1. The aggregate approach to innovation diffusion: the Bass Model

1.1 Introduction

Modelling and forecasting the diffusion of innovations is a broad research topic, whose importance is confirmed by the wealth of publications regarding it. With more than 4000 publications since the 1940, it has been said that “no other field of behavioural science research represents more effort by more scholars in more disciplines in more nations” (Rogers, 2003). This is a theme of both practical and academic interest as demonstrated, for example, by the considerable share of marketing and economic research devoted to it, which denotes not only the strategic importance of new products and technologies in triggering the growth of an economy, but also the role of diffusion in helping managers to plan more efficiently their strategies, by anticipating the development of product demand (Mahajan and Muller, 1979).

Mahajan and Muller (1979) have stated that the purpose of a diffusion model is to depict the successive increases in the number of adopters given a set of potential adopters over time and predict the development of a diffusion process already in progress. Shaikh, Rangaswamy and Balakrishnan (2005) have noticed that diffusion modelling is important both for firms that introduce new products and for firms that offer complementary or substitute products: for example, the time path of adoptions of iPod is important not only for Apple computer but also for competitors, like Sony, and firms that produce complementary goods, such as speakers, ear phones and carrying cases.
The success of an innovation depends on various factors that may be both internal and external. Given the great complexity of these factors, the rate of failure of new products is quite high: for example Mahajan, Muller and Wind (2000) have reported that this rate may vary in the range of 40 to 90 percent. Today, the shortening of product life cycles, the increasing level of competition between firms and products, the need to plan the likely existence of several successive generations of a new product and thus to manage resources and commitments, require a timely investigation on the features of a new product growth, in terms of its speed and dimension. The fundamental marketing concept underlying the employment of new product growth models is the product life cycle (see Wind, 1982): the product life cycle hypothesizes that sales of a new product are characterised by stages of launch, growth, maturity and decline, miming the life cycle of a biologic organism.

Diffusion models are typically concerned with the representation of this life cycle in the context of sales forecasting: however, they also may be used for descriptive and normative purposes. For example, according to the perspective presented by Muller, Mahajan and Peres (2007) the current trend on diffusion research seems to be increasingly focused on managerial diagnostics, able to reveal the basic structures of a market, to allow comparisons with other contexts, and to help firms to anticipate and prepare for possible scenarios in the future.

If the marketing and management fields played and still play a central role in defining the boundaries and the directions of this research, it is also true that the great interest towards innovation diffusion processes may be due to the opportunity to connect and involve many other scientific disciplines such as economics, technological
Indeed, the diffusion of an innovation is primarily a social phenomenon, whose complexity may be better understood through the contribution of various scientific areas. Traditionally, it has been defined as the process by which an innovation is communicated through certain channels among the members of a social system (Rogers, 2003). As such, it consists of four central elements, the innovation, the communication channels, time and the social system.

In one of the most famous reviews of diffusion models Mahajan, Muller and Bass (1990) argued that the main focus of diffusion theory is on communication channels, that are the means by which information about an innovation is transmitted to or within the social system. These means may be both formal, like mass media and advertising and informal, like interpersonal communication.

In particular, “interpersonal communications, including nonverbal observations, are important influences in determining the speed and shape of the diffusion process in a social system” (Mahajan, Muller and Bass, 1990).

The formal representation of these processes has historically used epidemic models borrowed from biology, like the logistic equation, in which the social contagion represents the driving factor of growth. The logistic equation was formulated for the first time by Verhulst in 1838 and was originally used in natural sciences for describing growth processes, like the spread of a disease. In 1925 the biologist Pearl called the attention on the fundamental fact that all growth processes can be adequately described through this equation, that gives rise to the typical s-shaped curve. Fisher and Pry
(1971) and Meade and Islam (1998) demonstrated the usefulness of the normalized logistic equation in representing the diffusion of basic technologies. Marchetti (1980) noted that innovation diffusion is basically a learning process, and learning, being a process of growth, may be conveniently represented through the logistic curve. Moreover, Modis (1992) demonstrated the complete equivalence between the logistic curve and the learning curve (learning by doing). Devezas (2005) has highlighted that the logistic equation represents one of the most powerful technological forecasting tools, almost a “natural law” of innovation diffusion, due to its success in representing dynamics of change within markets and industries. The s-curve is the link between a broad literature about economic dynamics of technical change in which innovations and the response to them are the consequences of market processes (Metcalfe, 2005).

The rationale behind the use of this equation in new product context is that an innovation spreads in a social system through communication like an epidemic disease through the mechanism of contagion between persons. A direct inspection of aggregate diffusion data sets suggests that this type of models is appropriate: in fact, the cumulative adoption of an innovation approximately follows an s-shape (or logistic) path. Many contributions helped to clarify that diffusion processes are essentially driven by learning through imitation. Probably one of the first to formalize this idea was the french lawyer Tarde, whose major work is “Le lois de l’imitation” (1890). In this work Tarde stated that the imitation process can represent a general law of social change. He clarified that invention (in his terminology indicates the origination of novelty) is a necessary condition of change but the actual change occurs only when a large number of persons begins the adoption process he termed imitation. Diffusion processes as the result of an imitative behaviour find a direct connection with a social psychological
theory called *social learning theory*. Founded by Bandura (1971), the social learning approach looks outside of the individual and tries to explain changes of behaviour as determined by information exchanges with others. Another noteworthy contribution in the economic reasoning was given by Veblen in “The theory of Leisure Class” (1971), recognising imitation as a structural element of individual economic behaviour.

The central role of imitation in explaining diffusion processes and the possibility to represent them with the logistic curve are common elements of all the research approaches on innovation. Rogers (2003) has stressed that this research started in a series of independent studies during its first several decades. Despite the specificity of these approaches to diffusion, each of these reached similar findings: in particular that the diffusion of an innovation follows an s-shaped curve over time.

Among these streams of research, the marketing diffusion has become particularly strong since the 1970s. Pioneering works in this area are those of Mansfield (1962), reproducing the Verhulst structure, Fourt and Woodlock (1960), Bass (1969). During the last 25 years, many reviews of diffusion models have been developed. Among these, we can remember Mahajan and Muller (1979), Mahajan, Muller and Bass (1990), Mahajan, Muller and Bass (1995), Mahajan, Muller and Wind (2000), Meade and Islam (2001), Meade and Islam (2006), Hauser, Tellis and Griffin (2006), Chandrasekaran and Tellis (2006), Muller, Peres, Mahajan (2007).

Interestingly, all these reviews are especially referred to the most known and employed diffusion model, the Bass model, BM, which offered the theoretical and empirical evidence for the existence of the s-shaped pattern to represent the first purchase growth of a new product in marketing (Mahajan, Muller and Wind, 2000). The purpose of this model is to depict and predict the development of this growth process.
through time, when it is already in progress. Since its publication in Management Science in 1969, the BM has been widely used both in academic research and practical applications, proving its reliability in forecasting the diffusion of new products in several industrial sectors, such as industrial technology, agriculture, pharmaceutics, durable goods sector. As declared in the article title “A new product growth for consumer durables”, this model was originally designed only for durable goods.

However, it has proven to be appliable to services too. The diffusion of services has been modeled as if they were durable goods, including the case of cellular phones (Krishnan, Bass and Kumar 2000), cable TV (Lilien, Rangaswamy and Van den Bulte 2000), online banking (Hogan, Lemon and Libai 2003), energy (Guseo, Dalla Valle and Guidolin 2007), email services (Montgomery, 2001). Showing that the Bass model may be applied with success also in the case of services’ diffusion has been very important, since most innovations today in fact are services. Libai, Muller and Peres (2006) remind that the service sector in the USA employs most of the work force, is responsible for more than 80 % of the GDP and is growing faster than the good sector.

This chapter is dedicated to an extensive treatment of the Bass model. Section 1.2 is dedicated to present the formal structure of the Bass model, highlighting some theoretical assumptions and some relevant aspects for strategic evaluations and forecasts. Section 1.3 deals with the issue of statistical implementation of the model and proposes two examples of its concrete application to time series data. Section 1.4 presents the most famous and useful generalization of the Bass model, the Generalized Bass model, that incorporates marketing mix variables and other exogenous factors by the means of a general intervention function \( x(t) \). Two applications are provided also for this model. Section 1.5 reviews the interesting themes of spatial diffusion and
successive generations of product, while section 1.6 summarises some proposals of refinement and extension of the Bass model, that according to the most recent reviews on innovation diffusion models would deserve a deeper investigation.

1.2 The Bass Model

The Bass model, BM, describes the life cycle of an innovation, depicting its characterising phases of launch, growth and maturity, decline. Its purpose is to forecast the development over time of a new product growth, as result of the purchase decisions of a given set of potential adopters (market potential).

These purchase decisions are assumed to be influenced by two sources of information, an external one, like mass media and advertising and an internal, namely social interactions and word-of-mouth. These are “competing” sources of information, whose effect creates two distinct groups of adopters. One group is influenced only by the external source and we call it innovators, the other only by the internal one and these are the imitators.

One of the great advantages associated with the BM is the concrete possibility to explain the initializing phase of diffusion, due to the presence of innovators. Indeed, there exists a huge literature on the role of innovators, also called “early adopters” (Rogers, 2003), “opinion leaders” (Katz and Lazarsfeld, 1995), but the first model formalizing their action is the BM.

In particular, it is assumed that there exists a constant level of adopters, innovators, buying the product at the beginning of the diffusion, even if other adopters influenced by external information are present during the whole product life cycle.
In this sense the BM has recognised the role of all the communication efforts realised by firms, whereas a pure logistic approach like that of the Mansfield model does not.

The formal representation of the BM is a first-order differential equation

\[ z'(t) = \left( p + q \frac{z}{m} \right) (m - z) \]  

(1)

or

\[ z'(t) = p(m - z) + q \frac{z}{m} (m - z) \]  

(1a)

This equation tells that the variation over time of instantaneous adoptions, \( z'(t) \), is proportional to the residual market, \( (m - z) \) where \( m \) is the market potential or carrying capacity and \( z(t) \) represents the cumulative number of adoptions at time \( t \). Notice that the market potential \( m \) depicts the maximum number of realizable adoptions within the life cycle and its value is assumed constant along the whole diffusion process.

The residual market is affected by two parameters, \( p \) and \( q \). Parameter \( p \) represents the effect of the external influence, due to the mass media communication, while parameter \( q \) is the so called coefficient of imitation, whose influence is modulated by the ratio \( \frac{z}{m} \), that at time \( t = 0 \) is clearly zero. Note that at time \( t = 0, z'(0) = pm \): this is the constant level of adopters (innovators) acting at the beginning of diffusion. Also notice in equation (1) that innovators are present at any stage of diffusion even if with a time decreasing share.

In equation (1a) the coexistence of the two groups of adopters is more evident: the first term refers to innovators, while the second one represents imitators.
The Bass model can also be interpreted as a *hazard function*, that is the probability that an event will occur at time \( t \) given that it has not occurred. We have that

\[
\frac{z'}{m - z} = p + q \frac{z}{m}.
\]  

(2)

Equation (2) describes the conditional probability of an adoption at time \( t \), resulting from the sum of the probabilities of two incompatible events, \( p \) and \( q \frac{z}{m} \): thus the model excludes adoptions due to both innovative and imitative effects, assuming that the final purchasing decision will be determined by only one influence (external or internal).

This separation of effects generates the two classes of adopters we have defined before. We shall observe that these are *latent categories*, since aggregate data on adoptions clearly do not provide evidence on this.

1.2.1 Solution of the Bass Model

If we denote \( y = \frac{z}{m} \) we can equivalently rewrite the Bass model with the following equation

\[
y' = (p + qy)(1 - y)
\]  

(3)

or

\[
y' + qy^2 + (p - q)y - p = 0
\]  

(3a)

Notice that equation (3a) represents a particular case of a more general Riccati equation, as analysed in Guseo (2004).
The real roots of the characteristic equation $ax^2 + bx + c = 0$ defined as

$$r_i = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

are $r_1 = -\frac{p}{q}$ and $r_2 = 1$ in the BM case. In general, the terms $y'$ and $1 - y$ are positive, $p$ and $q$ are positive too, so that $r_1 < 0 < r_2$. Therefore, the solution’s asymptotes are $-\frac{p}{q}$ and $1$ under initial condition $y(0) = 0$.

The proposed closed-form solution of the Bass model is a special cumulative distribution (see in particular Figure 1)

$$y(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}. \quad (4)$$

The proportion of adoptions $y(t)$ provided by equation (4), describes the dynamics of the diffusion process, in terms of adoption parameters, $p$ and $q$. We also can refer to the absolute scale representation, that is to the number of adoptions, $z(t)$, just multiplying equation (4) by the market potential $m$, acting as a scale parameter

$$z(t) = m \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}. \quad (4a)$$

Notice that $\lim_{t \to +\infty} y(t) = r_2$ and therefore $\lim_{t \to +\infty} z(t) = mr_2$, so that the asymptotic behaviour of diffusion in absolute terms, $z(t)$, is controlled by the size of the market potential $m$, since $r_2 = 1$. 
Figure 1. The Bass model, cumulative adoptions $y(t)$. The model describes a saturation.

Previous equations indicate cumulative adoptions at time $t$, but if we are more interested on period-by-period or instantaneous adoptions we will use the correspondent first order derivative, that is the density function (see Figure 2)

$$y'(t) = \frac{p(p+q)^2 e^{-(p+q) t}}{(p + q e^{-(p+q) t})^2}$$  \hspace{1cm} (5)

or the corresponding absolute version

$$z'(t) = m \frac{p(p+q)^2 e^{-(p+q) t}}{(p + q e^{-(p+q) t})^2}.$$  \hspace{1cm} (5a)
Figure 2. The Bass model: instantaneous adoptions highlight the existence of a peak, point of maximum growth of diffusion.

Instantaneous adoptions highlight the presence of a peak, that is the point of maximum growth of the diffusion, after which the process begins to decrease. It is easy to understand that from a strategic point of view the peak represents a very crucial stage of a diffusion process, indicating that maturity phase of the product life cycle, after which the decline begins. The time at which the peak occurs is given by

$$t^* = \frac{\ln(q/p)}{(p+q)}$$

(6)

where the cumulative function takes the value

$$z(t^*) = m \left(1/2 - p/2q\right).$$

(7)

Interestingly, we may observe in equation (7) that when the peak occurs, cumulative sales $z(t)$ are approximately equal to $\frac{m}{2}$, because $p$ is usually very small if compared with $q$. Equation (4a) depends on initial condition $z(t = 0) = 0$. However, if information and data about the very first stages of a diffusion process are not available, the model may be modified for overcoming this shortage.
\[ z(t) = m \frac{1 - e^{-(p+q)(t-c)}}{1 + \frac{q}{p} e^{-(p+q)(t-c)}}, \quad t \geq c; \ p, q > 0 \]  

where \( c \) is an unknown translation parameter to be estimated such that \( z(c) = 0 \).

1.3 Implementation of the Bass model

The use of the Bass model for forecasting the diffusion of an innovation requires the estimation of three parameters: external influence, \( p \), internal influence, \( q \), market potential, \( m \) (and possibly, \( c \)). These three parameters can be estimated using cumulative sales data: as reported by Sultan, Farley and Lehmann (1990) average values for \( p \) and \( q \) are respectively 0.03 and 0.38. The size of the market potential \( m \) is probably the most critical element in forecasting matters and a reliable estimation of it should be established as soon as possible. However, several empirical studies have demonstrated that parameter estimates, thus forecasts, are quite sensitive to the number of data available: in other words estimates suffer the fact that data are sequentially concentrated in the first part of the innovation life cycle. In particular, Srinivasan and Mason (1986) have maintained that reliable estimates may be obtained if non-cumulative data include the peak, which would imply, on the contrary, a considerable reduction of the model forecasting ability. Mahajan, Muller and Bass (1990) effectively synthesize the problem: “parameter estimation for diffusion models is primarily of historical interest; by the time sufficient observations have been developed for reliable estimation, it is too late to use the estimates for forecasting purposes”.

Van den Bulte and Lilien (1997) have considered some bias in parameter estimation, including the tendency to underestimate the market potential, whose value is generally close to the latest observed data. Given these estimation difficulties on the one hand and
the need of early forecasts on diffusion on the other, Meade and Islam (2006) suggest that the identification of factors determining the market potential would be a fruitful area of research. Estimation aspects are also discussed in Venkatesan and Kumar (2002), Venkatesan, Krishan and Kumar (2004) and Jiang, Bass and Bass (2006).

Empirical experience has shown that ordinary least squares technique (OLS) is non-optimal for estimating the Bass model, because of some shortcomings including the tendency to yield negative sign parameters (that is, negative probabilities).

Mahajan, Mason and Srinivasan and Srinivasan and Mason (1986) have proposed a non-linear least square approach (NLS), which is generally accepted as the more reliable non-parametric method of estimation of the Bass model (see Putsis and Srinivasan, 2000 and Muller, Peres and Mahajan, 2007). More recently, Venkatesan and Kumar (2002) have suggested the use of Genetic Algorithms (GAs) as an alternative to NLS approach: the claimed superiority of GAs with respect to NLS is questioned in Guseo and Guidolin (2007a).

Consistently with most of the literature on this issue, in this work it will be used a NLS approach (e.g. Levenberg-Marquardt, see Seber and Wild, 1989) to estimate the Bass model parameter: in doing so we may consider the structure of a non linear regression model, resulting from the sum of two components

$$z(t) = f(\beta, t) + \varepsilon(t) \quad (9)$$

where $z(t)$ is the observed response, $f(\beta, t)$ is the deterministic component, depending on parameter $\beta \in \mathbb{R}^k$ and time $t$. The second component, $\varepsilon(t)$, is defined as a stochastic process representing the residual term.

The BM regressive model is therefore
\[ z(t) = m \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} + \varepsilon(t). \]  

(10)

where \( z(t) \) is the observed data, namely cumulative number of adoptions or sales at time \( t \). The unknown constants \( m, p, q \) are the parameters to be estimated. In general \( \varepsilon(t) \) is a white noise process, so that residual mean is zero, \( M(\varepsilon(t)) = 0 \), the variance is constant, \( \text{Var}(\varepsilon(t)) = \sigma^2 \) and different error terms are incorrelated, \( \sigma_{\varepsilon(t),\varepsilon(t')} = 0 \), \( t \neq t' \). Nevertheless, the concrete application of the NLS procedure to several cases has shown that residuals cannot be always considered incorrelated and a better representation of \( \varepsilon(t) \) would be therefore required. A possible answer to this aspect may be given by ARMAX frameworks. See for instance Box and Jenkins (1976) and, among others, Guseo and Dalla Valle (2005).

In the following sub-sections two examples on the statistical implementation of the standard Bass model to new product diffusion are presented, in order to clarify some basic aspects of the concrete application of this model to time series. The first one concerns the diffusion of a new pharmaceutical drug in Italy, for which weekly cumulative sales data are available, while the second one considers the adoption path of photovoltaic solar cells in Japan. While the purpose of these two examples is purely an illustrative one, it is interesting to notice that two very different contexts like medical innovation and energy technologies have been chosen, to show the versatility of the Bass model, whose application ranges over a broad set of industrial sectors, like durable goods, services, entertainment products, medical and agricultural innovations, technologies.
1.3.1 A standard Bass model for the diffusion of a new pharmaceutical drug in Italy

This sub-section provides a simple example to describe the application of a standard Bass model to the diffusion of a new pharmaceutical drug in Italy. Several works have demonstrated the suitability of this model to new pharmaceuticals’ diffusion (see Mahajan, Muller and Bass, 1990). The markets for drugs are particularly attractive for diffusion research given the considerable level of competition between firms, the shortened life cycles of products and the consequent search for product innovations. In a recent review on new products’ diffusion Chandrasekaran and Tellis (2006) have stressed that medical innovations should constitute a typical field of investigation for diffusion research. Moreover, the acceptance of new drugs or new medical technologies by physicians represent a classical topic of research (see for instance Coleman, Katz and Menzel, 1966; Van den Bulte and Lilien, 2001), since it is still in doubt if the prescriptive behaviour of physicians is mainly influenced by advertising and marketing efforts or by contagion and network effects with other physicians.

The data analysed in this example are provided by IMS Health-Italia, cover the period between 2005 and 2007 and refer to the weekly cumulative number of sold packages of this drug in Italy. The product, introduced in August 2005, is normally prescribed by physicians to prevent fetus malformations and its assumption by expectant mothers has been also recommended by the Italian Department of Health with the sponsorizations of informative campaigns and advertisements. The Bass model combines the perspectives expressed in the literature on medical innovation, assuming that adoption of a new drug can occur either because of effective marketing activity or
for social contagion. The application of the Bass model to our time series yields the results summarised in Table 1.

<table>
<thead>
<tr>
<th>A new pharmaceutical drug in Italy (data source: IMS health)</th>
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<tbody>
<tr>
<td>m</td>
</tr>
<tr>
<td>------------------------------</td>
</tr>
<tr>
<td>1,69092E6</td>
</tr>
<tr>
<td>(1,60614E6)</td>
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<tr>
<td>(1,7757E6)</td>
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</tbody>
</table>

Table 1: Standard Bass model estimation results. Asymptotic 95% confidence intervals into parentheses

![A new drug's diffusion](image)

Figure 3. Cumulative observed vs. predicted values with a standard Bass model: good level of fitting.
A new drug's diffusion

Variables
Observed
Predicted

0 30 60 90 120 150
0
3
6
9
12
15
18

(X 1000)

Figure 4. Instantaneous sales vs. Bass model: the maximum growth of sales, the peak, has been attained in $t = 72$. Observe the overestimation of data in the first part of data.

Estimation results seem quite satisfactory: the involved parameters are significative and well identified, as confirmed by confidence intervals. Both innovative and imitative behaviour characterize the adoption pattern of this product, combining different points of view expressed in literature. The proposed results suggest that the peak of sales has been already reached in $t = 72$, that is, within the first week of January 2007, so that the product is entering the decline phase of its life cycle. Since the product was launched in 2005, this result would confirm the general perception of a shortening of drugs’ life cycles. The value of the $R^2$ index is good, though probably susceptible of improvement: in particular we may notice, observing both Figures 3 and 4, that the Bass model tends to overestimate the first part of data series, where the real process has evidently experienced some difficulty in “taking-off”, and, as a balancing effect, tends to underestimate the final part of data series. A possible explanation of this overestimation problem will be discussed in Chapter 3, with the proposal of a formal solution. In addition, some improvements in estimations may be obtained through
further assumptions on residuals, as suggested by the value of the Durbin-Watson statistic, whose value indicates the presence of residuals’ autocorrelation.

1.3.2 Modelling the diffusion of photovoltaic energy in Japan with a BM

Guidolin and Mortarino (2007) have studied the diffusion of photovoltaic systems across various world countries applying the Bass model or its extensions (here presented in section 1.4) to the annual data of cumulative installed power, provided by the International Energy Agency (IEA) for the period 1992-2006.

Studies and forecasts on impending oil and natural gas depletion, worsening climate change, increasing needs of security in energy provision are inducing many countries to put energy issues on top of their agendas and look for alternatives to fossil fuels with increasing pressure. Among all the viable solutions, photovoltaic solar energy (PV) is considered one of the most attractive for various reasons. The success of this energy source, that directly converts sunlight into electricity, is obviously driven by a widespread adoption of photovoltaic solar cells. However, the purchase of a PV system typically yields negative outcomes at the time of purchase, while positive outcomes are delayed, so that the final purchase decision appears particularly complex and risky to consumers. To overcome this problem in many countries incentive measures have been adopted.

In recent years, industry and markets for photovoltaic cells have experienced an unprecedented growth, so that evaluations and technological forecasts on the future development of this sector appear crucial. In fact, current technology for solar cells relies on silicon, whose limited availability is considered the major constraint for PV
growth. The innovation diffusion approach and specifically the Bass model have appeared an appropriate choice for analysing this technological context.

The most successful country in stimulating an adoption path is Japan, that, from being a PV producer just for small devices like calculators and watches, became the sector leader in less than ten years. Governmental and public institutions were required to install PV systems at facades and on roofs. The most important program for residential PV dissemination was the “70.000 Roofs” program, ending in 2002 after exceeding all objectives. Today the market for PV systems in Japan is largely self-supported and driven by market mechanisms. The most recent data on cumulative PV installations, provided by IEA, document an installed base of about 1700 MW (USA:624, Spain:118, Italy:50, for a direct comparison). Guidolin and Mortarino have applied a standard Bass model with parametric origin, because data are not available for the initializing phases of diffusion. The results of this application are summarised in Table 2.

| PV diffusion in Japan (data source: IEA 1992-2006) |
|---|---|---|---|---|
| m | p | q | c | R² |
| 2778 | 0.0001 | 0.420592 | 5.76226 | 99.9865 |
| (2535) | (-0.00074) | (0.404772) | -12.9056 | |
| (3020) | (0.00096) | (0.436412) | 24.4301 | |

Table 2. Parameter estimates. Asymptotic 95% confidence intervals into parentheses.

Estimation results are quite good, in spite of some uncertainty in confidence intervals for parameters $p$ and $c$. However, the estimate of the market potential $m$ seems rather stable, suggesting that Japan is probably going to saturate its domestic market in less
than ten years, as one may observe in Figure 6. Interestingly, we may see that parameter $q$ presents a quite high value, pointing out the importance of the imitative component in PV adoptions in Japan. Among the cases analysed in Guidolin and Mortarino (2007), that of Japan is one for which the standard Bass model fits well data, avoiding the use of more complex models. In other cases, the application of a Generalised Bass model, presented in section 1.4, is essential for recovering the impact on diffusion of external interventions, such as incentive measures and other forms of market stimulation. This does not indicate that the adoption process in Japan was not characterized by external interventions and incentive measures: on the contrary, it suggests that these actions represent a structural element of the whole diffusion from its origins, so that observed data, that is adopting behaviour, incorporate these as normal rules of the process.

Figure 5. Plot of fitted model: a standard Bass model yields satisfactory estimates, $R^2 = 99.98$ percent.
Figure 6. Predicted vs. observed cumulative data: Japan is going to saturate its domestic market in about ten years.

1.4 The introduction of marketing mix variables: the Generalized Bass Model

Reviews on diffusion models (Mahajan and Muller, 1979; Mahajan and Wind, 1986; Mahajan, Muller and Bass, 1990) had pointed out that a great limitation of the Bass model was not incorporating into the model marketing mix variables under managerial control, like price strategies and advertising. As clarified by Muller, Peres and Mahajan (2007) this omission raised a conceptual conflict since the model provides high level of fit and reliable forecasts just making some hypotheses on consumers’ behaviour and without marketing mix variables, but on the other side it is clear that marketing mix decisions exert a notable impact on new product growth. Besides, the shortening of life cycles due to the growth of successive generations (see Norton and Bass, 1987), especially for high technology products, increased the need of a model with control variables incorporated.
Bass, Jain and Krishnan (2000) provided a notable review on several attempts trying to incorporate control variables into diffusion models. Among these, we recall models including price effects alone, namely Robinson and Lakhani (1975), Bass (1980), Kalish (1985), Kamakura and Balasubramanian (1988), Jain and Rao (1990), Horsky (1990), and advertising alone, namely Horsky and Simon (1983) and Simon and Sebastian (1987). In particular Bass, Jain and Krishnan (2000) list some desirable properties of a diffusion model with decision variables: it should have empirical support and should be managerially useful, allowing a direct interpretation of parameters and comparisons with other situations, should have a closed-form solution and be easy to implement.

The model presenting all these properties, formalized by Bass, Krishnan and Jain (1994), is the Generalized Bass Model, GBM. Conceived for taking into account both price and advertising strategies, the Generalized Bass Model enlarges the basic structure of the Bass model by multiplying its basic structure by a very general intervention function \( x(t) = x(t, \vartheta), \vartheta \in R^k \), assumed to be essentially nonnegative and integrable.

The GBM presents a surprisingly simplified structure

\[
\frac{dz}{dt} = \left( p + q \frac{z}{m} \right) (m - z) x(t)
\]

and its closed-form solution is, under initial condition \( z(t = 0) = 0 \)

\[
z(t) = m \frac{1 - e^{-(p+q) \int_{t_0}^{t_1} x(\tau) d\tau}}{1 + \frac{q}{p} e^{-(p+q) \int_{t_0}^{t_1} x(\tau) d\tau}}
\]
The original form of function $x(t)$ as designed by Bass, Krishan and Jain (1994) jointly considers the percentage variation of prices and advertising efforts taking the form 

$$x(t) = 1 + \beta_1 \frac{Pr'(t)}{Pr(t)} + \beta_2 \frac{A'(t)}{A(t)},$$

where $Pr(t)$ and $A(t)$ are price and advertising at time $t$. One interesting feature of the GBM is that it reduces to the Bass model, when $x(t) = 1$, i.e. when there are no changes in price and advertising. Besides, if the percentage changes in price and advertising remain the same from one period to the next, then function $x(t)$ reduces to a constant, yielding again the Bass model. This would explain why the Bass model provides good parameter estimates, even without marketing mix variables. The GBM can be estimated by a NLS procedure, so that its implementation is quite easy: this generalization allows to test the effect of marketing mix strategies on diffusion and to make scenario simulations based on intervention function modulation. Interestingly, what was clarified with the publication of “Why the Bass model fits without decision variables” (1994) by Bass, Krishnan and Jain is that the model internal parameters $m, p, q$ are not modified by these external actions.

Function $x(t)$ acts on the natural shape of diffusion, modifying its temporal structure and not the value of its internal parameters: as a consequence, the important effect of $x(t)$ is to anticipate or delay adoptions, but not to increase or decrease them. In other words, function $x(t)$ may represent all those strategies applied to control the timing of a diffusion process, but not its size. Though this function was originally conceived to represent marketing mix variables, its structure is so general and simplified that it can take various forms, in order to depict external actions other than marketing strategies. For example, it has proven to be suitable for describing interventions that may interact with diffusion, like political, environmental and technological upheavals (see for
example Guseo 2004; Guseo and Dalla Valle, 2005; Guseo, Dalla Valle and Guidolin, 2007; Guidolin and Mortarino, 2007). A drastic perturbation, whose effect is strong and fast, may be modeled through exponential function components like

\[ x(t) = 1 + c_1 e^{b_1(t-a_1)} I_{t-a_1} + c_2 e^{b_2(t-a_2)} I_{t-a_2} + c_3 e^{b_3(t-a_3)} I_{t-a_3}, \]

where parameters \( c_i, i = 1, 2, 3 \) represent the depth and sign of interventions, \( b_i, i = 1, 2, 3 \) describe the persistency of the induced effects and are negative if the memory of these interventions is decaying to the stationary position (mean reverting), \( a_i, i = 1, 2, 3 \) denote the starting times of interventions, so that \( (t-a) \) must be positive. A more stable intervention acting on diffusion for a relatively long period, like institutional measures and policies, may be described by rectangular function components giving rise to intervention function,

\[ x(t) = 1 + c_1 I_{t-a_1} I_{t-b_1} + c_2 I_{t-a_2} I_{t-b_2}. \]

In this case, parameter \( c_i, i = 1, 2 \) describes the perturbation intensity and may be both positive and negative, while parameters \([a_i, b_i]\) define the temporal interval in which the shock occurs.

Interestingly, the possibility to define a flexible function \( x(t) \) has highlighted a much larger perspective on the usability of the Generalized Bass model, which may be applied as an efficient diagnostic for detecting all kinds of external actions affecting a diffusion process: in particular it has proven to be crucial for country level modelling, where innovation dynamics are significantly influenced by institutional aspects, policies, cultural and economic factors. Guidolin and Mortarino (2007) have applied the Generalized Bass model to describe the diffusion across several countries of photovoltaic solar cells: they have found that in many cases the process would not have begun without the start-up provided by policies and incentives, whose real effect should be inspected in adoption data and statistically identified for providing more reliable
analyses and better forecasts. Some applications of the GBM with a flexible intervention function in the field of energy technologies are presented in the following sub-sections.

1.4.2 A GBM with one exponential shock: the diffusion of photovoltaic energy in Germany

Together with Japan, Germany has been able to create a strong domestic market for photovoltaic cells from the early 1990s, when global warming issues led to consider solar energy as a suitable substitute for fuels for electricity needs. The good experience of Germany is likely due to the introduction of appropriate policies and incentive measures. Between 1990 and 1991 the German government passed an energy law, the “Electricity Feed in Law”, requiring all public utilities to buy electricity generated at a minimum guaranteed price. This law was replaced by the “Renewable Energy Sources Act” (EEG) in 2000, with important feed-in tariffs measures. As reported by IEA cumulative data, the installed base of PV power in Germany in 2006 was about 2800 MW and a direct inspection these historical data (IEA, 1992-2006) highlights a considerable acceleration in diffusion from 2002. This fact has suggested that a standard Bass model was not suitable for this situation, so that in Guidolin and Mortarino (2007) this adoption pattern has been modelled using a GBM with one exponential shock, to take into account the impact on growth reasonably due to favourable feed-in tariffs introduced with the EEG. The applied model is therefore
\[ z(t) = m \frac{1 - e^{-\int_{t_0}^{t} x(\tau)d\tau}}{1 + \frac{q}{p} e^{-\int_{t_0}^{t} x(\tau)d\tau}} + \epsilon(t) \] (13)

with \( x(t) = 1 + c e^{b(t-a)} \) in order to represent the exponential acceleration of the diffusion process occurred in 2003, as one may inspect by Figure 9.

The results of this application, summarised in Table 3, are particularly satisfactory. This is confirmed both by the high value of the R^2 index and by the statistical stability of all the involved parameters. The value of parameter \( q \) compared to that of \( p \) suggests that the PV adoption process has been characterised by a strong imitative component. The parameters describing the exponential perturbation are correctly identified: in particular we may observe that the intensity of this perturbation described by parameter \( c \) is positive, indicating the effectiveness of incentive measures, parameter \( a \) recognises the starting time of this exponential shock in \( t=12 \), that is in 2003, when the introduction of the EEG began to have effect on adoptions. Finally, the negative value of parameter \( b \) confirms a mean reverting situation, so that the memory of this external intervention is decreasing over time. One the most interesting results of this application clearly relates to the size of the market potential, \( m \), whose estimate slightly exceeds 6000 MW and therefore to the peak, which has been attained in 2006, as one may observe in Figure 10. These result would suggest that also in the case of Germany the domestic market for photovoltaic systems has reached a maturity stage: this is an interesting conclusion, especially if compared with many other countries, such as France, Spain and Italy that have begun to stimulate the growth of their domestic markets for photovoltaic systems only in recent years. In addition, the successful experience of
Germany apparently due to effective feed-in-tariffs, would confirm the central role of these incentive measures in PV markets’ deployment.

<table>
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<th>m</th>
<th>p</th>
<th>q</th>
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<th>b</th>
<th>c</th>
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<td>(0.4569)</td>
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<td>(-0.1011)</td>
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</tr>
</tbody>
</table>

Table 3. Parameter estimates of a GBM with one exponential shock (source: Guidolin and Mortarino, 2007), asymptotic 95% confidence intervals into parentheses.

Figure 9. PV historical diffusion in Germany 1992-2006: a considerable acceleration of the process is observed in t=12 (time origin: 1992).
1.4.3 A GBM with three exponential shocks: the case of world oil production

In Guseo, Dalla Valle and Guidolin (2007) it has been proposed the use of a GBM for modelling the world production of crude oil.
The fast economic growth that involved many countries since the 1950s was indeed sustained by a large availability of energy by hydrocarbon fuels, crude oil in particular. The development and massive diffusion of innovative technologies in transport, electricity, electric appliances, plastic materials, chemical and pharmaceutical products, artificial manures for agriculture are essentially based on crude oil transformations. The adoption and retention into society of all these oil-consuming technologies has been possible thanks to an increasing production of oil.

Oil production is at the same time the driver and the consequence of the diffusion of such energy based technologies. Since all these innovations have followed diffusion processes with limited life cycles, it has appeared a reasonable choice to model oil production itself as a diffusion process associated to them.

In this case the carrying capacity \( m \) represents the Ultimate Recoverable Resource, that is the total amount of finite resource obtainable at the end of the extraction or production process. The assumption of a constant value of \( m \), typical of the BM and GBM, seems particularly suitable for this context, since oil is a finite, not renewable resource. The proposed modelling choices have tended to incorporate the drastic changes in production implied by the historical oil crises of the 1970s and the GBM has had a prominent role in this sense. Among various modeling options proposed in Guseo, Dalla Valle and Guidolin, the most convincing, as well as statistically significant, has been a GBM with an intervention function characterized by three exponential shocks, namely 

\[
x(t) = 1 + c_1 e^b_1 (t-a_1) I_{t \geq a_1} + c_2 e^b_2 (t-a_2) I_{t \geq a_2} + c_3 e^b_3 (t-a_3) I_{t \geq a_3},
\]
Table 4 summarizes the estimates of the applied model, a GBM with three exponential shocks, which presents a very high fitting, $R^2 = 0.999994$.

Parameters $m$, $p$, $q$ describe the basic structure of oil production, represented as a diffusion process in which $m$ is the total amount of recoverable resource (oil) and parameter $p$ and $q$ describe the speed of the process. Notice the high value of $q$ with respect to $p$, denoting a dominance of the imitative behaviour. The role of function $x(t)$ has been crucial in modelling terms.

Parameters $a_i, i = 1, 2, 3$ correctly identify the historical timing of the two oil crises occurred in the 1970s (time origin: 1900) and highlight the presence of a third positive shock arising in 1951 (parameter $a_2$). A notable aspect refers to the sign of parameters $b_i, i = 1, 2, 3$ which is always positive, as can be checked in Table 1. This is a surprising fact, since the memory of a perturbation is generally negative.

The intensity of the perturbations, expressed through parameters $c_i, i = 1, 2, 3$, is negative in $c_1$ and $c_3$, denoting a decrease in production caused by the Yom Kippur war and consequent embargo in 1973 and by OPEC limitations starting in 1979; on the
contrary, it is positive in $c_2$, suggesting an exponential increase of oil consumption beginning in 1951, due to the strong economic growth after World War II. The fast diffusion of oil consuming technologies has allowed the development of an economic and social system, whose structure is largely dependent on energy availability. This may explain the positive value of parameters $b_i, i = 1, 2, 3$ which is indicative of a persistent memory of the positive shock of 1951, whose effect has not been completely balanced by the successive ones. Figure 12 may help to appreciate the role of these perturbations in modifying the natural structure of the diffusion process.

![Figure 12. World oil production: Generalized Bass Model with three shocks vs. Bass Model (Guseo, Dalla Valle, Guidolin, 2007).](image)

In Figure 12 the dots represent the daily oil production per year, the continuous line a Bass model without intervention, the broken line the GBM with three shocks.

It is easy to verify the improvement in terms of fitting obtained through the application of a GBM with respect to a simple s-shaped approach, without interventions, here represented with a standard Bass model, as generally applied in oil depletion models following the pioneering work of Hubbert.
Besides, the deviation in production started in 1951 is particularly evident, generating a strong contraction of the diffusion process and an anticipation of the saturation point, that is the depletion of oil.

The case of oil production well illustrates the effect exerted by function \( x(t) \), which is able to modify the structure of diffusion, but not its characteristic parameters, namely \( m, p, q \). As is clear observing Figure 12, an acceleration in production and consumption implies less available resource for the future, since the amount of recoverable resource (carrying capacity \( m \)) is fixed.

Moreover, it shows that the diffusion of knowledge and of corresponding oil based technologies play a conclusive role in driving this particular life cycle. Adoption decisions may be partially governed and controlled, but eventually they are the result of an independent learning process, that has asserted certain cultures and lifestyles into social systems. The imitative component in adoptions expressed through a word-of-mouth effect has proven to be largely dominant, as evidence of the importance of this learning process.

In predictive terms, the application of a GBM to world oil production positions the peak date in 2007 and the 90% depletion time in 2019, assuming no external perturbations, i.e. \( x(t) = 1 \). Different hypotheses have been also considered, in order to take into account external perturbations. As a first hypothetical scenario Guseo, Dalla Valle and Guidolin (2007) have supposed that after the oil peak in 2007 some international political actions may be positively introduced: assuming two interventions trying to limit production similar to those of 1973 and 1979 and located around 2008
and 2013 yields a shift of oil production of 3 or 4 years. These limitations may be useful for improving current energy alternative solutions.

On the contrary, increasing energy consumption in developing countries such as China and India would motivate the hypothesis of a positive shock to oil demand and production similar to that occurred in 1951: under this scenario we obtain that the peak is shifted of one year (2008) but is followed by a contraction of 1 year of depletion time (2018). In this perspective, technlogical transitions to other energy sources seem particularly pressing.

Another noteworthy aspect of this application proposed by Guseo, Dalla Valle and Guidolin (2007) refers to the role of prices in crude oil adoption process: several model performances have excluded a central role of prices in defining the dynamics of this particular diffusion process.

1.5 Modelling diffusion across space and time

1.5.1 Cross-country and spatial diffusion

The initial application of the Bass model was limited to the study of the diffusion of new products within the United States. A huge body of subsequent research has focused on country level-diffusion, trying to understand to what extent diffusion can vary between nations and which are the factors determining different growth patterns among countries. A salient result of this research is that diffusion processes can present strong differences among countries even for the same products and even within the same continent (for a review on this topic see Muller, Peres and Mahajan, 2007). It has been documented that country specific characteristics, like income, life style, health status, urbanization and access to media clearly have an impact on diffusion. In addition,
market related sources, such as regulation, competition and price levels will reasonably affect diffusion, though few studies have so far investigated this aspect. In general, it may be argued that country-differences that result in variations in diffusion parameters may be explained through cultural sources, economic sources and market structure sources. A particular effort seems to be due in order to understand if the same patterns and concepts employed for developed countries also apply for developing countries, since the knowledge about these ones is still quite limited. One benefit of modelling diffusion across several countries is the possibility to whether later adopting countries adopt more quickly than earlier adopters. For example, Takada and Jain (1991) have used the Bass model to study the diffusion of various durable goods among several countries, finding that those with different cultures such as USA and Korea are characterized by different coefficients of imitation. In addition, they have found that a lagged product introduction leads to accelerated diffusion. In this perspective, Kalish, Mahajan and Muller (1995) have argued that potential adopters in lagging countries observe the introduction and diffusion of technology in the lead country: if the product is succesful in leading countries, then the risk associated with the innovation is reduced, thus inducing an accelerated diffusion in lagging countries. Muller, Peres and Mahajan (2007) have noticed that the level of acceptance of an innovation in a country acts a signal for others.

Considered from a broader perspective, cross-country diffusion is a special case of spatial diffusion. The main question concerning the study of diffusion from spatial perspective is whether additional information about spatial aspects of diffusion may help forecasts and analyses. Indeed, research on spatial diffusion is quite scarce in the field of marketing, while it has a long tradition in the field of geography and agricultural
history, originating in the pioneering work of Hagerstrand (1953). In marketing research, some efforts have been made by Redmond (1994), who has argued that diffusion models typically assume spatial homogeneity by examining the process at the national level. Applying the Bass model to the diffusion of two consumer durables across nine regions within the USA, he has found that differing local and demographic conditions across regions lead to differences in diffusion patterns within the same country. Garber, Goldenberg, Libai and Muller (2004) have faced the issue of predicting innovation success from the spatial conditions of potential adopters. Using Complex Systems Analysis, they have reported that spatial proximity seems to facilitate the formation of clusters and therefore stimulate positive word-of-mouth. As indicated by Muller, Peres and Mahajan (2007) further research effort on spatial diffusion is due to formalize the intuition that spatial conditions can both facilitate or impede diffusion, so that this factor cannot be neglected when studying new product growth processes. In this sense, a methodological framework, including definitions, measures and tools is certainly needed.

1.5.2 Some aspects on diffusion across generations of technologies

Since the publication of the diffusion model for successive generations by Norton and Bass (1987), there has been a considerable interest in analysing growth processes across technology generations, for example with the works of Norton and Bass (1992), Mahajan and Muller (1996), Bass and Bass (2001; 2004). Because newer technologies are continually replacing older ones at decreasing time intervals, the importance to understand the impact of new technologies on older ones increases (see Norton and Bass, 1987). In the Norton and Bass model, the substitution effect diminishes the potential of earlier technologies in two ways: first, customers who would have adopted
the earlier generation, eventually will choose to adopt the later one and second, customers that have already adopted the old generation may disadopt it and, in turn, adopt the new one. In any case, newer technologies are not chosen immediately by all potential buyers: as a consequence, the diffusion process of an earlier technology may continue, even if substitution dynamics are occurring, especially when the time interval between technologies is short.

The Norton and Bass approach effectively succeeded the models on technological substitution, where one technology replaced its predecessor (see, for instance, Fisher and Pry, 1971; Sharif and Kabir, 1976). Norton and Bass (1992) applied their model to several data series, taken from electronics, pharmaceutical and industrial sectors. Mahajan and Muller (1996) extended the Norton and Bass model in order to consider the case in which consumers skip generations and demonstrated the validity of this extension using data on generations of IBM mainframe computers.

As pointed out by Muller, Peres and Mahajan (2007) research on successive generations is relevant because it is possible to forecast the temporal pattern of future generations of technology, based on diffusion models of past generations. Indeed, the Bass model cannot be applied for predicting the growth pattern of technologies prior to their launch or when just few data are available, so that it has been proposed to use parameters of past generations and apply them to the newest one, except for the market potential. Clearly, this may be done under the hypothesis that diffusion parameters remain constant across technology generations. Several studies have confirmed this hypothesis and have showed that assuming constant parameters across generations yields excellent results in fitting terms. At the same time, other studies have showed that the temporal development of diffusion accelerates across generations of technology.
(Van den Bulte and Stremersch 2004, 2006; Kohli, Leman and Pae, 1999). As pointed out by Muller, Peres and Mahajan (2007) these two branches of research have reached contradictory conclusions: in fact, while the successive generation branch has shown that parameters remain constant across generations, that of temporal growth of diffusion has documented that the speed of diffusion of new products accelerates over time. Apparently, this raises an interesting paradox, that would probably deserve to be investigated in depth. A possible and preliminary explanation of this paradox will be proposed in this work at the end of Chapter 3.

1.6 Refinements and extensions of the Bass model: some directions of research

This chapter has been dedicated to introduce the theme of innovation diffusion modelling and in particular to present the most famous and employed innovation diffusion model, the Bass model. Some applications of this model in its standard and generalized versions have been proposed to show that the innovation diffusion approach is suitable for studying a broad set of contexts, including that fundamental of energy sources.

This conclusive section is dedicated to present some directions of research suggested in the most recent reviews of innovation diffusion models, with a particular focus on Mahajan, Muller and Bass (1990), Meade and Islam (2006), Mahajan, Muller and Wind (2000), Muller, Peres and Mahajan (2007). In fact, these reviews constitute an essential reference for the work proposed in this thesis.

Mahajan, Muller and Bass (1990) have pointed out that several assumptions underlie the Bass model. Though most of them are simplifying hypotheses that allow a
parsimonious representation of diffusion, they nonetheless deserve to be discussed. An important assumption relates to the market potential, \( m \), whose size is determined at the time of introduction of an innovation and remains constant along the whole diffusion process. The authors observe that there is no theoretical rationale for this assumption to apply, so that modelling a dynamic market potential may be a reasonable purpose of research. Since establishing market potential as early as possible is a priority for forecasts and evaluations, Meade and Islam (2006) have stressed that the identification of factors determining the market potential would be a fruitful idea of research.

Another assumption characterizing the Bass model is that it describes diffusion as a binary process: indeed, in the BM potential adopters either adopt or not adopt. Consequently, stages in adoption, like awareness and knowledge, are not considered. Though attempts to model multistage diffusion were made in the past, the final implementation of these models was rather cumbersome, requiring further effort to describe diffusion with a multistage structure. Moreover, Mahajan, Muller and Bass (1990) have observed that not all products are accepted by consumers at the time of their introduction. In other words, some products are much slower than others in “taking-off”. Since the “take-off” phenomenon is not explicitly considered in the Bass model, extensions that try to incorporate this phenomenon would be desirable. Chandrasekaran and Tellis (2006) and Muller, Peres and Mahajan (2006) have focused on stages of product life cycle, reporting that this is characterized by two turning points, take-off and saddle, needing to be carefully examined. The take-off has been defined as the first dramatic and sustained increase in a new product’s sales, while the saddle is the beginning of a period of slowly increasing or temporarily decreasing product sales. The presence of a saddle was first documented by Moore (1991), who noticed that
innovative high-tech products may experience a sudden cut-off in sales after an initial rapid growth: this intuition was empirically tested and formalized independently by Goldenberg, Libai and Muller (2002) and Golder and Tellis (2004). Goldenberg, Libai and Muller (2002) defined the saddle as an initial peak that predates a trough of sufficient depth and duration, followed by sales that eventually exceed the initial peak. Indeed, there is still no consensus on its importance and its drivers. Chandrasekaran and Tellis (2004) maintain that if the pattern proves to be regular, it represents a challenge for research to model it and integrate it in basic diffusion models.

Mahajan, Muller and Bass (1990) devoted particular attention to the issue of understanding diffusion processes at the micro level, maintaining that empirical evidence provided by Chatterjee and Eliashberg (1989) on the development of aggregate diffusion models from individual level adoption decisions, was encouraging. Interestingly, individual level modelling is reproposed as a central topic of research in Muller, Peres and Mahajan (2007), but from a different perspective, aimed at describing adoption behaviour of the single agent with the recent tools of Complex Systems Analysis, namely Agent-Based models and Network models. The application of such models to diffusion processes is intended at describing an aggregate behaviour with a bottom-up view, mapping the formation and the evolution of networks of interacting agents. Indeed, diffusion of innovations is strongly connected to the existence of networks of agents, that share information between them.

Various types of information sharing create interdependences between customers. It is the importance of such interdependences in determining consumers’ decisions that has suggested to re-define the diffusion of innovations as the growth of new products and services driven by consumer interdependences that tie the utilities of various
market players together even without their explicit knowledge (see Muller, Peres and Mahajan, 2007). Recognizing the role and the characteristics of these ties is therefore considered a major issue in current diffusion research. Muller, Peres and Mahajan (2007) argue that these interdependencies take essentially three forms: word-of-mouth communications, signals and network externalities.

Following this distinction, word-of-mouth communications represent all those situations in which consumers collect and process information about a given product through verbal communications, like conversations, e-mails, virtual communities. In this perspective, word-of-mouth communications are intended as deliberate actions to gain relevant information about a product and possibly reduce the uncertainty associated with a purchase decision. In this sense, signals are considered a kind of market information other than personal recommendation. They refer to what a consumer may perceive just observing the number of individuals that have made a certain choice. Observation may be a strong carrier of information, inspiring imitative actions. The argued difference between word-of-mouth communications and signals is that signals do not need any interpersonal tie to be effective. Network externalities are a property of those goods and services that become more valuable as the number of their users increases. Interactive innovations, like telecommunications products and services are generally characterized by strong network externalities. These effects may be direct, when the utility of a consumer is directly affected by the number of other users, or indirect, if there is a market mediation. The presence of network externalities may have a great influence in the diffusion of an interactive innovation.

Muller, Peres and Mahajan have pointed out the need of further modelling efforts to distinguish between these effects in a clear manner, both theoretically and empirically.
This would provide a wider range of tools for treating different mechanisms in different ways and better understanding their effect on growth.

Another noteworthy research topic suggested by Muller, Peres and Mahajan is modelling the diffusion of services: one of the characterising aspects of services that can have a great impact on diffusion is that of disadoption or churn. Not considering the importance of this phenomenon in services’ context may introduce considerable bias in parameter estimates and forecasts, so that this element should be always included in analysing the diffusion of services. More in general, shadow diffusion, that is, any diffusion process accompanying and influencing the major one, but not captured in sales data, like piracy or negative word-of-mouth, deserves to be investigated with increasing detail (Muller, Peres and Mahajan, 2007).