not a good way for escaping the problems of a type-distinction in knowledge's case.

The alternative to the content-based distinction is to consider the same nature of knowledge as including some feature able to justify the introduction of types. In particular, we should firstly find a reason for distinguishing different kinds of knowledge through some substantial feature of it, and, secondly, link such kinds of knowledge to specific type-levels. This link should be able to give a "substantialization" of the typification of knowledge, providing in this way the justification of the

[14] The example follows one Rosenkranz proposed in (Rosenkranz 2003), pp. 354-356. Despite Rosenkranz's criticism has other purposes than those concerned in this paper, the criticism is whatever valid in the present case.

[15] Another problem is that if we distinguish types on the base of the epistemic and non-epistemic content of the K-operator, the distinction of types is effective only between type-levels 1 and 2; in fact, every level higher that 1 has under its scope an epistemic proposition and the distinction is no more available.

type-distinction that we are searching for. For example, knowledge requires that there are knowing subjects. So, it is possible to distinguish kinds of knowledge by its respective subjects, and then to link each subject to a type-level. This move could motivate the introduction of types in knowledge's case giving the needed "substantialization" of them. However, it does not work in the case of the *Knowability Paradox*, where the reading of K, "someone at some time knows that", generalizes over subjects preventing the type-distinction.[16]

In our opinion, a better proposal in this direction is to distinguish kinds of knowledge through different kinds of justification, and then to link each kind to a type-level. For example, consider two kinds of knowledge distinguishable for their respective kind of justification: knowledge by acquaintance and knowledge by testimony. If we link those two kinds of knowledge to type-levels of K (K_{ac} for knowledge obtained by acquaintance and K_{te} for knowledge by testimony) we could coherently claim: (7^j) ($K_{te}p \wedge \neg K_{ac}p$). In fact, we could know by testimony of a very reliable person that p without, at the same time, having knowledge by acquaintance that p: that's not contradictory. The example shows that, assuming that different kinds of knowledge can be distinguished through different kinds of justification, and linking each type-level of K to a different kind of knowledge, the type-solution can be used and Fitch's argument blocked at step (7).

This proposal can be perfectly adapted to the case of scientific knowledge: in fact scientific knowledge can be differentiated in specific scientific fields (e.g. physics, biology, chemistry…) and those fields are distinguishable between them by their proper kinds of justification: a specific, well defined, kind of justification corresponds to each specific scientific field. In the case of the *Knowability Paradox* we obtain that the two types of K at step (7*) ($K_2p \wedge \neg K_1p$) can be distinguished through their

[16] Notice that the same problem follows if we identify different types with different times.

different kinds of scientific justification, and hence (7*) is consistent, and Fitch's argument blocked. Therefore, given the above interpretation of types, a solution to the paradox based on a type-distinction in scientific knowledge's case seems to be effective. [17]

IV. Is the given solution compatible with reductionism? The price to pay

Our suggested proposal can be summarized as follows: the *Knowability Paradox* arises only in a "context-free" situation (where the type-distinction is not allowed). On the contrary, each specific scientific field is conceived as closed under a specific epistemic context that is dependent on some peculiar kind of justification. If we accept such contextual dependence for scientific knowledge, the type-distinction seems to be justified, it can be used for blocking the *Knowability Paradox* and preventing Rescher's conclusion for the imperfectibility of science.

Therefore, in the last section we have suggested a possible answer to the first question concerning the justification of a type-introduction in the case of knowledge: the type-introduction can be done on the base of an effective distinction between knowledge's justifications. Now we should square with the second question: what in the suggested case does prevent the collapse of higher types into lower ones?

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[17] A clarification: a solution based on such interpretation of the type-distinction is effective only on condition of a further premise. Let we call it the *double epistemic access* requirement: we need that there are at least two different epistemic accesses (two different kinds of justification) for each proposition. In fact, if we have a proposition q such that it can be known through only one kind of justification (say K_1), we have that the proposition $(q \wedge \neg K_1 q)$ cannot be known, because we could not know the conjunction $(q \wedge \neg K_1 q)$ neither through a different kind of justification (for the assumption that there is only an epistemic access to q), nor through the same kind of justification (otherwise the paradox regains), and the type-solution is not available in this case. So, our argument is based on the assumption that there are at least two different epistemic accesses to each scientific proposition.

To answer the above question we start observing that in the given interpretation of types, they are identified with kinds of justification. In this context, the collapse of a higher type into a lower one is equivalent to the reduction of a kind of justification into another one. So, the question about the possibility of a type-collapse can be turned into the question about the possibility of a reduction between kinds of scientific justification, i.e. a reduction between scientific fields. However the answer to the above questions is not easy, because the mentioned reduction is nothing but the debated issue between reductionists and anti-reductionists in science. Hence, in order to prevent the collapse, and consequently the conclusion of Fitch's argument, we must adopt a non-reductionist position in science according to which kinds of scientific justification are independent and irreducible to each other.

For clarifying the point at issue, consider the following example:

(7*) $(K_2p \wedge \neg K_1p)$,

'K_1' stands for *physical knowledge*, and 'K_2' stands for *biological knowledge*. (7*) says that we acquire biological knowledge that p but we do not acquire physical knowledge that p, i.e. we have good biological justifications for believing that p but not good physical justifications for believing that p. All that seems, *prima facie*, sound (and (7*), in this specific case, consistent). The problem is that, if physical and biological justifications are not independent of each other, but one of them is reducible to the other, a knowledge-level (say biological knowledge) is reducible to the other (say physical knowledge). If so, $(K_2p \rightarrow K_1p)$, and

(7*) $(K_2p \wedge \neg K_1p)$

implies

(7**) $(K_1p \wedge \neg K_1p)$.

(7**) is inconsistent, the type-solution is undermined, and the *Knowability Paradox* remains.

Hence, given the possibility of a reduction of a kind of scientific knowledge into another one, the type-collapse is not prevented and our interpretation of the type-distinction is not an effective solution to the paradox.

More precisely, if *every* kind of scientific justification can be reduced to a basic kind – if there is complete reductionism in science – every higher level type of knowledge can be reduced to the lowest one, the type-distinction is not able to stop the *Knowability Paradox* at step (7), and some propositions remain unknowable. On the other hand, if every scientific truth can be known through at least two mutually irreducible kinds of justification, then the collapse of higher types into lower ones does not always occur, and does permit the knowability of each proposition: we escape the case in which $(K_{n+1}p \rightarrow K_np)$ for every n-level, and, at least for a type-level, we cannot deduce the contradictory $(K_np \wedge \neg K_np)$ from $(K_{n+1}p \wedge \neg K_np)$.

For this reason, a certain degree of reductionism can be compatible with the given solution of the paradox offered in the scientific case, but not a complete reductionism.

V. Conclusion

To conclude, in section III we have suggested a way of justifying a type-solution of the *Knowability Paradox* in the scientific field. This solution links type-levels to kinds of knowledge distinguished through their respective kinds of justifications. But the effectiveness of this solution strictly depends on the degree of reductionism adopted in science: in the last section (IV), we have shown that a complete reductionist position in science, according to which every kind of scientific knowledge can be reduced into a basic one and, consequently, every kind of scientific justification can be reduced into a unique basic kind, is not compatible with the here suggested solution of the *Knowability Paradox*.

If we do not accept a complete reductionism, the given solution of the *Knowability Paradox* is still available, at least in the scientific case. Otherwise, with a complete reductionism, Rescher's claim – according to which Fitch's argument shows a limit of knowledge and constitutes a sort of clue of the imperfectibility of science – witholds its stability.

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