A Mathematical Model for Bluetooth with Enhanced Data Rate in Fading Channels

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SHORT ABSTRACT

In this report, we present an accurate mathematical model for performance analysis of Bluetooth v2.0 with Enhanced Data Rates in fading channels. The model captures the details of the activities performed over an asynchronous data link and permits an accurate estimation of the average throughput and energy efficiency achieved by the different frame formats in fading radio channels.

Conversely to most part of the literature concerning Bluetooth performance, the model takes into consideration the microscopic level power–saving features defined by the Bluetooth standard for reducing the energy consumption during packet handling operations.

The model permits to investigate the potential performance tradeoff between different frame formats and to determine which format yields higher performance in each channel condition.

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I. INTRODUCTION

The SIG [1] has recently released the specifications for the enhanced data rate version of the standard, Bluetooth v2.0 + EDR (Enhanced Data Rate), that will permit higher bit rates and faster node connections [2]. The enhanced data rates are obtained by using Differential Phase Shift keying modulations, which are able to transmit two or three bits per symbol, thus providing data rates of 2 Mbps and 3 Mbps, respectively. However, to preserve backward compatibility, the control part of the packet is still transmitted at the basic rate of 1 Mbps, while the enhanced data rates can be used only over the payload field.

One of the most attractive features of Bluetooth is its energy efficiency. Bluetooth, indeed, was designed to be integrated in portable, battery driven electronic devices, for which energy saving is a key issue. In order to meet this goal, Bluetooth standard defines four operational modes, namely Active, Hold, Sniff, and Parked. These modes correspond to different degrees of activity of the Bluetooth devices which, in turn, lead to different levels of power consumption. Besides these high–level mechanisms, energy–saving is also pursued at a microscopic level, by means of a suitable packet reception mechanism [2]. In fact, a Bluetooth device is allowed to switch off the receiver circuitry as soon as it realizes that the incoming signal cannot be correctly decoded or it is addressed to another device. In this way, a unit that is not addressed by any valid packet is active for less than 10% of the time.

Although the reception mechanism is well defined by the Bluetooth standard, many aspects related to the performance achieved by the system in different operating conditions still need to be investigated.

In this Technical Report, we propose a mathematical framework for the analysis of Bluetooth piconets performance in terms of both throughput and energy efficiency in Active mode. The novelty of the work lies in the detailed analysis of the reception mechanism defined by the Bluetooth standard, which leads to a simple but effective system model that permits an accurate performance analysis in active state, i.e., when the units are actively exchanging data.

The model provides useful indications for the design of advanced segmentation and reassembly strategies and proves to be a valuable tool to gain insights on the aspects that have a major impact on the system performance.

II. BLUETOOTH V2.0 + ENHANCED DATA RATE

This section shortly overviews the features of the Bluetooth v2.0+EDR standard that are of specific interest for our analysis. An extensive description of the standard may be found, for instance, in [2]–[4].

A. Physical layer: basic and enhanced rates

The Bluetooth v2.0 with Enhanced Data Rate specifications encompass three transmission rates, namely [2]: the basic rate at $R_1 = 1$ Mbps, the enhanced data rate at $R_2 = 2$ Mbps (EDR2) and the enhanced data rate at $R_3 = 3$ Mbps (EDR3). The Basic Rate makes use of a binary Gaussian–shape Frequency Shift Keying scheme (GFSK), while EDR2 and EDR3 are obtained by using Differential encoded Phase Shift Keying (DPSK) modulations, with a constellation of four symbols ($\pi/4$–DQPSK) and eight symbols ($8$DPSK), respectively. In all the cases, the symbol period remains equal to $T_s = 1\mu s$, so that the frequency band of the signal is not significantly modified by the introduction of the EDR schemes.

The expressions of the bit error rate (BER) for the three modulation schemes can be found, for instance, in [5]–[7].

B. ACL data packet formats

The Bluetooth standard encompasses two types of links: Synchronous Connection Oriented (SCO) and Asynchronous ConnectionLess (ACL). SCO links are aimed at the transport of delay–sensitive traffic (mainly voice) and make use of a periodical time–reservation scheme. ACL links are intended for the transport of asynchronous data traffic, typically generated by elastic services. Examples of applications that require this service include the transfer of digital pictures from a digital camera to a storage device, the file synchronization between portable and fixed computer, and so on. In the following of this paper we focus on asynchronous data traffic, so that only ACL links will be considered.

Bluetooth v1.0 defined six different ACL frame formats, each structured into three fields: Access Code (AC), Packet Header (HEAD) and Payload (PAYL).

The AC field is used for synchronization, DC offset compensation and piconet identification. AC is followed by the packet header field (HEAD), which contains link control information, including packet type, destination address, sequence number, acknowledgment flag. A Header Checksum field (HEC) is used to verify the integrity of the decoded HEAD field. The HEAD field is immediately followed by the payload (PAYL) field that carries the upper layer data together with a 1– or 2–byte header and a 2–byte cyclic redundancy code (CRC), which is used for integrity check after decoding. Furthermore, the PAYL field can be optionally protected by a 2/3 forward error correction code. Finally, frames can extend over one, three or five consecutive slots of $T_{slot} = 625\mu s$ each. Unprotected data formats are denoted by $DH_n$, where $n = 1, 3, 5$ denotes the frame extension (in number of slots). Similarly, $DM_n$ is used for protected formats. Notice that Basic Rate frames are always transmitted at the rate of $R_1 = 1$ Mbps.

Bluetooth v2.0+EDR adds other six frame formats to the original ones. For backward compatibility, the EDR frame formats inherit the same AC and HEAD fields of BR formats, which are still transmitted at the basic rate. However, the HEAD field
is now followed by a guard time of approx. 5\,\mu s, which is used to let the transceiver circuitry switching to the appropriate DPSK modulation scheme. The guard time is followed by a synchronization field (SYNC) of 10 DPSK modulated symbols that is used for signal acquisition at the receiver. The SYN is followed by a variable length PAYL field, which still includes a 2-byte header and a 2-byte CRC. Finally, an Enhanced Data Rate trailer field of 2 symbols tails the frame. Notice, that SYNC, PAYL and trailer fields are transmitted at the selected EDR rate, by using the associated DPSK modulation scheme. Nevertheless, the time occupancy of EDR frames is still limited to 1, 3 or 5 consecutive slots. Furthermore, EDR specs do not encompass any error protection code for the PAYL field, so that only high-data rate formats are supplied. The six EDR frame formats are denoted by $jDHn$, where $j = 2, 3$ is the transmission rate (in Mbps), while $n = 1, 3, 5$ is the slot occupancy.

The characteristics of the twelve different ACL frame formats provided by Bluetooth v2.0+EDR are summarized in Tab. I.

<table>
<thead>
<tr>
<th>Type</th>
<th>Slots</th>
<th>Rate [Mbps]</th>
<th>PAYLOAD [bytes]</th>
<th>FEC rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DM1$</td>
<td>1</td>
<td>1</td>
<td>1-11-2</td>
<td>2/3</td>
</tr>
<tr>
<td>$DM3$</td>
<td>3</td>
<td>1</td>
<td>2-121-2</td>
<td>2/3</td>
</tr>
<tr>
<td>$DM5$</td>
<td>5</td>
<td>1</td>
<td>2-224-2</td>
<td>2/3</td>
</tr>
<tr>
<td>$DH1$</td>
<td>1</td>
<td>1</td>
<td>1-27-2</td>
<td>–</td>
</tr>
<tr>
<td>$DH3$</td>
<td>3</td>
<td>1</td>
<td>2-183-2</td>
<td>–</td>
</tr>
<tr>
<td>$DH5$</td>
<td>5</td>
<td>1</td>
<td>2-339-2</td>
<td>–</td>
</tr>
<tr>
<td>$3DH1$</td>
<td>1</td>
<td>3</td>
<td>2-83-2</td>
<td>–</td>
</tr>
<tr>
<td>$3DH3$</td>
<td>3</td>
<td>3</td>
<td>2-552-2</td>
<td>–</td>
</tr>
<tr>
<td>$3DH5$</td>
<td>5</td>
<td>3</td>
<td>2-1021-2</td>
<td>–</td>
</tr>
</tbody>
</table>

C. Baseband

The basic Bluetooth network configuration is the so-called piconet, a cluster of no more than eight devices sharing a common frequency–hopping radio channel: all the devices in the same piconet are synchronized on a common frequency hop sequence and concurrently change the carrier frequency at every new packet transmission/reception slot.

The access to the shared medium is regulated by one of the units, called master, which cyclically polls the other devices, named slaves. The master can poll the slave implicitly, by sending a useful data packet addressed to the slave, or explicitly, by transmitting a short control packet (POLL) that contains only AC and HEAD fields. The addressed slave is required to immediately reply by transmitting a data packet or a control packet (NULL), which again contains only AC and HEAD fields. Full–duplex communication is achieved by means of a slot–based time–division duplex (TDD) mechanism, according to which master and slave transmissions alternate in the channel access.

Bluetooth provides a reliable data connection by using an Automatic Retransmission Query (ARQ) mechanism at the baseband layer. Each data packet is transmitted and retransmitted until the transmitter receives a notification of successful reception from the destination. The acknowledgment (ACK) information is embedded in the HEAD field of the data or control packets sent by the destination (piggy–backing). Notice that, since negative acknowledgment is assumed by default, retransmissions can also occur because the return packet carrying a positive ACK is lost. In this case, the destination might receive multiple copies of the same frame, which are called duplicate packets (DUPCKs).

Notice that DUPCKs can be transmitted by the master node only. In fact, a slave is permitted to transmit a frame only upon receipt of a valid frame from the master that, in turn, will also carry the (positive or negative) acknowledgment for the last slave–to–master transmission. Thus, slave retransmissions occur only when solicited by an explicit not acknowledgment sent by the master.

D. Micro–level energy saving mechanisms

Energy–saving was a key feature in the design of the Bluetooth technology. On the basis of this perspective, a receiving unit stops reception and enters a low–power doze mode as soon as it determines that the incoming packet is addressed to another unit or the signal strength is too low to guarantee good reception (see [3], pg. 124, and [2], Vol. 3, pg. 182).

At the beginning of each receive slot, the Bluetooth receiver correlates the incoming bit stream against the expected synchronization word.

In case AC is not recognized, reception stops and the unit enters a low–power doze mode until the beginning of the following receive slot. Conversely, after the recognition of the AC field, the receiver decodes the HEAD field and, at the end, it performs the integrity check using the HEC field. If the check fails, the device enters doze–mode, otherwise the HEAD field is inspected to determine the packet format and the destination address. Slaves not addressed by the packet may remain in doze–mode until the end of the frame transmission, while the intended receiver may go on decoding the PAYL field (if any). Finally, if the
HEAD field of the incoming packet carries the same sequence number of the last decoded frame, then the receiver recognizes it as a DUPCK and returns a positive acknowledgment to the sender, irrespective of the actual reception status of the PAYL field.

III. ACL DATA LINK MODEL

In this section, we define the mathematical model for a simple Bluetooth connection. We first introduce some notations and hypothesis. Then, we define the Markovian model used to describe the system evolution and we briefly outline the basis of the renewal reward theory. Finally, we define the reward functions in each state of the Markov Chain and derive the performance indexes.

A. Notation

The reception of a baseband frame consists of three consecutive steps: 1) the acquisition of the Access code; 2) the recognition of the frame Header and 3) the reception of the payload field. Let us denote the three steps with $A$ (for Access code), $H$ (for Header), and $D$ (for Data). In case one of such steps fails, the remaining steps are not performed and the unit enters power saving modes. We will use the subscripts $s$ and $f$ to discriminate between success and failure of a given reception step, respectively. Furthermore, we will denote by $P$ the probability of the event. For instance, $P_{As}$ gives the probability of successful AC acquisition, whereas $P_{Hf}$ is the probability that the AC is recognized but the frame header is corrupted by unrecoverable errors. Notice that the following relations must hold among the different event probabilities

\[ 1 = P_{As} + P_{Af} ; \]  
\[ P_{As} = P_{Hs} + P_{Hf} ; \]  
\[ P_{Hs} = P_{Ds} + P_{Df} . \]

When necessary, we use the superscript $(M)$ and $(S)$ to distinguish between master and slave units. For example, $P_{Df}^{(M)}$ denotes the probability that a master frame is received with unrecoverable errors in the PAYL field only.

For space constraints, we do not report the expressions of such probabilities, which can be found in [8]. Notice, that the reception of AC and HEAD fields, which are always transmitted at the basic rate, is not affected by the frame format. Conversely, the probabilities $P_{Ds}$ and $P_{Df}$ clearly depend on the considered frame format, though this dependency is not explicit in the notation.

B. Hypotheses

For the sake of simplicity, we limit the study to the case of a piconet with only two units: one master and one slave. (The extension of the analysis to the multi–slave case would complicate the exposition without adding any relevant concept.) We consider a heavy traffic scenario, where master and slave have always packets waiting for transmission. We assume infinite retransmission timeout: packets are retransmitted over and over again until the sender receives a positive acknowledgement. In order to determine the performance achieved by the different packet formats, we consider a static Segmentation and Reassembly (SAR) policy, so that a single frame format per connection is used.

For the radio channel, we assume the classical WSSUS (Wide–Sense Stationary Uncorrelated Scattering) slow flat Rician fading model [9], so that, by virtue of the frequency hopping mechanism, packets are subject to statistically independent flat fading.

Finally, we assume that units are not capable of any carrier sensing functionality. Notice that, under this hypothesis, a unit can determine the end of an ongoing transmission only inspecting the information contained in the HEAD field of the transmitted frame. Therefore, if AC or HEAD are received with unrecoverable errors, the receiver does not have any way to determine the actual time extension of the frame. This situation may give rise to a sort of master–slave collision, which occurs when the master does not recognize a multi–slot frame transmitted by the slave. In fact, the master may start transmitting its frame when the slave transmission is stil ongoing, so that the two transmissions will partially overlap in time (not in frequency). Clearly, the slave node cannot receive any new frame while transmitting its own frame, so that the master transmission will be wasted.

Such kind of master–slave collisions might be avoided by providing the units with carrier sensing functionalities [10]. Indeed, the master could defer its retransmission until the uplink channel would be sensed idle again. Moreover, carrier sensing may also lead to some advantages in terms of energy efficiency. Indeed, slave units that are not interested by the current communication could recognize the end of each transmission by sensing the medium, rather than waking up every two slots to listen for a valid AC, thus saving some energy.

In this paper, however, we do not consider the carrier–sensing mechanism.
C. Markovian Model

Under the considered hypotheses, the dynamic of the system can be captured by means of a Two–State Markov Chain (MC) with event space \( E = \{N, D\} \). In Normal state \((N)\), the master transmits new downlink packets or retransmit packets that have never been correctly received by the slave. The MAC leaves the \( N \) state to enter the Duplicate state \((D)\) whenever the master does not recognize an uplink packet carrying a positive acknowledgment. Therefore, the transition probability \( \pi_{ND} \) from state \( N \) to \( D \) is given by

\[
\pi_{ND} = P_{D,i}^{(m)}(1 - P_{H,i}^{(m)}) \; .
\]  

(4)

In state \( D \), the master keeps transmitting duplicate packets. State \( D \) is left when the master finally gets a positive acknowledgment from the slave. Since the slave disregards the PAYL field of DUPCKs, the transition probability \( \pi_{DN} \) from state \( D \) to \( N \) is given by

\[
\pi_{DN} = P_{H,i}^{(m)} P_{H,i}^{(s)} \; .
\]  

(5)

Hence, state transitions occur in discrete steps. Each step begins when the master starts transmitting a downlink packet and the slave is ready for reception. Notice, that in case of master–slave collisions, a transition step covers multiple master/slave transmissions.

The steady state probabilities \( \pi_N \) and \( \pi_D \) of the MC being in states \( N \) and \( D \), respectively, are then given by

\[
\pi_N = \frac{\pi_{DN}}{\pi_{ND} + \pi_{DN}} ;
\]  

(6)

\[
\pi_D = \frac{\pi_{ND}}{\pi_{ND} + \pi_{DN}} .
\]  

(7)

D. Reward functions

Following the approach suggested in [11], Bluetooth performance can be investigated by resorting to the classical theory of renewal reward processes [12]. Consider two generic reward functions, \( R^{(1)} \) and \( R^{(2)} \), such that \( R^{(1)}_j \) and \( R^{(2)}_j \) are the average reward earned each time the Markov chain enters in state \( j \in E \). Furthermore, let \( R^{(1)}(\tau) \) and \( R^{(2)}(\tau) \) be the total reward earned through the system evolution in the interval \([0, \tau]\). Then, from renewal theory [12], we have:

\[
\lim_{\tau \to \infty} \frac{\tau}{(\tau)} = \frac{\sum_{j \in E} \pi_j R^{(1)}_j}{\sum_{j \in E} \pi_j R^{(2)}_j} = \frac{R^{(1)}}{R^{(2)}} ;
\]  

(8)

where \( \pi_j \) is the steady state probability of the chain being in state \( j \), while \( R^{(1)} \) and \( R^{(2)} \) are the expected rewards per state transition.

A proper choice of the reward functions will allow us to derive a number of performance indexes. In particular, we consider the following functions:

- state transition time \( T_i \);
- average number of successfully delivered data bits, \( D_j \);
- amount of consumed energy, \( W \).

In order to derive the expected values of these reward functions, we need to introduce some further notations. Let \( w_{rx} \), \( w_{ss} \) and \( w_{rs} \) be the amount of energy consumed by a unit for transmitting, receiving and sensing, respectively, the generic packet field \( X \). Let \( jDHn \) and \( iDHm \), with \( n, m \in \{1, 3, 5\} \) and \( i, j \in \{2, 3\} \), be the packet types used by the master and slave units, respectively. Finally, let \( \mathbb{D}(j, n) \) be the number of useful data bits carried by the \( jDHn \) packet format, as reported in the data column of Tab. I.

State Transition Time

The transmission of a \( jDHn \) frame by the master always takes \( n \) time slots. If the slave does not successfully decode the AC or the HEAD fields of the incoming frame, then it is not allowed to reply and the step is then concluded in \( n + 1 \) slots. On the contrary case, the slave is allowed to return an \( iDHm \) frame. If the master successfully decodes the AC and HEAD field of the slave’s frame and, then, receives the entire frame, or the slave frame is single–slot long, then the step is concluded in \( m + n \) slots. However, if \( m > 1 \) and either AC or HEAD fields of the slave frame contain unrecoverable errors, a master–slave collision occurs. The collision will be solved after \( c(n, m) \) transmission attempts by the master, which is given by

\[
c(n, m) = \left\lfloor \frac{m - 1}{n + 1} \right\rfloor + 1 .
\]
Therefore, the average time reward earned per MC transition is equal to
\[
T = (n + 1) \left( cn,mP_{H_j}^{(M)}(1 - P_{H_i}^{(S)}) + 1 - P_{H_i}^{(M)} \right) + (m + n) P_{H_j}^{(M)} P_{H_i}^{(S)} .
\]

**Delivered Data**

In state \(N\), the master transmits packets that have never been correctly received by the slave. Therefore, starting from state \(N\), a master transmission delivers \(D(j,n)\) data whenever the frame is successfully decoded by the slave. In state \(D\), the master transmits DUPCKs that do not carry useful information. The slave unit, in turn, will deliver \(D(i,m)\) data whenever it gets the chance to transmit a frame and this frame is successfully decoded by the master. Thus, the average number of data bits successfully delivered by the master and slave units, respectively, in a MC step is given by
\[
\overline{D}^{(M)} = \pi_N D(j,n) P_{D_s}^{(M)} ;
\]
\[
\overline{D}^{(S)} = D(i,m) P_{H_j}^{(M)} P_{D_s}^{(S)} .
\]

**Consumed Energy**

The computation of the energy spent by the master and slave units for each transition step of the MC, though cumbersome, is not complicated. Let us first focus on the master unit. At each step of the MC, the master spends \(w_{Tx}(jDHn)\) energy units by transmitting its \(jDHn\) downlink frame. If the slave gets polled by the master frame, which occurs with probability \(P_{H_j}^{(M)}\), then it will return a \(iDHm\) frame. Therefore, the master will go through the three reception steps, unless one of such steps fails. If the AC and HEAD fields are correctly received, which occurs with probability \(P_{H_i}^{(S)}\), the master receives the entire data frame, otherwise a collision occurs (provided that \(m > 1\)). In case of collision, the master transmits the same frame \(c(n,m)\) times before the collision is resolved and listens for a valid AC as many times. Finally, if the slave misses the master poll, no frames are returned and, thus, the master turns off its receiver immediately after sensing an idle channel for a period equal to the AC duration. Therefore, the average amount of energy spent by the master is given by
\[
W^{(M)} = w_{TX}(jDHn) \left[ 1 - P_{H_i}^{(M)}(1 - P_{H_j}^{(S)}) \right] + w_{RX}(iDHm) P_{H_i}^{(M)} P_{H_j}^{(S)} + (w_{TX}(jDHn) + w_{RX}(AC)) c(n,m) P_{H_i}^{(M)}(1 - P_{H_j}^{(S)}) + w_{RX}(HEAD) P_{H_i}^{(M)} P_{H_j}^{(S)} + w_{RX}(AC)(1 - P_{H_j}^{(M)}). \]

The energy spent by the slave unit depends also on the MC state. Indeed, as explained in Sec. II-D, the slave does not listen for the PAYL field of DUPCKs. Hence, if the system is in state \(D\) and the slave does recognize the HEAD field of the upcoming packet, it enters sleep mode till the end of the incoming packet, saving energy. However, if the AC or HEAD fields are not recognized, the slave keeps waking up every two slots to check for a valid AC. On the basis of the rationale discussed for the master case, it is easy to realize that the average amount of energy spent by the slave unit can be expressed, after some algebra, as follows
\[
W^{(S)} = P_{H_j}^{(M)} \left[ w_{RX}(jDHn) \pi_N + (w_{RX}(AC) + w_{RX}(HEAD)) \pi_D + w_{TX}(iDHm) + (1 - P_{H_i}^{(S)}) w_{RX}(AC)i_s(n,m) \right] + (1 - P_{H_i}^{(M)}) w_{RX}(AC)[n/2] + P_{H_i}^{(M)} w_{RX}(HEAD) \]
where \(i_s(jn,m)\) is the number of times the slave unit wakes up looking for a valid AC in case of collision and it is given by
\[
i_s(n,m) = \begin{cases} 1 & n = m, n > 1 \\ 2 & n = 5, m = 3 \\ 0 & \text{otherwise} \end{cases}
\]

**IV. CONCLUDING REMARKS**

In this paper, provided a mathematical model for the performance analysis of a Bluetooth data link, in terms of goodput, delay, energy efficiency and system lifetime. The model is based on an accurate analysis of the microscopic energy-saving mechanisms provided by Bluetooth standard, which have a non marginal impact on the overall performance figure of the system.

**REFERENCES**


