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Measurements of Partial Branching Fractions for Charmless Semileptonic B Decays with the BaBar Experiment and Determination of $|V_{ub}|$

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Introduzione

L’argomento della tesi è la misura del modulo dell’elemento $V_{ub}$ della matrice di Cabibbo-Kobaishi-Maskawa (CKM). La misura del parametro $|V_{ub}|$ è particolarmente importante per testare i meccanismi della violazione della simmetria di CP nel Modello Standard, perché $|V_{ub}|$ è proporzionale alla lunghezza del lato del Triangolo Unitario opposto all’angolo $\beta$. Mentre la misura dell’angolo beta viene effettuata con processi che coinvolgono diagrammi a loops (attraverso l’oscillazione $B^0 - \bar{B}^0$), il parametro $|V_{ub}|$ viene estratto dallo studio dei decadimenti semileptonici del mesone $B$, che essendo processi dominati dai diagrammi albero sono, con ottima accuratezza, liberi dalla presenza di Nuova Fisica. Il confronto diretto tra la misura di $\sin 2\beta$ e quella di $|V_{ub}|$, fornisce quindi un test diretto dell’unitarita’ della matrice CKM e puo’ mettere in evidenza la presenza di Nuova Fisica nei loops. Attualmente, $\sin 2\beta$ è conosciuto con errore del 3%, mentre $|V_{ub}|$ ha un errore del 10%, per cui è cruciale ridurre l’errore sul parametro $|V_{ub}|$.

La larghezza del decadimento $B \to X_u \ell \nu$, (dove $X_u$ è uno stato finale inclusivo in cui non sono presenti adroni con charm) è proporzionale a $|V_{ub}|^2$. La difficoltà nello studio del decadimento $B \to \ell \nu$ inclusivo, è data dalla presenza del fondo dovuto all’altro processo di decadimento semileptonico (con charm nello stato finale) $B \to X_c \ell \nu$. Il rapporto segane su rumore è dunque $R \sim |V_{ub}|^2/|V_{cb}|^2 \sim 1.50$. Il fondo fisico da $B \to X_u \ell \nu$ viene soppresso sfuttendo la differenza nelle variabili cinematiche tra le transizioni Cabibbo-soppresse $b \to u$ e quelle Cabibbo-favorite $b \to c$, indotte dalla grande differenza di massa tra il quark $u$ e il quark $c$. Si vietano inoltre quegli eventi in cui la presenza di Kaoni puo’ indicare la produzione intermedia di adroni charmati. Per effettuare questo tipo di studi, si è rivelato molto utile utilizzare un campione di dati in cui un mesone $B$ fosse completamente ricostruito attraverso i suoi prodotti di decadimento adronici, detto campione di $B_{reco}$. In questo modo tutte le altre particelle dell’evento che non contribuiscono alla ricostruzione del $B_{reco}$ devono
essere associate al mesone che decade semileptonicamente nel lato di segnale. Questa tecnica, anche se riduce l’efficienza di ricostruzione del segnale (l’efficienza di ricostruzione del $B_{reco}$ e’ dell’ordine di 0.5\%), permette di ridurre notevolmente i fondi combinatori, e fornisce ulteriori vincoli cinematici per la ricostruzione inclusiva del decadimento semileptonico. Il lavoro di tesi e’ organizzato come segue.

Nel primo capitolo viene presentata una trattazione teorica della violazione di CP e dei decadimenti semileptonici inclusivi senza stati charmati del mesone B.

Il secondo capitolo e’ dedicato alla descrizione dell’esperimento BABAR.

Il terzo capitolo e’ basato sulle tecniche sperimentali utilizzate per la ricostruzione completa di un mesone B attraverso i suoi decadimenti adronici.

Nel quarto capitolo vengono mostrati i diversi campioni con cui si e’ effettuata l’analisi.

Il quinto capitolo e’ dedicato allo studio di particolari criteri di selezione del segnale in modo da ridurre la componente di $B \rightarrow X_c \ell \nu$ che rappresenta il fondo dominante.

Nel sesto capitolo viene presentata la strategia di fit che verrà utilizzata per misurare i vari partial branching ratio di $B \rightarrow X_u \ell \nu$ nelle regioni cinematiche che presentano il migliore rapporto $S/B$.

Il settimo capitolo e’ concentrato sullo studio delle incertezze teoriche che caratterizzano la varie misure effettuate.

L’ottavo capitolo e’ dedicato alla estrazione di $|V_{ub}|$ dalle varie misure di partial branching fraction svolte.
Introduction

The study of the B meson properties offers crucial tests of the flavor physics in the Standard Model and the source of CP violation in nature.

The \textit{BaBar} experiment at PEP-II and Belle at KEK-B in the 2001 observed with 4 sigma significance the CP violation in the interference of the $B^0$ mixing and the B decays, which also allows a precise measurement of the angle $\beta$ of the Unitary Triangle. The most recent measurements of the CP, allows to measure $\beta$ with a relative uncertainty of only 4%. In the Unitary Triangle picture, the CKM matrix element $|V_{ub}|$, the strength of the coupling of the $b$ quark to $u$ quark, play a peculiar role because its magnitude is the length of the side of the UT opposite to the angle $\beta$. This means that a precise determination of $|V_{ub}|$ is crucial to enhance the constraints of the unitarity of the CKM matrix and the Standard Model consistency.

The theoretically cleanest determination of $|V_{ub}|$ is obtained using charmless semileptonic $b \rightarrow u\ell\nu$ transition. In particular the inclusive decay $B \rightarrow X_u\ell\nu$, where no attempt is made to completely reconstruct the final state $X_u$, are particularly interesting since the relevant contributions to the dominant theoretical uncertainty can be factorized and computed systematically to a high level of accuracy.

The experimental approach presented in this thesis uses $Y(4S) \rightarrow BB$ transitions to study $B \rightarrow X_u\ell\nu$ decays recoiling against a $\bar{B}$ meson which is fully reconstructed in an hadronic final state. The dominant background due to semileptonic decays with charm, about a factor 50 larger than charmless decays, is reduced by kinematic requirements which allow to identify phase space regions with a manageable signal over background ratio. In general, these requirements introduce sizeable theoretical uncertainties when the extrapolation to the full phase space, needed in order to determine $|V_{ub}|$, is performed. In order to avoid that also a full phase space measurement has been performed. The determination of $|V_{ub}|$ therefore depends on the interplay between a careful optimization of signal over background ratio and
the minimization of theoretical uncertainties.

This thesis is organized as follows. The theory of inclusive charmless semileptonic $B$ decays is reviewed in Chapter 1, together with a brief reminder of the electroweak sector of the standard model and the CKM mechanism. Detector is presented in Chapter 2, where characteristics and specific performances of each $\text{Babar}$ sub-detector are outlined and briefly discussed. The experimental techniques used to reconstruct events, identify particles and resonances, and fully reconstruct a $B$ meson into hadronic final states, are presented in Chapter 3. The event samples used in this thesis, consisting of both real and simulated data, are detailed in Chapter 4. Chapter 5 shows the selection criteria applied in order to select the signal sample. Measurements of partial branching fraction in regions of restricted phase space and total branching fraction for inclusive charmless decays are reported in Chapter 6, the associated systematic uncertainties are discussed in Chapter 7. Determinations of the CKM matrix element $|V_{ub}|$ and conclusions are presented in Chapter 8.
Chapter 1

CKM Matrix and Semileptonic $B$ Decays

The theory of inclusive charmless semileptonic $B$ decay is reviewed in this chapter, together with a brief reminder of the electroweak sector of the standard model and the CKM mechanism. In section 1.3 the theoretical framework needed for $|V_{ub}|$ extraction is presented while in section 1.3.4 a short review of the existing inclusive measurements is shown.

1.1 The Electroweak Sector of the Standard Model

The electroweak sector of the SM is a gauge theory based on the local group $SU_L(2) \otimes U_Y(1)$, which describes the symmetries of the matter field. The Yang-Mills electroweak Lagrangian is [1]

$$\mathcal{L} = -\frac{1}{4} \Sigma_A W_{\mu \nu}^A W^{A\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} + \bar{\Psi}_L i \gamma^\mu D_\mu \Psi_L + \bar{\Psi}_R i \gamma^\mu D_\mu \Psi_R$$

where the spinors $\Psi_L$ and $\Psi_R$ represent the matter fields in their chiral components, and the field strength tensors are given by:

$$W_{\mu \nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g \epsilon_{ABC} W^B_\mu W^C_\nu$$ and $$B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Here $W^A$ and $B$ are the SU(2) and U(1) gauge fields, with the coupling constants $g$ and $g'$ and $\epsilon_{ABC}$ is the totally anti-symmetric Levi-Civita tensor. The corresponding covariant derivative
is:
\[
D_\mu \Psi_{L,R} = \left[ \partial_\mu + ig \Sigma t^A_{L,R} W_{A_\mu} + ig' \frac{1}{2} Y_{L,R} B_\mu \right] \Psi_{L,R},
\]
where \( t^A_{L,R} \) and \( \frac{1}{2} Y_{L,R} \) are the SU(2) (weak isospin) and U(1) (hypercharge) generators. The electric charge generator is related to the isospin and hypercharge by:
\[
Q = t^A_L + \frac{1}{2} Y_L = t^A_R + \frac{1}{2} Y_R.
\]
The left and the right fermion components have different properties under the gauge group. The left component behave as doublets while the right as singlets. In the symmetric limit the two chiral component cannot interact each other, and thus mass term for fermions (of the form \( \bar{\Psi}_L \Psi_R \)) are forbidden. To give mass terms to fermions as well as to gauge bosons without loosing the symmetry properties, the electroweak theory is realized with a vacuum state invariant only under the \( U_{EM}(1) \) electric charge gauge transformation. This phenomenon is known as “spontaneous symmetry breaking”. The gauge theories spontaneous broken allow to introduce mass terms for the gauge boson and the fermion fields without loosing the gauge invariance, and the renormalizability of the theory. The mechanism by which, starting from a degenerate vacuum state, mass terms are introduced is known as Higgs mechanism [2]. The Higgs Lagrangian term is:
\[
\mathcal{L}_{Higgs} = (D_\mu \phi) \dagger (D^\mu \phi) - V(\phi^\dagger \phi) - \bar{\Psi}_L \Gamma \Psi_R \phi - \bar{\Psi}_R \Gamma^\dagger \Psi_L \phi^\dagger,
\]
where \( \phi \) is the isospin doublet of the Higgs scalar fields and the quantities \( \Gamma \) (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz gauge groups. The general form of the Higgs potential is:
\[
V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2,
\]
and it is not possibile to include terms with higher dimension without breaking the renormalizability of the SM. To have a vacuum state (the minimum of the potential) degenerate, the \( \mu^2 \) coefficient should be negative, while the coefficient \( \lambda \) should be positive to guarantee the potential bound from below. Under these hypotheses the vacuum state of the Higgs field satisfies \( |\phi|^2 = -\mu^2/2\lambda = v^2 \). The field \( \phi \) can be expanded around one of its ground states; in choosing a particular ground state \( (\phi_0 = (0 \ \ \nu) \) the \( SU_{L,R}(2) \otimes U_Y(1) \) symmetry is spontaneously broken.

The mass terms for the gauge bosons are coming from the kinetic part of the Higgs Lagrangian once it is expanded around the Higgs vacuum state. The correct quantum numbers of the Higgs field are fixed by the requirement that the Lagrangian (1.5) is gauge invariant.
1.1 The Electroweak Sector of the Standard Model

Since the $SU_L(2) \otimes U_Y(1)$ is spontaneously broken into $U_{EM}(1)$, only the linear combination of gauge fields with the quantum numbers of the photon remains mass-less. A general linear combination between the gauge bosons associated to the generator in Eq. 1.4 can be written:

$$
\begin{pmatrix}
A_\mu \\
Z_\mu
\end{pmatrix} =
\begin{pmatrix}
-\sin \theta_W & \cos \theta_W \\
\cos \theta_W & \sin \theta_W
\end{pmatrix}
\begin{pmatrix}
W^3_\mu \\
B_\mu
\end{pmatrix}
$$

(1.7)

where the angle $\theta_W$ is known as the Weak or Weinberg mixing angle. Once the symmetry is spontaneously broken through the interaction with the Higgs field, $A_\mu$ remains mass-less while $Z_\mu$, $W^+_\mu$ and $W^-_\mu$ acquire a mass term. $W^+_\mu$ and $W^-_\mu$ are defined as:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^3_\mu \pm iW^2_\mu).$$

(1.8)

The bi-linear terms in the fields $Z_\mu$ and $W^\pm_\mu$ in Eq. 1.5 can be identified as the mass terms:

$$M^2_Z = \frac{v^2 g^2}{2\cos^2 \theta_W}$$

(1.9)

$$M^2_W = \cos^2 \theta_W M^2_Z$$

(1.10)

which implies $\tan \theta_W = g'/g \phi$. In terms of these new fields the fermionic currents are:

$$J^\pm_\mu = \Sigma_f \bar{\Psi}^f (1 - \gamma_5) \gamma_\mu t^\pm \Psi^f$$

(1.11)

$$J^0_\mu = \Sigma_f \bar{\Psi}^f \gamma_\mu [(1 - \gamma_5)t^3 - 2Q \sin^2 \theta_W] \Psi^f,$$

(1.12)

$$J^{em}_\mu = \Sigma_f \bar{\Psi}^f \gamma_\mu Q \Psi^f,$$

(1.13)

where $\bar{\Psi}^f$ represents the isospin doublet for the fermion fields (see Tab. 1.1) with $f$ acting as a family index, $(1 - \gamma_5)$ is the left-handed chiral projector, and $t^\pm$ are the isospin generator associated to the fields $W^\pm$. The first current describes interactions which change the electric charge, while the other two, produce transitions charge-conserving. The Lagrangian (1.1) could be rewritten in two terms: one including interactions between the neutral current and the $A_\mu$ and $Z_\mu$ bosons, and another describing the interactions of the $W^\pm_\mu$ with the charged current:

$$\mathcal{L}_{ED} = \mathcal{L}_{CC} + \mathcal{L}_{NC},$$

(1.14)

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} (J^+_\mu W^+_\mu + J^-_\mu W^-_\mu),$$

(1.15)

$$\mathcal{L}_{NC} = -e J^{em}_\mu A^\mu + \frac{g_2}{2\cos \theta_W} J^0_\mu Z^\mu,$$

(1.16)

where $e$ is defined as $e = g_2 \sin \theta_W.$
starting from the same doublet which gives masses to the gauge bosons it is possible to introduce mass terms for the fermion fields. This imposes others restrictions on the Higgs field. To obtain fermion mass terms like:

$$-\bar{\psi}_L \Gamma \Psi_R \phi - \bar{\Psi}_R \Gamma \bar{\psi}_L \tilde{\phi} \quad \text{where} \quad \tilde{\phi} = i \sigma_2 \phi^\dagger,$$

invariant under $SU_{L,R}(2)$ transformations, the Higgs field is required to have isospin equal to 1/2. The $\Gamma$ matrices contain the Yukawa constants, which determine the strength of the fermion couplings to the Higgs fields.

The fermion mass matrix is obtained from the Yukawa couplings expanding $\phi$ around the vacuum state:

$$M = \bar{\psi}_L \mathcal{M} \psi_R + \bar{\psi}_R \mathcal{M}^\dagger \psi_L ,$$

with

$$\mathcal{M} = \Gamma \cdot v .$$

It is important to observe that by a suitable change of basis we can always make the matrix $\mathcal{M}$ Hermitian, $\gamma_5$-free, and diagonal. In fact, we can make separate unitary transformations...
on $\psi_L$ and $\psi_R$ according to
\[
\psi'_L = L\psi_L, \quad \psi'_R = R\psi_R,
\]
and consequently
\[
\mathcal{M} \rightarrow \mathcal{M}' = \mathcal{L}^\dagger \mathcal{M} R.
\]
This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}$.

Weak charged currents are the only tree level interactions in the SM that may induce a change of flavor. By emission of a W boson an up-type quark is turned into a down-type quark, or a $\nu_\ell$ neutrino is turned into a $\ell^-$ charged lepton. If we start from an up quark that is a mass eigenstate, emission of a W turns it into a down-type quark state $d'$ (the weak isospin partner of $u$) that in general is not a mass eigenstate. In general, the mass eigenstates and the weak eigenstates do not in fact coincide and a unitary transformation connects the two sets:
\[
(\ d \ )' s'b' = V (\ d \ ) s_b,
\]
where $V$ is the Cabibbo-Kobayashi-Maskawa matrix (CKM)[3]. Thus in terms of mass eigenstates the charged weak current of quarks has the form:
\[
J^\pm_\mu \propto \bar{u}_\gamma \mu (1 - \gamma_5) t^\dagger V d,
\]
Since $V$ is unitary (i.e. $V V^\dagger = V^\dagger V = 1$) and commutes with $T^2$, $T_3$ and $Q$ (because all $d$-type quarks have the same isospin and charge) the neutral current couplings are diagonal both in the primed and unprimed basis. If the $Z$ down-type quark current is abbreviated as $d' \Gamma d'$ then, by changing basis we get $d V^\dagger \Gamma V d$ and $V$ and $\Gamma$ commute; it follows that $d' \Gamma d' = d \Gamma d'$. This is the Glashow - Iliopoulos - Maiani (GIM) mechanism [4] that ensures natural flavor conservation of the neutral current couplings at the tree level.

With three fermion generations the matrix $V$ could be expressed in terms of three angles and one irremovable complex phase [5]. The CKM matrix is usually represented as:
\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\]

The irremovable phase in the CKM matrix allows possible CP violation.

The measurement of the elements of the CKM matrix is fundamental to test the validity of the SM. Many of them (actually the first two rows of the matrix) are measured directly,
namely by tree-level processes. Using unitary relations one can put constraints on the top mixing $|V_{tb}|$. Moreover the $B$ mixing measurements, that involve box diagrams, can give information also about $V_{td}$ and $V_{tb}$.

The CKM-matrix can be expressed in terms of four Wolfenstein parameters $(\lambda, A, \rho, \eta)$ with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and $\eta$ representing the $CP$-violating phase [6]:

$$V = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4). \quad (1.25)$$

$\lambda$ is small, and for each element in $V$, the expansion parameter is actually $\lambda^2$.

The Wolfenstein parametrization offers a transparent geometrical representation of the structure of the CKM matrix. The unitarity of the matrix implies various relations among its elements. Three of them are very useful for understanding the SM predictions for $CP$ violation:

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \quad (1.26)$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \quad (1.27)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (1.28)$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are “the Unitarity Triangles”. If the $CP$ symmetry is violated the area of the triangles is not zero. The $B$ physics is related to the third triangle at least for what the $B$ factory can access. The study of the parameters of this triangle encompasses the physics of $CP$ violation in the SM. The openness of this triangle, due to the fact that all the three sides are of the same order of magnitude, predicts large $CP$ asymmetries.

It should be remarked that the Wolfenstein parametrization is an approximation and neglecting $O(\lambda^4)$ terms could be wrong in particular processes. An improved approximated parametrization of the original Wolfenstein matrix is given in [7]. Defining

$$V_{us} = \lambda, \quad V_{cb} = A\lambda^2, \quad V_{ub} = A\lambda^3(\rho - i\eta), \quad (1.29)$$

defining one can then write

$$V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta}), \quad (1.30)$$
\[ \Im(V_{cd}) = -A^2 \lambda^5 \eta, \quad \Im(V_{ts}) = -A\lambda^4 \eta, \] (1.31)

where

\[ \bar{\rho} = \rho(1 - \lambda^2 / 2), \quad \bar{\eta} = \eta(1 - \lambda^2 / 2), \] (1.32)

turn out to be excellent approximations to the exact expressions. Depicting the rescaled Unitarity Triangle in the \((\bar{\rho}, \bar{\eta})\) plane, the lengths of the two complex sides are

\[ R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = 1 - \frac{\lambda^2 / 2}{\lambda} \frac{|V_{ub}|}{V_{cb}}, \] (1.33)

\[ R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \frac{|V_{td}|}{V_{cb}}. \] (1.34)

Figure 1.1: The rescaled Unitarity Triangle, all sides divided by \(V_{cb}^* V_{cd}\).

The rescaled Unitarity Triangle (Fig. 1.1) is derived from Eq. 1.28 by choosing a phase convention such that \((V_{cd} V_{cb}^*)\) is real, dividing the lengths of all sides by \(|V_{cd} V_{cb}^*|\), aligns one side of the triangle with the real axis and makes the length of this side equal to 1.
The form of the triangle is unchanged. Two vertexes of the rescaled Unitarity Triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex are denoted by \((\bar{\rho}, \bar{\eta})\). Both angles and sides can be measured in a \(B\) factory and they can offer two independent tests of the Standard Model. Inconsistencies between these two tests would indicate the presence of

Figure 1.2: Unitarity Triangle constrains from different measurements of the Standard Model parameters in the \((\bar{\rho}, \bar{\eta})\) plane, updated to the Summer 2008 results [8]. Allowed region for \((\bar{\rho}, \bar{\eta})\), using all available measurements, is shown with closed contours at 68\% and 95\% probability. The full lines correspond to 95\% probability regions for the constraints, given by the measurements of \(|V_{ub}|/|V_{cb}|\), \(\epsilon_K\), \(\Delta m_d/\Delta m_s\), \(\alpha\), \(\beta\), \(\gamma\), \(\Delta \Gamma_d/\Gamma_d\), \(\Delta \Gamma_s/\Gamma_s\), \(A_{SL}^d\), and the dimuon asymmetry from \(D^0\).
1.2 Inclusive Semileptonic $B$ Decays

New Physics. The constraints on the apex of the Unitarity Triangle, obtained from different measurements, now overlap in one small area in the first quadrant in the $(\bar{\rho}, \bar{\eta})$ plane [8, 9]. The constraints from the lengths of the sides and independently those from CP violating processes indicate the consistent regions on the $(\bar{\rho}, \bar{\eta})$ plane. The $|V_{ub}|/|V_{cb}|$ constraint shown in Fig. 1.2 can be directly derived from Eq. 1.33. The CKM elements $|V_{ub}|$ and $|V_{cb}|$ therefore provide a test of the SM by over-constraining the $(\bar{\rho}, \bar{\eta})$ plane vertex with other measurements. These elements can be directly determined from $b \to u\ell\nu$ and $b \to c\ell\nu$ decays respectively.

While $|V_{cb}|$ has been measured with a 2\% uncertainty [10], $|V_{ub}|$ still remains one of the least known elements of the CKM matrix and dominates the error on the length of the side opposite to the angle $\beta$.

The Unitarity Triangle analysis shows the impressive success of the CKM picture in describing CP violation in the SM, but, with the increasing precision of the experimental results, a slight disagreement between $\sin 2\beta$ and $|V_{ub}|$ is appeared in the UT fit, as shown in Fig. 1.2. This disagreement could be due to some problem with theoretical calculation and/or with the estimate of the uncertainties on the $|V_{ub}|$ measurements. So an effort must be done for a substantial improvement of the theoretical and experimental accuracy for this quantity. In the future, if the $|V_{ub}|$ value will be confirmed by more precise data and theory calculations, the disagreement in the UT fit might reveal a bound of New Physics phase in $B_d$ mixing [8].

1.2 Inclusive Semileptonic $B$ Decays

Semileptonic $B$ decays (Fig. 1.3 shows a Feynman diagram of a charmless semileptonic $B$ decay), due to their simplicity, provide an excellent laboratory in which to measure $|V_{cb}|$ and $|V_{ub}|$. These processes also allow us to study the effects of non-perturbative QCD interactions on the weak-decay process. These goals may sound contradictory: “how can be measured standard-model parameters if complicated hadronic effects are present?"

First of all, it is notable that even if the effects of strong interactions on semileptonic decays are difficult to calculate, they are isolated to the hadronic current. As a consequence, these effects can be rigorously parametrized in terms of a small number of form factors, which are functions of the Lorentz-invariant quantity $q^2$, the square of the mass of the virtual $W$.

Particular and specific care is needed in case it is intended to study $|V_{cb}|$ or $|V_{ub}|$. For $b \to c\ell\nu$ decays, the large masses of both the $b$ and $c$ quarks provide the key to reliable
Feynman diagram for a charmless semileptonic decay $\bar{B} \to X_u \ell \bar{\nu}$.

While the determination of $|V_{ub}|$ is improving, both experimentally and theoretically, there are still large uncertainties. The problem with exclusive decays is that the strong hadronic dynamics can not be calculated from first principles, and it has to resort to models, light-cone sum rules, or lattice QCD calculations to obtain the form factor. So inclusive decays should provide a straightforward means to measure $|V_{ub}|$: one considers the sum over all possible final-state hadrons, ignoring the detailed breakdown among the individual decay modes that contribute to the semileptonic rate. Experimentally, it would be desirable to observe only the lepton, eliminating reconstruction difficulties that are often complex decay sequences of the daughter hadrons. Theoretical calculations of inclusive properties have certain advantages of simplicity as well, since calculations in which the heavy quark is assumed to decay as a free particle (with the light quark acting merely as a spectator) provide a good starting point for predictions.
1.3 $|V_{ub}|$ Extraction from Charmless Semileptonic B Decays

The theoretical tools that we have for calculating decay rates and spectra in heavy-quark physics have as underlying theme the separation of short-distance from long-distance physics, which is natural due to the presence of the large mass scale $m_Q \gg A_{QCD}$, which is far above the scale of nonperturbative hadronic physics. Factorization theorems state that short and long-distance contributions to a given observable can be separated into Wilson coefficient functions $C_i$ and nonperturbative matrix element $M_i$. Generically, one has

$$\text{Observable} \sim \sum_i C_i(m_Q, \mu)M_i(\mu) + \ldots \quad (1.35)$$

Such a factorization formula is useful, since by virtue of it the dependence on the high scale $m_Q$ is calculable, and often the number of matrix elements $M_i$ is smaller than the number of observables that can be expressed in the form shown above. The factorization scale $\mu$ serves as an auxiliary separator between the domains of short and long-distance physics. Observables are formally independent of the choice of $\mu$; however, they inherit some residual dependence once the Wilson coefficient $C_i$ are computed at finite order in perturbation theory. The dependence gets weaker as higher orders in the perturbative expansion are included.

Processes involving energetic light partons require a more sophisticated form of the factorization theorem, which generally can be expressed as

$$\text{Observable} \sim \sum_i H_i(Q, \mu)J_i(\sqrt{Q/\Lambda}, \mu) \otimes S_i(\mu) + \ldots \quad (1.36)$$

Here $Q \gg A_{QCD}$ is the hard scale of the process, e.g. a heavy-quark mass or the center-of-mass energy. The hard function $H_i$ capture virtual effects from quantum fluctuation at the hard scale. The jet functions $J_i$ describe the properties of the emitted collinear particles, whose characteristic virtuality or invariant mass scales as an intermediate scale $\sqrt{Q/\Lambda}$ between $Q$ and QCD scale $A_{QCD}$. For the inclusive processes the jet scale is in the perturbative domain ($\sqrt{Q/\Lambda} \gg A_{QCD}$). Finally, the soft functions $S_i$ describe the (nonperturbative) physics associated with soft radiation in the process. The symbol $\otimes$ implies a convolution, which arises since the jet and soft functions share some common variables, such as some small momentum components of order $A_{QCD}$.

The theoretical basis of the factorization theorem (Eq. 1.36) is a generalization of the euclidean operator product expansion to the time-like domain derived in the soft-collinear effective theory [14, 15, 16, 17].
1.3.1 Calculation of the Total Decay Rate

The total decay rate $B \to X_u \ell \bar{\nu}$ is directly proportional to $|V_{ub}|^2$, and can be calculated reliably and with small uncertainties using the Operators Product Expansion (OPE), as a double expansion in powers of $\Lambda_{QCD}/m_b$ and $\alpha_s(m_b)$ [11], according to Equation 1.35.

The $b$-quark decay mediated by weak interactions takes place on a time scale that is much shorter than the time it takes the quarks in the final state to form physical hadronic states. Once the $b$-quark has decayed on a time scale $t \ll \Lambda_{QCD}^{-1}$, the probability that the final states will hadronize somehow is unity, and it is needed not to know the probability of hadronization into specific final states. Moreover, since the energy release in the decay is much larger than the hadronic scale, the decays is largely insensitive to the details of the initial state hadronic structure.

This intuitive picture is formalized by the OPE, presented in [18], which expresses the inclusive rate as an expansion in inverse power of the heavy quark mass with the leading term corresponding to the free quark decay. Let us consider, as an example, the inclusive semileptonic $b \to c$ decay, mediated by the operator $O_{sl} = -4G_F/\sqrt{2}V_{cb}(J_{bc})^\alpha(J_{\nu})_\alpha$, where $J_{bc}^\alpha = (\tau^\alpha P_L b)$ and $J_{\nu}^\beta = (\bar{\nu}^\beta P_L \nu)$. The decay rate is given by the square of the matrix element, integrated over phase space ($\Phi$) and summed over final states ($X_c$),

$$\Gamma(B \to X_c \ell \bar{\nu}) \sim \sum_{X_c} \int d[\Phi]|\langle X_c | J_{\nu} \rangle|^2$$

(1.37)

Since the leptons have no strong interactions, it is convenient to factorize the phase space into $B \to X_c W^*$ and a perturbative calculable leptonic part, $W^* \to \ell \bar{\nu}$. The nontrivial part is the hadronic tensor,

$$W^{\alpha\beta} \sim \sum_{X_c} \delta^4(p_B - q - p_{X_c})|\langle B | J_{bc}^\alpha | X_c \rangle |\langle X_c | J_{bc}^\beta | B \rangle|^2$$

(1.38)

$$\sim \Im \int dx \ e^{-iq \cdot x} \langle B | T J_{bc}^\dagger(x), J_{bc}^\beta(0) | B \rangle,$$

(1.39)

where the second line is obtained using the optical theorem, and $T$ denotes the time ordered product of the two operators. This is convenient because the time ordered product can be expanded in local operators in the $m_b \gg \Lambda_{QCD}$ limit. In this limit the time ordered product is dominated by short distances, $x \ll \Lambda_{QCD}^{-1}$, and one can express the nonlocal hadronic tensor $W^{\alpha\beta}$ as a sum of local operators.
Schematically,

\[
\begin{align*}
\Gamma_{\bar{b} \to c} & = \Gamma_0 + \frac{G_F^2 m_b^5}{192\pi^3} (1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln \rho), \\
\end{align*}
\]

where \( \rho = m_q^2/m_b^2 \). Non-perturbative corrections are suppressed by at least two powers of \( m_b \). There are no \( \mathcal{O}(A_{QCD}/m_b) \) corrections, because the \( b \) meson matrix element of any dimension-4 operator vanishes. The resulting expression for the total rate of the semileptonic \( B \to X_c \ell \bar{\nu} \) has the form

\[
\Gamma_{b \to c} = \Gamma_0 \left[ 1 + A \left( \frac{\alpha_S}{\pi} \right) + B \left( \frac{\alpha_S}{\pi}^2 \beta_0 \right) + C \left( \frac{\alpha_{QCD}^2}{m_b^2} \right) + D \left( \frac{\alpha_{QCD}^3}{m_b^3}, \frac{\alpha_S}{m_b^2} \right) \right]
\]

where the coefficients \( A, B, C \) depend on the quark masses \( m_c, b \). The perturbative corrections are known up to order \( \alpha_S^3 \beta_0 \). Non-perturbative corrections are parametrized by matrix elements of local operators. The \( \mathcal{O}(A_{QCD}/m_b^2) \) corrections are given in terms of the two terms matrix elements

\[
\begin{align*}
\lambda_1 & = \frac{1}{2M_B} \langle B | h_v (iD)^2 h_v | B \rangle, & \lambda_2 & = \frac{1}{6M_B} \langle B | h_v g_{\mu\nu} G^{\mu\nu} h_v | B \rangle \end{align*}
\]

The dependence on these matrix elements is contained in the coefficient \( C \equiv C(\lambda_1, \lambda_2) \). Up to higher order corrections, the connection to an alternative notation is \( \lambda_1 = -\mu_b^2 \) and \( \lambda_2 = \mu_b^2/3 \). At order \( 1/m_b^3 \) there are two additional matrix elements. Thus, the total decay rate depends on a set of non-perturbative parameters, including the quark masses with the number of such parameters depending on the order in \( A_{QCD}/m_b \) one is working.

The same expansion can be written in case of \( b \to u \) semileptonic transitions, with the Equation 1.41 and 1.42 that have perturbative corrections, known through order \( \alpha_S^3 \) [65], as follows:

\[
\Gamma_{\bar{b} \to u} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[ A_0 \left( 1 - \frac{m_b^2}{2m_b^2} \right) - 2 \frac{\mu_b^2}{2m_b^2} + \mathcal{O} \left( \frac{1}{m_b^3} \right) \right].
\]
with the leading order corrections given in terms of $\mu^2 e \mu^2 G$. The leading term includes purely perturbative corrections (embedded in the coefficient $A_0$).

These perturbative effects have been calculated, and no significant uncertainties are expected from yet uncalculated higher order perturbative effects. The corrections depend on $\alpha_S$ and $m_b$ which are scale dependent. The largest term is proportional to $1/m_b^2$ and thus is of order 5%, leading to a reduction in the decay width by $\approx 4\%$. Similar results can be derived for differential distributions, as long as the distributions are sufficiently inclusive. To quantify this last statement, it is crucial to remember that the OPE does not apply to fully differential distributions but requires that such distributions be smeared over enough final state phase space. The size of the smearing region $\Delta$ introduces a new scale into the expressions for differential rates and can lead to non-perturbative corrections being suppressed by powers of $A_{QCD}^n/\Delta^n$ rather than $A_{QCD}^n/m_b^n$. Thus, a necessary requirement for the OPE to converge is $\Delta \gg A_{QCD}$, although a quantitative understanding of how experimental cuts affect the size of smearing regions is difficult.

1.3.2 Weak Annihilation

The size of the contribution from four quark operators of dimension six to the total $B \rightarrow X_u \ell \bar{\nu}$ decay rate presented in the previous section has been shown to be non negligible. This contribution, or weak annihilation, corresponds to a small portion of the momentum of order $O(\frac{1}{m_b})$ carried away by the final light quark, and all the momentum of the initial heavy quark flowing through the lepton pair. Practically weak annihilation processes are those $B^+ \rightarrow X_u \ell^+ \nu$ decays in which the anti-$b$ and the spectator $u$ quarks forming the $B^+$ meson annihilate into a charmless hadronic final state. Thus, this contribution appears only around $q^2 \simeq m_b^2$, and constitutes a fixed fraction of the total rate of the $B \rightarrow X_u \ell \bar{\nu}$ decays, about $\sim 3\%$. However, this effect may be enhanced since kinematic constraint must be applied to remove charm background and typically result in selecting a portion of the phase space with higher $q^2$-value.

1.3.3 Decay Rates in Restricted Phase Space Region

The measurement of $B \rightarrow X_u \ell \bar{\nu}$ suffers from a large background from the decay $B \rightarrow X_c \ell \bar{\nu}$. The reason for the large background is that $|V_{cb}| \gg |V_{ub}|$, meaning that the $b$ quarks “prefers” to decay to a $c$ quark much more than to a $u$ quark. In order to eliminate the background we have to look at regions of phase space where particles containing charm cannot be produced.

In order to analyze inclusive decays one can start answering to a simple question “How many kinematical variables are needed to describe an inclusive event?”. We are looking at a decay of a $B$ meson into $n$ particles, where $n - 1$ of them have known mass. The “particle” $X$ has unspecified invariant mass. In general for a decay into $n$ particles there are $3n - 4$ kinematical variables, but not
all of them are determined from the dynamics. Since we do not have information about the spin of the hadronic state \( X \) or the spin/polarization of the other decay products, the only four vectors we have at our disposal are \( P_B \), the four momentum of the \( B \) meson, \( p_1 \equiv P_X \) the four momentum of \( X \) and \( p_2, \ldots, p_n \). Lorentz invariance implies then that the possible variables are the \((n + 1)(n + 2)/2\) scalar products of these \( n + 1 \) vectors. Since the masses of all the particles apart from \( p_1 \) are known, we have \( n \) constraints of the form \( p_i^2 = m_i^2, \ (i = B, 2, \ldots, n) \). Conservation of momentum would allow us to eliminate all the pairs that contain \( P_B \) (since \( P_B = \sum_{i=1}^{n} p_i \), i.e. eliminating \( n + 1 \) variables. That leaves us with \( n(n - 1)/2 \) independent variables. The rest of the variables are undetermined from the dynamics and can be integrated over. This arguments holds for \( n \leq 3 \), since at 4 dimensions at most 4 vectors (corresponding to \( n = 3 \)) can be linearly independent. For \( n > 3 \) the argument needs to be modified. For \( B \to X, \ell \nu \) we have \( n = 3 \) and three relevant variables (see Table 1.2). There are various choices in the literature for these variables, and in order to understand them we have to say a few words about the dynamic of the decay. First we need to distinguish between the hadronic level, which

<table>
<thead>
<tr>
<th>Scalar Products</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_B^2 )</td>
<td>( P_B = P_X + P_\ell + P_\nu )</td>
</tr>
<tr>
<td>( P_{B} \bar{X} )</td>
<td>( (P_X + P_\ell + P_\nu)^2 = M_B^2 ), ( P_\ell^2 = 0 )</td>
</tr>
<tr>
<td>( P_X P_\ell )</td>
<td>( P_\nu^2 = 0 )</td>
</tr>
<tr>
<td>( P_{B} \bar{X} )</td>
<td>( P_\nu^2 = 0 )</td>
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Table 1.2: Number of kinematic variables for inclusive \( B \) decays. Using the constraints on the right column we can eliminate some of the variables on the left column.

looks at the decay as a decay of hadrons, and the partonic level, which looks at the decay as a decay of quarks. At the hadronic level we have a \( B \) meson carrying momentum \( P_B \equiv M_B v \) decaying into a leptonic pair (the lepton and anti-neutrino) carrying momentum total \( q \), and a hadronic jet carrying momentum \( P_X \). Conservation of momentum implies \( M_B v - q = P_X \). At the partonic level we look at the decay as a decay of a \( b \) quark, carrying momentum \( m_b v \), into a \( u \) quark, carrying momentum \( p \) (at tree level), and a virtual \( W \) carrying momentum \( q \). The \( W \) in turn decays into a lepton \( \ell \) and an anti-neutrino \( \bar{\nu}_\ell \) (more accurately we write the momentum of the \( b \) quark as \( m_b v + k \) where \( k \) is \( O(\Lambda_{QCD}) \) and expand in powers of \( k \). Conservation of momentum implies \( m_b v - q = p \). If we define \( A \equiv M_B - m_b \) we find that \( P_X = p + A v \). Beyond tree level \( p \) would be the momentum of the jet of light partons created in the decay.

There are generically two common choices of variables:

- **Leptonic**: the energy of the lepton \( E_\ell \), the energy of the neutrino \( E_\nu \) and the invariant mass of virtual \( W \) boson \( q^2 \), which is also the invariant mass of the leptonic pair. This choice focuses on the leptons created in the decay.
• **Partonic:** the energy of the lepton \( E_\ell \), the energy of the partonic jet \( v \cdot p \) and the invariant mass of the partonic jet \( p^2 \). This choice focuses on the partons created in the decay.

For example we can only look at events for which \( P_X^2 < M_D^2 \). This fact implies that the “typical” \( X_u \) state will have large energy of order \( m_b \), since it originates from a decay of a heavy quark and intermediate invariant mass, because of the experimental cut. Since \( M_D \sim \sqrt{m_b \Lambda_{QCD}} \) we can write \( P_X^2 \sim m_b \Lambda_{QCD} \). This implies that some of the components of \( P_X \) are larger than others. We would like the choice of our variables to reflect that. A pair of variables which satisfy this condition is:

\[
P_+ = E_X + |\vec{P}_X|, \quad P_- = E_X - |\vec{P}_X|,
\]

where \( E_X \) and \( P_X \) are the energy and momentum of the hadronic jet, respectively. Note that \( P_+ P_- = P^2 \), \( P_+ + P_- = 2E_X \) and \( P_+ < P_- \). The scaling of these variables will therefore be \( P_- \sim m_b \) and \( P_+ \sim \Lambda_{QCD} \). Specifying \( P_+ \) and \( P_- \) would determine the energy and the invariant mass of the lepton pair, but not the individual energies of the lepton and neutrino. We therefore have to add another variable that would distinguish between the two:

\[
P_\ell = M_B - 2E_\ell
\]

We can determine now the phase space region in terms of \( P_+ \), \( P_\ell \) and \( P_- \). In the rest frame of \( B \) meson, conservation of energy and momentum gives us the following equations:

\[
E_\ell + E_\nu + E_X = M_B
\]

\[
\vec{P}_\ell + \vec{P}_\nu + \vec{P}_X = 0.
\]

Considering the lepton massless implies \( E_\ell = |\vec{P}_\ell| \) and \( E_\nu = |\vec{P}_\nu| \). The limits of phase space are determined by the extremal values of angles between the momenta. Because of conservation of momentum (Eq. 1.48) we have only two independent angles.

Let \( \theta \) be the angle between \( \vec{P}_\ell \) and \( \vec{P}_\nu \). Equation 1.48 then implies \( \vec{P}_X^2 = E_\ell^2 + E_\nu^2 + 2E_\ell E_\nu \cos \theta \).

Since \( -1 \leq \cos \theta \leq 1 \), we have \( (E_\ell - E_\nu)^2 \leq |\vec{P}_X|^2 \leq (E_\ell + E_\nu)^2 \). Using \( P_+ < P_- \), the upper limits gives us \( P_- \leq M_B \), and the lower limit \( P_+ \leq P_\ell \leq P_- \).

Let \( \alpha \) be the angle between \( \vec{P}_\ell \) and \( \vec{P}_X \). Equation 1.48 then implies \( E_\ell^2 = (M_B - E_\ell - E_X)^2 = E_\ell^2 + |\vec{P}_X|^2 + 2E_\ell |\vec{P}_X| \cos \alpha \). Since \( -1 \leq \cos \alpha \leq 1 \), the upper limit gives \((M_B - P_+)(P_- - P_\ell) \geq 0 \), and the lower limit \((M_B - P_-)(P_- - P_\ell) \geq 0 \). Another constraint comes from the QCD spectrum: the lightest state containing a \( u \) quark is the pion, so we must have \( M_\pi^2 \leq P_X^2 = P_+ P_- \). Combining all of these constraints we finally have:

\[
\frac{M_\pi^2}{P_-} \leq P_+ \leq P_\ell \leq P_- \leq M_B.
\]

One of the benefits of this choice of variables (apart from the easy derivation) is that phase space (shown in Fig. 1.4) has an extremely simple form. By examining the phase-space plane we can find
Figure 1.4: The $(P_+ P_-)$ phase space for $\bar{B} \rightarrow X_u \ell \bar{\nu}$. Top: The charm free region is the dark grey region below the black hyperbola, which correspond to $M_X^2 = P_+ P_- = M_D^2$. The solid blue line is $q^2 = (M_B - P_+)(M_B - P_-) = (M_B - M_D)^2$. The red dashed line is $P_+ = M_D^2/M_B$. Bottom: same as left, but with the actual distribution of $b \rightarrow u \ell \bar{\nu}$ superimposed.
regions where the CKM favored $b \to c$ transitions are forbidden and therefore select cuts that can be used for the experimental measurements:

- cut on the charged lepton energy $E_\ell$. If $E_\ell \geq (M_B^2 - M_D^2)/2M_B \approx 2.31$ GeV, the final hadronic state will have invariant mass smaller than $M_D$. For this cut $P_+ \leq 0.66$ GeV, which implies that $P_+$ is of order or $A_{QCD}$;

- cut on the hadronic invariant mass $M_X^2$. To eliminate charm background we need $M_X^2 \leq M_D^2$. The cut $M_X^2 = M_D^2$ is depicted as a solid hyperbola in Figure 1.4;

- cut on the leptonic invariant mass $q^2$. Any cut of the form $q^2 \geq (M_B - M_D)^2$ would not contain charm events. The cut $q^2 = (M_B - P_+)(M_B - P_-) = (M_B - M_D)^2$ is depicted as a solid line in Figure 1.4;

- cut on $P_+$. Red line in Figure 1.4 shows $P_+ = M_D^2/M_B$.

Figure 1.5: The shape of the electron energy (left), hadronic invariant mass (center) and leptonic invariant mass (right) spectra. The dashed curves are the $b$ quark decay results to $O(\alpha_s)$, while the solid curves are obtained by convoluting the parton-level rate with the Fermi motion model with typical parameters. The vertical line marks a reasonable kinematic cut that can be used in order to discriminate $b \to u\ell\nu$ from $b \to c\ell\nu$ transitions. The singularities in $m_X^2$ spectrum reflect the unphysical nature of the parton level distributions. The differences vanish once the Fermi motion is implemented and the parton level variables are replaced with observable quantities.

As we said above, the experimental extraction of $B \to X_u\ell\nu$ signal events have to be performed in restricted region ($\zeta(\Delta \Phi)$ of phase space where $B \to X_c\ell\nu$ are suppressed. The charmless semileptonic partial branching fraction, $\Delta B(B \to X_u\ell\nu)$ is translated in to measurement of $|V_{ub}|$ using:

$$|V_{ub}| = \sqrt{\frac{\Delta B(B \to X_u(\ell\nu)) \tau_B \cdot \zeta(\Delta \Phi)}{\zeta(\Delta \Phi)}}.$$

(1.50)
where $\tau_B$ is the B meson lifetime and $\zeta(\Delta \Phi)$ is predicted by theory and depends on the phase space region defined by kinematic cuts.

From the theoretical point of view, calculating the partial decay rate in restricted regions of phase space is very challenging because the HQE convergence is spoiled requiring the introduction of a non-perturbative distribution function, called shape function which describes the distribution of the $b$ quark momentum inside the $B$ meson, whose form is unknown.

The shape function is a universal property of $B$ mesons at leading order. It has been recognized for over a decade [19, 20] that the leading shape function can be measured in $\bar{B} \to X_s\gamma$ decays. However, sub-leading shape functions [21, 22, 23, 24, 25] arise at each order in $1/m_b$ and differ in semileptonic and radiative $B$ decays. As previously said, the form of the shape function cannot be calculated but can be constrained by moment relations, which relate weighted integrals over the shape function to the heavy-quark parameters $m_b$ ($b$ quark mass), $\mu_b^2$ (kinetic energy of the $b$ quark in the $B$ meson) and $\mu_G$ (chromomagnetic moment of the $b$ quark). The inclusive charmed semileptonic decay rate depends on these parameters so constraints to their values and, therefore, to the shape function, can be applied studying $\bar{B} \to X_c\ell\bar{\nu}$ decays. The same arguments hold for the “2 body” $\bar{B} \to X_s\gamma$ decay, which is directly sensitive to $m_b$. An alternative way is to employ shape-function independent relations between weighted $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_c\ell\bar{\nu}$ spectra [20, 26, 27, 28, 29]. Both approaches are equivalent; yet, not all predictions that have been obtained using the shape function independent relations are up to the standard of present-day calculations.

A fairly complete theoretical analysis of inclusive $\bar{B} \to X_u\ell\bar{\nu}$ spectra was based on calculation described in [28, 30] and referred to as BLNP approach. It includes complete perturbative calculations at NLO with Sudakov resummations, subleading shape functions at three level, and kinematical power corrections at $O(\alpha_s)$. An alternative scheme called “Dressed Gluon Exponentiation” (DGE) [31, 32, 33] employs a renormalon-inspired model for the leading shape function, which is less flexible in its functional form that the forms used by BLNP. Also, no attempt is made to include subleading shape functions, which among other things means that the predictions for more inclusive partial rates would not be in accord with the operator product expansion beyond the leading power in $A_{QCD}/m_b$. The DGE model is therefore less rigorous than the BLNP approach, even though it leads to numerical results that are compatible with those of BLNP. Very recently, Gambino et al. (GGOU) [34] released a work in which authors include the recently calculated $O(\beta_0\alpha_s^2)$ corrections to the decay rates [35] which were not available for BLNP. They use moment constraints to model the subleading shape function differently for the various structures in hadronic tensor. However, no attempt is made to resum the Sudakov logarithms. The numerical results obtained using the GGOU code are consistent with those of BLNP in both their central values and error estimates. Another calculation based on pure OPE was proposed some time ago by Bauer, Ligeti and Luke [36]; their idea presupposes the extraction of $|V_{ub}|$ from inclusive semileptonic decays $b \to u\ell\bar{\nu}$ by applying a cut on the squared lepton-neutrino invariant mass $q^2$. Such a cut forbids the hadronic final state from moving fast in the
$B$ rest frame, and so the light-cone expansion which gives rise to the shape function is not relevant in this region of phase space. It is shown that the BLL calculation is most predictive, also in terms of uncertainties, in phase space regions defined by a combination of $M_X$ and $q^2$ requirements.

1.3.4 Experimental Approaches

The experimental approaches commonly used to study charmless semileptonic $B$ decays fall into three categories:

1. Charged lepton momentum “endpoint” measurements. In these analyses, a single charged electron is used to determine a partial decay rate for $B \rightarrow X_u \ell \bar{\nu}$ and no neutrino reconstruction is employed, resulting in a $\sim 50\%$ selection efficiency. The decay rate can be cleanly extracted for $E_\ell > 2.3$ GeV, but this is deep in the shape function region, where theoretical uncertainties are large. Recent measurements push down to 2.0 or 1.9 GeV, but the cost is a lower ($< 1/10$) signal-to-background (S/B) ratio;

2. “untagged” neutrino reconstruction measurements. In this case, both the charged electron and the missing momentum are measured, allowing the determination of $q^2$ and providing additional background rejection. This allows a much higher S/B $\sim 0.7$ at the same $E_\ell$ cut and a selection efficiency $\sim 5\%$, but at the cost of smaller accepted phase space for $B \rightarrow X_u \ell \bar{\nu}$ decays and uncertainties associated with the determination of the missing momentum;

3. “tagged” measurements in which one $B$ meson is fully reconstructed. The S/B ratio can be quite high ($\sim 2$) but the selection efficiency is $\sim 10^{-3}$. The $E_\ell$ cut is typically 1.0 GeV and the full range of signal-side variables ($q^2, M_X, P_+$) is available for study, each having its own pros and cons. A cut on the hadronic invariant mass (region below the black hyperbola in Fig. 1.4) has the advantage to include $\approx 80\%$ of $b \rightarrow u$ transition, but has a strong dependence on the shape function. The highest cut that is possible to set on the light-cone momentum is $P_+ < 0.66$ (region below the red line in Fig. 1.4) which provides access to $\approx 70\%$ of charmless semileptonic $b$ transition, but the dependence on the shape function is high as well. The $q^2$ distribution is less sensitive to non perturbative effects and less dependent on the calculation of theoretical acceptance, but only a small fraction of events is usable with a pure $q^2$ cut (region below the blue line in Fig. 1.4). Combined $M_X$ and $q^2$ cuts mitigate the drawbacks of the two methods while retaining good statistical and systematic sensitivities. The fraction of $b \rightarrow u$ event selected with these combined cuts is $\approx 45\%$.

Recent results based on the above methods performed by CLEO, BaBar and BELLE are presented in figure 1.6
Figure 1.6: Status of $|V_{ub}|$ measurements prior to this work (winter 2008).
1.3.5 Latest Inclusive $|V_{ub}|$ extraction

The latest Inclusive $|V_{ub}|$ measurements was recently published from BABAR, after being presented at the 2007 Lepton Photon Conference. It’s based on the study of the recoil to fully reconstructed $B$ mesons in restricted regions of phase space, $M_X < 1.55 \text{ GeV}/c^2, P_+ < 0.66 \text{ GeV}/c^2,$ and $(M_X < 1.7 \text{ GeV}/c^2, q^2 > 8.0 \text{ GeV}^2/c^4$ [37]. The dataset correspond to about $347 \text{ fb}^{-1}$ luminosity integrated on the $T(4S)$ peak. The published values for the partial branching fractions and $|V_{ub}|$ using various theoretical computations are in Table 1.3.

| Method   | $\Delta B(B \rightarrow X_u \ell \nu)(10^{-3})$ | $|V_{ub}| \times (10^{-3})$ |
|----------|-----------------------------------------------|-----------------------------|
| $M_X$    | $1.18 \pm 0.09 \pm 0.07 \pm 0.01$            | $4.27 \pm 0.16 \pm 0.13 \pm 0.30$ [38][39][40] |
|          |                                               | $4.56 \pm 0.17 \pm 0.14 \pm 0.32$ [41] |
| $P_+$    | $0.95 \pm 0.10 \pm 0.08 \pm 0.01$            | $3.88 \pm 0.19 \pm 0.16 \pm 0.28$ [38][39][40] |
|          |                                               | $3.99 \pm 0.20 \pm 0.16 \pm 0.24$ [41] |
| $M_X, q^2$ | $0.81 \pm 0.08 \pm 0.07 \pm 0.02$          | $4.57 \pm 0.22 \pm 0.19 \pm 0.30$ [38][39][40] |
|          |                                               | $4.64 \pm 0.23 \pm 0.19 \pm 0.25$ [41] |
|          |                                               | $4.93 \pm 0.24 \pm 0.20 \pm 0.36$ [36] |

Table 1.3: Summary of the fitted number of events and efficiencies, $\Delta B(B \rightarrow X_u \ell \nu)$, and extracted $|V_{ub}|$ for the three kinematic cuts. The first uncertainty is statistical, the second systematic. For $\Delta B$, the third uncertainty is due to the theoretical knowledge of the signal efficiency; for the $|V_{ub}|$ values, it comes from the theoretical uncertainty on $\zeta(\Delta \Phi)$. For Ref. [38][39][40] are used the exponential parametrization of the shape function has been used.

1.3.6 New inclusive $|V_{ub}|$ measurements

In this thesis, the tagged approach has been chosen and different kinematic regions are exploited, in order to compare measurements with different sensitivities to theoretical uncertainties;

- three regions are defined by pure $M_X$ cuts, pure $P_+$ cuts, and combined $(M_X, q^2)$ cuts. Table 1.4 shows a summary of the features of each kinematic requirement.

- A method to reduce the model dependence is to measure the $B \rightarrow X_u \ell \nu$ rate over the entire $m_X$ and $q^2$ spectrum. Since no extrapolation is necessary to obtain the full rate, systematic uncertainties from $m_b$ and Fermi motion are much reduced.

- In section 1.3.2 we have argued that a cut on high $q^2$ ( $q^2 < q_0^2$) will eliminate the effect of weak annihilation and remove the uncertainty associated with this contribution. The cutoff should be
1.3 $|V_{ub}|$ Extraction from Charmless Semileptonic $B$ Decays

<table>
<thead>
<tr>
<th>Kinematic region</th>
<th>% of rate</th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_X &lt; M_D$</td>
<td>~ 80%</td>
<td>large rate</td>
<td>depends on shape function (and subleading corrections)</td>
</tr>
<tr>
<td>$P_+ &lt; M_B^2/M_B$</td>
<td>~ 70%</td>
<td>lots of rate</td>
<td>depends on shape function (and subleading corrections)</td>
</tr>
</tbody>
</table>
| $q^2 > (M_B - M_D)^2$ | ~ 20% | insensitive to shape function | - very sensitive to $m_b$
|                  |           |      | - Weak Annihilation may be substantial
|                  |           |      | - effective expansion parameter is $1/m_c$
| $(M_X, q^2)$ combined | ~ 45% | insensitive to shape function | smaller rate than $M_X$ and $P_+$ cuts |

Table 1.4: Comparison of different kinematic cuts.

small enough to exclude the region around $q^2 = m_b^2$ where this contribution is concentrated.

In order to do this we have performed a combined $m_X, q^2$ analysis and four different regions $(m_X, q^2)$ are exploited. The cuts used and the amount of $B 	o X_u \ell \nu$ phase space considered are reported in table 1.5.

<table>
<thead>
<tr>
<th>$q^2$ Cut ($GeV^2/c^4$)</th>
<th>$m_X$ Cut ($GeV/c$)</th>
<th>Phase Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 &lt; 8$</td>
<td>$m_X &lt; 1.55$</td>
<td>~ 33%</td>
</tr>
<tr>
<td>$q^2 &lt; 10$</td>
<td>$m_X &lt; 1.55$</td>
<td>~ 40%</td>
</tr>
<tr>
<td>$q^2 &lt; 12$</td>
<td>$m_X &lt; 1.55$</td>
<td>~ 33%</td>
</tr>
<tr>
<td>$q^2 &lt; 14$</td>
<td>$m_X &lt; 1.55$</td>
<td>~ 40%</td>
</tr>
</tbody>
</table>

Table 1.5: Different kinematic cuts used in order to eliminate weak annihilation contribute.

The detailed impact of the theoretical models presented above on the determination on $|V_{ub}|$ from the experimental measurements discussed in this thesis are shown in Chapter8.
Chapter 2

The BABAR Experiment

The BABAR experiment, located at the Stanford Linear Accelerator Center (SLAC) in California, has been optimized for the systematic study of CP violation in the B meson system. It involves a large international collaboration of more than 500 physicists. The experiment consists of a detector [42] built around the interaction region of the high luminosity $e^+e^-$ asymmetric collider PEP-II [43]. The geometry of the detector as well as the technical requirements of the main components have been designed in order to obtain the cleanest environment and the best efficiency to reconstruct the $B$ meson decays.

In this chapter we describe the main features and performances of PEP-II and the BABAR detector

2.1 The PEP-II accelerator

The PEP-II B-Factory is an asymmetric-energy $e^+e^-$ collider designed to operate at a center of mass energy of $E_{CM} = 10.58$ GeV, corresponding to the mass of the $\Upsilon(4S)$ vector meson resonance. The $\Upsilon(4S)$ has a mass slightly above the $BB$ threshold, and thus it decays almost exclusively into $B^+\bar{B}^0$ or $B^+B^-$ pairs. If the $\Upsilon(4S)$ is produced at rest, then the $B$ mesons would have an average residual momentum of the order of $\sqrt{(M_{\Upsilon(4S)}/2)^2 - M_B^2} \sim 325$ MeV/$c$. With this momentum, the average distance covered by a $B$ meson would be of the order of $2 \beta\gamma c \tau_B \sim 30\mu m$ and it would be experimentally very difficult to measure the separation between the decay points of the two $B$ mesons.

\footnote{We use $M_{\Upsilon(4S)} = 10.58$ GeV/$c^2$ and $M_B = 5.28$ GeV/$c^2$.}

\footnote{The factor $\beta\gamma$ arising from a momentum of the $B$ of 325 MeV/$c$ is $\beta\gamma \sim 0.061$ and the $B$ meson lifetime is $\tau_B = (1.530 \pm 0.009) \times 10^{-12}$s.}
The PEP-II machine collides a 9.0 GeV electron beam head-on with a 3.1 GeV positron beam, in this way the Lorentz boost of the T(4S) is \( \beta \gamma = \frac{E_p - E_e}{E_{CM}} \sim 0.56 \), resulting in an average separation between the two B meson of the order of 250 \( \mu \)m, compatible with the \( \text{BABAR} \) vertex resolution, as it will be shown in the following.

![PEP-II B-Factor](image)

\textbf{Figure 2.1: Overview of the PEP-II B-Factor.}

Electrons and positrons are accelerated in the 3.2 km long SLAC linac and accumulated into two 2.2 km long storage rings, called HER (high-energy ring, in which the electrons circulate) and LER (low-energy ring, in which the positrons, produced in the linac by collisions of 30 GeV electrons on a target, circulate).

In proximity of the interaction region the beams are focused by a series of offset quadrupoles (labelled Qx) and bent by means of a pair of dipole magnets, which allow the bunches to collide head-on and then to separate. The tapered B1 dipoles, located at \( \pm 21 \) cm on either side of the interaction point (IP), and the Q1 quadrupoles operate inside the field of the \( \text{BABAR} \) superconducting solenoid, while Q2, Q4, and Q5, are located outside or in the fringe field of the solenoid. The interaction region is enclosed in a water-cooled beam pipe consisting of two thin layers of beryllium with a water channel in between. Its outer radius is about 28 mm. The total thickness of the central beam pipe section at normal incidence corresponds to 1.06 % of a radiation length.

The beam pipe, the permanent magnets and the Silicon Vertex Tracker (SVT) are assembled, aligned and then enclosed in a 4.5 m long support tube. This rigid structure is inserted into the \( \text{BABAR} \) detector, spanning the IP. The \( \text{BABAR} \) data taking, started with the first collisions in PEP-II at the end of 1999 and ended in the first days of April 2008. \( \text{BABAR} \) has recorded an integrated luminosity of about 531 fb\(^{-1}\), including about 54 fb\(^{-1}\) just below the \( \Upsilon(4S) \) resonance, 433 fb\(^{-1}\) recorded at the \( \Upsilon(4S) \) and 44 fb\(^{-1}\) at other \( \Upsilon(4S) \) resonances. The \( \text{BABAR} \) recorded luminosity until the end of data taking is shown in Fig. 2.2. PEP-II surpassed its design performances, both in terms...
of the instantaneous luminosity and the daily integrated luminosity (see Tab. 2.1), achieving the peak value of $1.2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ during Run 6. A significant improvement to the integrated luminosity has been achieved between December 2003 and March 2004 with the implementation of a novel mode of operation of PEP-II, called “trickle injection”.

Until the end of 2003, PEP-II typically operated in a series of 40-minute fills during which the colliding beams coasted: at the end of each fill, it took about three to five minutes to replenish the beams for the next fill, and during this period the BaBar data acquisition system had to be turned off for detector safety. With the new technique, the BaBar detector can keep taking data virtually uninterrupted while the linac continuously injects electron and positron bunches (at a rate up to 10 Hz in the HER and 20Hz in the LER) into the two PEP-II storage rings. This novel mode of operation allows an increase of 20 to 30% of the
Table 2.1: PEP beam parameters. Values are given both for the design and for the records achieved during 2007.

integrated luminosity. Moreover, the continuous injection makes the storage of particles more stable, so that PEP-II rings are easier to operate and beam losses are far less frequent than with the previous operational mode. This result is very important since, after a loss of the stored beams, it takes approximately 15 minutes to refill the two beams during which obviously no data taking is allowed.

2.2 The BABAR detector

The design of the BABAR detector is optimized for CP violation studies, but it is also well suited to do precision measurements in other $B$ and non-$B$ physics. To achieve the goal of performing accurate measurements there are many requirements:

- a large and uniform acceptance, in particular down to small polar angles relative to the boost direction, to avoid particle losses. Although the boost originated by the asymmetric beams is not a big one, optimizing the detector acceptance leads to an asymmetric detector;
- a good vertex resolution;
- an excellent detection efficiency and an excellent precision on the momentum measurement for charged particles with transverse momentum ranging between 60 MeV/c and 4 GeV/c;
- an excellent energy and angular resolution for photons and $\pi^0$ s with energy down to 20 MeV and up to 5 GeV;
- a good discrimination between $e, \mu, \pi, K, p$ over a wide kinematic range;
- neutral hadrons identification capability.

Since the average momentum of charged particles produced in $B$ meson decays is below 1 GeV/c, the errors on the measured track parameters are dominated by multiple Coulomb scattering, rather
than intrinsic spatial resolution of the detectors. Similarly, the detection efficiency and energy resolution of low energy photons are severely impacted by material in front of the calorimeter. Thus, special care has been given to keep the material in the active volume of the detector to a minimum.

A schematic view of the BABAR detector is shown in Fig. 2.3. The BABAR superconducting solenoid, which produces a 1.5 T axial magnetic field, contains a set of nested detectors, which are going from inside to outside—a five layers Silicon Vertex Tracker (SVT), a central Drift Chamber (DCH) for charged particles detection and momentum measurement, a fused-silica Cerenkov radiation detector (DIRC) for particle identification, and a CsI(Tl) crystal electromagnetic calorimeter for detection of photons and electrons. The calorimeter has a barrel and an end-cap which extends asymmetrically into the forward direction ($e^-$ beam direction), where many of the collision products emerge. All the detectors located inside the magnet have practically full acceptance in azimuth ($\phi$). The flux return outside the cryostat is composed of 18 layers of steel, which increase in thickness outwards, and are instrumented (the IFR) with 19 layers of planar resistive plate chambers (RPCs) or limited streamer tubes (LSTs) in the barrel and 18 in the end-caps. The IFR allows the muon identification, and also detects penetrating neutral hadrons. The right-handed coordinate system is indicated in Fig. 2.3. The $z$ axis corresponds to the magnetic field axis and is offset relative to the beam axis by about 20 mrad in the horizontal plane. It is oriented in the direction of electrons. The positive $y$-axis points upward and the positive $x$-axis points away from the center of the PEP-II storage rings. A schematic view of the interaction region is shown in Figure 2.4. The next sections are dedicated to a description of each subsystem.

### 2.2.1 The Silicon Vertex Tracker

The Silicon Vertex Tracker (SVT) provides a precise measurement of the decay vertices and of the charged particle trajectories near the interaction region. The mean vertex resolution along the $z$-axis for a fully reconstructed $B$ decay must be better than 80 $\mu$m in order to avoid a significant impact on the time-dependent $CP$ asymmetry measurement precision; a 100 $\mu$m resolution in the $x-y$ transverse plane is necessary in reconstructing decays of bottom and charm mesons, as well as $\tau$ leptons. The SVT also provides standalone tracking for particles with transverse momentum too low to reach the drift chamber, like soft pions from $D^*$ decays and many charged particles produced in multi-body $B$ meson decays. Finally, the SVT supplies particle identification (PID) information both for low and high momentum tracks. For low momentum tracks the SVT $dE/dx$ measurement is the only PID information available, for high momentum tracks the SVT provides the best measurement of the track angles, required to achieve the design resolution on the Cerenkov angle measured by the DIRC.

The design of the SVT is constrained by the components of the storage ring which have been arranged so as to allow maximum SVT coverage in the forward direction: the SVT extends down to $20^\circ$ ($30^\circ$) in polar angle from the beam line in the forward (backward) direction. Furthermore, it
Figure 2.3: \textsc{babar} detector front view (top) and side view (bottom).

must have a small amount of material, so to reduce the multiple scattering which would affect the performance of the outer subdetectors. The solution which was adopted is a five-layer device with 340 double-sided silicon wafers mounted on a carbon-fiber frame (see Figure 2.5). On the inner (outer) face of each wafer, strip sensors are located running orthogonal (parallel) to the beam direction, measuring the \( z \) (\( \phi \)) coordinate of the tracks. The wafers are organized in modules split into forward and backward sections: they are read out on their respective ends and the charge deposited by a particle is determined by the time over threshold of the signal on each strip. In total, 150,000 read-out channels are present. The inner three layers, containing six modules each, are placed close to the
beam pipe (at 3.3, 4 and 5.9 cm from it) and dominate the determination of tracks position and angles. The outer two layers, containing 16 and 18 modules respectively, are arch-shaped, thus minimizing the amount of silicon needed to cover the solid angle, and placed close to the DCH (between 9.1 and 14.6 cm from the beam pipe) to help the track matching between the two detectors. The total active silicon area is 0.96 $m^2$ and the geometrical acceptance is 90% of the solid angle in the center-of-mass frame. The material traversed by particles corresponds to $\sim 4\%$ of a radiation length.

The SVT efficiency is calculated for each section of the modules by comparing the number of associated hits to the number of tracks crossing the active area of the module and is found to be 97%. The spatial resolution of SVT hits is determined by measuring the distance between the track trajectory and the hit for high-momentum tracks in two-prong events: it is generally better than 40 $\mu m$ in all layers for all track angles, allowing a precise determination of decay vertices to better than 70 $\mu m$ (see Figure 2.6).
Figure 2.5: Schematic view of the SVT, transverse section (upper plot) and longitudinal section (bottom plot).

The SVT provides stand-alone tracking for low momentum particles that do not reach the drift chamber, with an efficiency estimated to be 20% for particles with transverse momenta of 50 MeV/c, rapidly increasing to over 80% at 70 MeV/c. Limited particle ID information for low momentum particles that do not reach the drift chamber and the Čerenkov detector is provided by the SVT through the measurement of the specific ionization loss, $dE/dx$, as derived from the total charge deposited in each silicon layer.
Figure 2.6: SVT resolution (layer 1) on the single hit, as a function of the track angle.

2.2.2 The Drift Chamber

The Drift Chamber (DCH) is the main tracking device for charged particles with transverse momenta $p_T$ above $\sim 120$ MeV/$c$, providing the measurement of $p_T$ from the curvature of the particle’s trajectory inside the 1.5 T solenoidal magnetic field. The DCH also allows the reconstruction of secondary vertices located outside the silicon detector volume, such as those from $\bar{K}_S^0 \rightarrow \pi^+ \pi^-$ decays. For this purpose, the chamber is able to measure not only the transverse coordinate, but also the longitudinal ($z$) position of tracks with good resolution (about 1 mm). Good $z$ resolution also aids in matching DCH and SVT tracks, and in projecting tracks to the DIRC and the calorimeter.
For low momentum particles the DCH provides particle identification by measurement of ionization loss \( (dE/dx) \), thus allowing for \( K/\pi \) separation up to \( \approx 600 \text{ MeV}/c \). This capability is complementary to that of the DIRC in the barrel region, while it is the only mean to discriminate between different particle hypotheses in the extreme backward and forward directions which fall outside of the geometric acceptance of the DIRC. Finally, the DCH provides real-time information used in the first level trigger system. The DCH is a 2.80 m long cylinder with an inner radius of 23.6 cm and an outer radius of 80.9 cm (Figure 2.7). Given the asymmetry of the beam energies, the DCH center is displaced by about 37 cm with respect to the interaction point in the forward direction. The active volume provides charged particle tracking over the polar angle range \(-0.92 < \cos \theta < 0.96\).

![Figure 2.7: Schematic view of the DCH (longitudinal section).](image)

The drift system consists of 7104 hexagonal cells, approximately 1.8 cm wide by 1.2 cm high, arranged in 10 superlayers of 4 layers each, for a total of 40 concentric layers (Fig. 2.8). Each cell consists of one sense wire surrounded by six field wires. The sense wires are 20 \( \mu \text{m} \) Rh-W gold-plated wires operating nominally in the range 1900-1980 V;

the field wires are 120 \( \mu \text{m} \) Al wires operating at 340 V. Within a given superlayer, the sense and field wires are organized with the same orientation. For measuring also the \( z \) coordinate, the superlayers alternate in orientation: first an axial view, then a pair of small angle stereo views (one with positive, one with negative angle), as indicated in Fig. 2.8.

The layers are housed between a 1 mm beryllium inner wall and a 9 mm carbonfiber outer wall (corresponding to 0.28% and 1.5% radiation lengths, respectively) both to facilitate the matching between the SVT and DCH tracks and to minimize the amount of material in front of the DIRC and the calorimeter. The counting gas is a 80:20 mixture of helium: isobutane, which again satisfies the requirement of keeping the multiple scattering at minimum. Overall, the multiple scattering inside the DCH is limited by less than 0.2% radiation lengths of material.

The drift chamber reconstruction efficiency has been measured on data in selected samples of
multi-track events by exploiting the fact that tracks can be reconstructed independently in the SVT and the DCH. The absolute drift chamber tracking efficiency is determined as the fraction of all tracks detected in the SVT which are also reconstructed by the DCH when they fall within its acceptance. Its dependency on the transverse momentum and polar angle is shown in Figure [42]. At the design voltage of 1900V the reconstruction efficiency of the drift chamber averages 98 ± 1% for tracks above 200 MeV/c and polar angle \( \theta > 500 \text{ mrad} \) (29°).

The \( p_T \) resolution is measured as a function of \( p_T \) in cosmic ray studies:

\[
\frac{\sigma_{p_T}}{p_T} = (0.13 \pm 0.01)\% \cdot p_T + (0.45 \pm 0.03)\% ,
\]

where \( p_T \) is expressed in GeV/c. The first contribution, dominating at high \( p_T \), comes from the curvature error due to finite spatial measurement resolution; the second contribution, dominating at low momenta, is due to multiple Coulomb scattering.

The specific ionization loss \( dE/dx \) for charged particles traversing the drift chamber is derived from the total charge deposited in each drift cell. The resolution achieved to date is typically about
Figure 2.9: Track reconstruction efficiency in the drift chamber at operating voltages of 1900 V and 1960 V, as a function of transverse momentum (a) and polar angle (b).

7.5% (as shown in Fig. 2.10 for $e^\pm$ from Bhabha scattering). A 3σ separation between kaons and pions can be achieved up to momenta of about 700 MeV/c [45].

Figure 2.10: Resolution on $dE/dx$ for $e^\pm$ from Bhabha scattering.
2.2 The **BaBar** detector

2.2.3 The Cerenkov detector

The particle identification (PID) at low momenta exploits primarily the $dE/dx$ measurements in the DCH and SVT. However, above the threshold of 700 MeV/c, the $dE/dx$ information does not allow to separate pions and kaons. The Detector of Internally Reflected Cerenkov radiation (DIRC) is employed primarily for the separation of pions and kaons from about 500 MeV/c to the kinematic limit of $4 \text{ GeV}/c$ reached in rare $B$ decays like $B \rightarrow \pi^+\pi^-/K^+K^-$. The principle of the DIRC is based on the detection of Cerenkov light generated by a charged particle in a medium of refractive index $n$, when its velocity $v$ is greater than $c/n$. The photons are emitted on a cone of half-angle $\theta_c$ with respect to the particle direction, where $\cos \theta_c = 1/\beta n$, $\beta = v/c$. Knowing the particle momentum thanks to the SVT and the DCH, the measurement of $\theta_c$ allows the mass measurement, so the particle identification, with the relation:

$$m^2 c^2 = \frac{1-\beta^2}{\beta^2} - \frac{p^2}{c^2}$$

(2.2)

Figure 2.11 illustrates the principles of light production, transport, and imaging.

![Diagram of the DIRC fused silica radiator bar and imaging region.](image)

Figure 2.11: Schematics of the DIRC fused silica radiator bar and imaging region.

The radiator material of the DIRC is synthetic fused silica (refraction index $n = 1.473$) in the form of 144 long, thin bars with regular rectangular cross section. The bars, which are 17 mm thick, 35 mm wide and 4.9 m long, are arranged in a 12-sided polygonal barrel, each side being composed of 12 adjacent bars placed into sealed containers called bar boxes. Dry nitrogen gas flows through each bar box, and humidity levels are measured to monitor that the bar box to water interface remains sealed. The solid angle subtended by the radiator bars corresponds to 94% of the azimuth and 83% of the cosine of the polar angle in the center-of-mass system. The bars serve both as radiators and as light pipes for the portion of the light trapped in the radiator by total internal reflection. For particles with $\beta \approx 1$, some photons will always lie within the total internal reflection limit, and will be transported
to either one or both ends of the bar, depending on the particle incident angle. To avoid having to instrument both bar ends with photon detectors, a mirror is placed at the forward end, perpendicular to the bar axis, to reflect incident photons to the backward (instrumented) bar end.

Once photons arrive at the instrumented end, most of them emerge into an expansion region filled with 6000 litres of purified water ($n = 1.346$), called the stand-off box (see Figure 2.12). A fused silica wedge at the exit of the bar reflects photons at large angles and thereby reduces the size of the required detection surface. The photons are detected by an array of densely packed photomultiplier tubes (PMTs), each surrounded by reflecting “light catcher” cones to capture light which would otherwise miss the PMT active area. The PMTs, arranged in 12 sectors of 896 phototubes each, have a diameter of 29 mm and are placed at a distance of about 1.2 m from the bar end. The expected Čerenkov light pattern at this surface is essentially a conic section, whose cone opening-angle is the Čerenkov production angle modified by refraction at the exit from the fused silica window. By knowing the location of the PMT that observes a Čerenkov photon and the charged particle direction from the tracking system, the Čerenkov angle can be determined. In addition, the time taken for the photon to travel from its point of origin to the PMT is used to effectively suppress hits from beam-generated background and from other tracks in the same event, and also to resolve some ambiguities in the association between the PMT hits and the track (for instance, the forward-backward ambiguity between photons that have or haven’t been reflected by the mirror at the forward end of the bars). The

![Figure 2.12: Schematic view of the DIRC.](image)

relevant observable to distinguish between signal and background photons is the difference between the measured and expected photon time, $\delta t_\gamma$. It is calculated for each photon using the track time-
2.2 The $\text{BaBar}$ detector

of-flight, the measured time of the candidate signal in the PMT and the photon propagation time within the bar and the water filled standoff box. The resolution on this quantity, as measured in dimuon events is $1.7\text{ ns}$, close to the intrinsic $1.5\text{ ns}$ transit time spread of the photoelectrons in the PMTs. Applying the time information substantially improves the correct matching of photons with tracks and reduces the number of accelerator induced background hits by approximately a factor 40, as can be seen in Fig. 2.13 [46]. The reconstruction routine provides a likelihood value for each of the five stable particle types ($e, \mu, \pi, K, p$) if the track passes through the active volume of the DIRC. These likelihoods are calculated in an iterative process by maximising the likelihood value for the entire event while testing different hypotheses for each track. If enough photons are found, a fit of $\theta_c$ and the number of observed signal and background photons are calculated for each track.

Figure 2.13: Display of one $e^+e^- \rightarrow \mu^+\mu^-$ event reconstructed in $\text{BaBar}$ with two different time cuts. On the left, all DIRC PMTs that were hit within the $\pm300\text{ ns}$ trigger window are shown. On the right, only those PMTs that were hit within $8\text{ ns}$ of the expected Cerenkov photon arrival time are displayed.

Figure 2.14 shows the number of photons detected as a function of the polar angle in di-muons events. It increases from a minimum of about 20 at the center of the barrel ($\theta \approx 90^\circ$) to well over 50 in the forward and backward directions, corresponding to the fact that the path-length in the radiator is longer for tracks emitted at large dip angles (therefore the number of Cerenkov photons produced in the bars is greater) and the fraction of photons trapped by total internal reflection rises. This feature is very useful in the $\text{BaBar}$ environment, where, due to the boost of the center-of-mass, particles are emitted preferentially in the forward direction. The bump at $\cos \theta = 0$ is a result of the fact that for tracks at small angles internal reflection of the Cerenkov photons occurs in both the forward and
backward direction. The small decrease of the number of photons from the backward direction to the forward one is a consequence of the photon absorption along the bar before reaching the stand-off box in the backward end. The combination of the single photon Cerenkov angle resolution, the distribution of the number of detected photons versus polar angle and the polar angle distribution of charged tracks yields a typical track Cerenkov angle resolution which is about 2.5 mrad in di-muon events. The pion-kaon separation power is defined as the difference of the mean Cerenkov angles for pions and kaons assuming a Gaussian-like distribution, divided by the measured track Cerenkov angle resolution. As shown in Fig. 2.15, left, the separation between kaons and pions at 3 GeV/c is about 4.3 $\sigma$. The efficiency for correctly identifying a charged kaon hitting a radiator bar and the probability of wrongly identifying a pion as a kaon are determined using $D^0$ decays kinematically selected from inclusive $D^*$ meson production (Fig. 2.15): the kaon identification efficiency and pion mis-identification probability are about 96% and 2%, respectively.

### 2.2.4 The Electromagnetic Calorimeter

The $\text{BABAR}$ electromagnetic calorimeter (EMC) is designed to detect and measure electromagnetic showers with high efficiency and very good energy and angular resolution over a wide energy range.
Figure 2.15: Left plot: average difference between the expected value of $\theta_C$ for kaons and pions, divided by the uncertainty, as a function of momentum. Right plot: efficiency and misidentification probability for the selection of charged kaons as a function of track momentum.

between 20 MeV and 9 GeV. This allows the reconstruction of $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ decays where the photons can have very low energy, as well as the reconstruction of Bhabha events and processes like $e^+e^- \rightarrow \gamma\gamma$, important for luminosity monitoring and calibration, where the electron and photon energies can be as large as 9 GeV. The EMC also provides the primary information for electron identification and electron-hadron separation.

Energy deposit clusters in the EMC with lateral shape consistent with the expected pattern from an electromagnetic shower are identified as photons when they are not associated to any charged tracks extrapolated from the SVT and the drift chamber, and as electrons if they are matched to a charged track and if the ratio between the energy $E$ measured in the EMC and the momentum $p$ measured by the tracking system is $E/p \approx 1$.

The EMC contains 6580 CsI crystals doped with Tl (Fig. 2.16). CsI(Tl) has a high light yield (50,000 photons/MeV) and a small Molière radius (3.8 cm), which provide the required energy and angular resolution; its radiation length of 1.86 cm guarantees complete shower containment at the BaBar energies.

Each crystal is a truncated trapezoidal pyramid and ranges from 16 to 17.5 radiation lengths in thickness. The front faces are typically about 5 cm in each dimension. The crystals are arranged to form a barrel and a forward endcap giving a 90% solid-angle coverage in the center-of-mass frame. The barrel has 48 rows of crystals in $\theta$ and 120 in $\phi$; the forward endcap contains 8 rings in $\theta$. Overall the EMC extends from an inner radius of 91 cm to an outer radius of 136 cm and is displaced asymmetrically with respect to the interaction point.
Figure 2.16: Longitudinal section of the top half of the EMC. Dimensions are in mm.

The crystals are read out by two independent 1 cm² PIN photodiodes, glued to their rear faces, which are connected to low-noise preamplifiers that shape the signal with a short shaping time (400 ns) so to reduce soft beam-related photon backgrounds.

For the purpose of precise calibration and monitoring, use is made of a neutron activated fluorocarbon fluid, which produces a radioactive source ($^{16}$N) originating a 6.1 MeV photon peak in each crystal. A light pulser system injecting light into the rear of each crystal is also used. In addition, signals from data, including $\pi^0$ decays and $e^+e^- \rightarrow e^+e^-/\gamma\gamma/\mu^+\mu^-$ events, provide an energy calibration and resolution determination.

The efficiency of the EMC exceeds 96% for the detection of photons with energy above 20 MeV. The energy resolution is usually parametrized by

$$\frac{\sigma_E}{E} = \frac{\sigma_1}{E^{3/4}(\text{GeV})} \oplus \sigma_2,$$

where $\sigma_1 = 2.32\pm0.30\%$ and $\sigma_2 = 1.85\pm0.12\%$, as determined using the above mentioned sources. The first term in Equation 2.3 arises from fluctuations in photon statistics and is dominant for energies below about 2.5 GeV, while the constant term takes into account several effects, such as fluctuations in shower containment, non-uniformities, calibration uncertainties and electronic noise. The decays of $\pi^0$ and $\eta$ candidates in which the two photons have approximately equal energy are used to infer angular resolution. It varies between about 12 mrad at low energies and 3 mrad at high energy. The data fit the empirical parametrization:

$$\sigma_{\theta,\phi} = \left(\frac{3.87 \pm 0.07}{\sqrt{E(\text{GeV})}} + (0.00 \pm 0.04)\right) \text{ mrad}$$

(2.4)

Fig. 2.17 [47] shows the energy and angular resolution measured as a function of the photon energy.
2.2 The \textit{BaBar} detector

Figure 2.17: Energy (left) and angular (right) resolutions measured using a variety of data. The solid curves represent a fit to the data using Equation 2.3 and 2.4 respectively.

2.2.5 The Instrumented Flux Return

The Instrumented Flux Return (IFR) is designed to identify muons and neutral hadrons (primarily $K_L$ and neutrons). Muons are important for tagging the flavor of neutral $B$ mesons via semi-leptonic decays, for the reconstruction of vector mesons, like the $J/\psi$, and the study of semi-leptonic and rare decays involving leptons from $B$ and $D$ mesons and $\tau$ leptons. $K_L$ detection allows for the study of exclusive $B$ decays, in particular $CP$ eigenstates. The principal requirements for IFR are large solid angle coverage, good efficiency and high background rejection for muons down to momenta below 1 GeV/c. For neutral hadrons, high efficiency and good angular resolution are most important. The IFR uses the steel flux return of the magnet as muon filter and hadron absorber, limiting pion contamination in the muon identification. Originally single gap resistive plate chambers (RPC) with two-coordinate readout, operated in limited streamer mode constituted the active part of the detector [48], with 19 layers in the barrel and 18 in each endcap. The RPC were installed in the gaps of the finely segmented steel of the six barrel sectors and the two end-doors of the flux return, as illustrated in Fig. 2.18. The steel segmentation has been optimized on the basis of Monte Carlo studies of muon penetration and charged and neutral hadron interactions. In addition, two layers of cylindrical RPCs were installed between the EMC and the magnet cryostat to detect particles exiting the EMC. RPCs contain a 2 mm Bakelite gap with $\sim 8$ kV across it. Ionizing particles which cross the gap create streamers of ions and electrons in the gas mixture (Argon, freon and isobutane), which in turn creates signals via capacitive coupling on the strips mounted on each side of the RPC. Soon after the installation (which took place in Summer 1999), the efficiency of a significant fraction of the chambers (initially greater than 90\%) has started to deteriorate at a rate of 0.5-1\%/month. In order to solve some of the inefficiency problems, an extensive improvement program has been developed. The forward endcap
was retrofitted with new improved RPCs in 2002, their efficiency has not significantly decreased since then. In the barrel, the RPCs have been replaced in 2004 and 2006 by 12 layers of limited streamer tube (LST) detectors and 6 layers of brass have been added to improve hadron absorption. The tubes have performed well since their installation with an efficiency of all layers at the geometrically expected level of 90%. The pion rejection versus muon efficiency is shown in Figure 2.19 for the LSTs and RPCs. The LSTs efficiency is better than the efficiency that the RPCs had, even during the Run1.

2.2.6 Trigger

The BABar trigger is designed to select a large variety of physics processes (efficiency greater than 99% for $B\bar{B}$ events) while keeping the output rate below 400 Hz to satisfy computing limitations of the offline processing farms (beam induced background rates with at least one track with $p_t > 120$ MeV/c or at least one EMC cluster with $E > 100$ MeV are typically 20 kHz). The trigger accepts also 95% of continuum hadronic events and more than 90% of $\tau^+\tau^-$ events. It is implemented as a two level hierarchy, the hardware Level 1 (L1) followed by the software Level 3 (L3).

The L1 trigger has an output rate of the order of 1 kHz to 3 kHz, depending on the luminosity and background conditions. It is based on charged tracks in the DCH above a preset transverse momentum, showers in the EMC, and track detected in the IFR. L3 operates by refining and augmenting the selection methods used in L1. Based on both the complete event and L1 trigger information, the L3
Figure 2.19: Pion rejection versus muon efficiency for two different momentum ranges. (left: $2 < p < 4$ GeV/$c$, right $0.5 < p < 2$ GeV/$c$)

<table>
<thead>
<tr>
<th>L3 Trigger</th>
<th>$\epsilon_{l\beta}$</th>
<th>$\epsilon_{B \rightarrow \pi^{0}\pi^{0}}$</th>
<th>$\epsilon_{B \rightarrow \tau\nu}$</th>
<th>$\epsilon_{\tau\tau}$</th>
<th>$\epsilon_{uds}$</th>
<th>$\epsilon_{T}\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined DCH filters</td>
<td>99.4</td>
<td>89.1</td>
<td>96.6</td>
<td>97.1</td>
<td>95.4</td>
<td>95.5</td>
</tr>
<tr>
<td>Combined EMC filters</td>
<td>93.5</td>
<td>95.7</td>
<td>62.3</td>
<td>87.4</td>
<td>85.6</td>
<td>46.3</td>
</tr>
<tr>
<td>Combined DCH + EMC filters</td>
<td>&gt;99.9</td>
<td>99.3</td>
<td>98.1</td>
<td>99.0</td>
<td>97.6</td>
<td>97.3</td>
</tr>
<tr>
<td>Combined L1+L3</td>
<td>&gt;99.9</td>
<td>99.1</td>
<td>97.8</td>
<td>98.9</td>
<td>95.8</td>
<td>92.0</td>
</tr>
</tbody>
</table>

Table 2.2: L3 trigger efficiency (%) for various physics processes, derived from Monte Carlo simulation.

Software algorithm selects events of interest allowing them to be transferred to mass storage data for further analysis. It uses an algorithm based on the drift chamber tracking, which rejects beam-induced charged particle background produced in the material close to the IP, and a second algorithm based on the calorimeter clustering. Then, based on the L3 tracks and clusters, a variety of filters perform event classification and background reduction. Tab. 2.2 shows the L3 and L1+L3 trigger efficiency for some relevant physics processes, derived from simulated events.
Chapter 3

Event Reconstruction

This analysis is based on the study of the recoil of $B$ mesons fully reconstructed in a hadronic mode ($B_{\text{reco}}$). One of the two $B$ mesons from the decay of the $\Upsilon(4S)$ is reconstructed in a fully hadronic mode (see fig. 3.1). The remaining particles of the event are then supposed to belong to the other $B$ ($B_{\text{recoil}}$ see Chapter 5).

Due to the exclusive reconstruction of the $B_{\text{reco}}$ the properties of the recoil can be studied in a very clean environment. As it will be shown in detail in Chapter 5, charge conservation can be required and the missing mass of the event should be compatible with zero. Moreover, since the kinematics is over-constrained, the resolution on the reconstructed quantities, such as the mass $M_X$ of the hadronic system $X$, is improved. The momentum of the recoiling $B$ is also known and therefore it's possible to apply a Lorentz transformation to the charged lepton four-momentum and compute it in the $B$ rest frame. The charge of the $B$ is known, so $B^0$ and $B^+$ decays can be studied separately. The flavor of the $B$ is known, therefore the correlation between the charge of the lepton and the flavor of the $B$ can be used to reject $B \rightarrow D \rightarrow \text{lepton}$ cascade events, as described in Sec.6.1 The only drawback is that the efficiency of this method is quite low and is dominated by the $B$ reconstruction efficiency. The charged and neutral particles reconstruction is described in this chapter together with the algorithm used for the full hadronic $B$ reconstruction, the so-called Semi-exclusive reconstruction [49].

3.1 Charged Particle Reconstruction

The charged particle tracks are reconstructed by processing the information from both tracking systems, the SVT and the DCH. Charged tracks are defined by five parameters ($d_0, \phi_0, \omega, z_0, \tan \lambda$) and their associated error matrix, measured at the point of closest approach to the z-axis. $d_0$ and $z_0$ are the distances between the point and the origin of the coordinate system in the $x-y$ plane and
along the \( z \)-axis respectively. The angle \( \phi_0 \) is the azimuth of the track, \( \lambda \) is the angle between the transverse plane and the track tangent vector at the point of closest approach and the \( x \)-axis, and
\[
\omega = 1/p_t
\]
is the curvature of the track. \( d_0 \) and \( \omega \) are signed variables and their sign depends on the charge of the track. The track finding and the fitting procedures use the Kalman filter algorithm \cite{50} that takes into account the detailed distribution of material in the detector and the full magnetic field map.

For what concerns this analysis, the definition of charged track is based on some specific quantities:

- **distance of closest approach to the beam spot** measured in the \( x \) - \( y \) plane (\( |d_{xy}| \)) and along the \( z \) axis (\( |d_z| \)). A cut on these variables rejects fake tracks and background tracks not originating near the beam-beam interaction point. We require \( |d_{xy}| < 1.5 \text{ cm} \) and \( |d_z| < 5 \text{ cm} \);

- **maximum momentum**: to remove tracks not compatible with the beam energy we require \( p_{\text{lab}} < 10 \text{ GeV/c} \), where \( p_{\text{lab}} \) refers to the laboratory momentum of the track, against misreconstructed tracks;

- **polar angle acceptance**: the polar angle, in the laboratory frame, is required to be \( 0.41 < \theta_{\text{lab}} < 2.54 \) in order to match the acceptance of the detector. This ensures a well-understood tracking efficiency and systematics.

No restrictions on the impact parameter have been imposed for secondary tracks from \( K_s \) decays. No cut on the minimum number of hits on track is used in order to maximize the efficiency for low momenta tracks.

In addition, special criteria are used to reject tracks due to specified tracking errors. Tracks with a transverse momentum \( p_{\perp} < 0.18 \text{ GeV/c} \) don’t reach the EMC and therefore they will spiral inside the DCH ("loopers"). The tracking algorithms of \( B\bar{B}\text{aR} \) will not combine the different fragments of these tracks into a single track. Therefore dedicated cuts have been developed to reject track fragments compatible with originating from looper based on their distance from the beam spot. Looper candidates are identified as two tracks with a small difference in \( p_{\perp}, \phi \) and \( \theta \). Of such a pair only the track fragments with the smallest distance \( |d_z| \) to the beam interaction point is retained. These cuts
3.2 Particle Identification

remove roughly 13% of all low-momentum tracks in the central part of the detector. On average, the mean observed charged multiplicity per \( B \) meson is reduced by less than 1%.

Two tracks very closely aligned to each other are called "ghosts". These cases arise when the tracking algorithms splits the DCH hits belonging to a single track in two track fragments. If two tracks are very close in phase space only the track with the largest number of DCH hits is retained. This ensures that the fragment with the better momentum measurement is kept in the analysis.

A summary of the track selection criteria is shown in Table 3.1.

<table>
<thead>
<tr>
<th>Track selection</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance in ( x - y ) plane</td>
<td>(</td>
</tr>
<tr>
<td>distance in ( z ) axis</td>
<td>(p_{lab} &lt; 10 \text{ GeV/c}) (P_{\perp,lab} &gt; 0.06 \text{ GeV/c})</td>
</tr>
<tr>
<td>maximum momentum</td>
<td>(p_{lab} &lt; 0.2 \text{ GeV/c}) if (N_{DCH} = 0)</td>
</tr>
<tr>
<td>minimum momentum</td>
<td></td>
</tr>
<tr>
<td>maximum momentum for SVT-only</td>
<td></td>
</tr>
<tr>
<td>tracks</td>
<td></td>
</tr>
<tr>
<td>geometrical acceptance</td>
<td>(0.410 &lt; \theta_{lab} &lt; 2.54 \text{ rad})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reject tracks if</th>
<th>(\Delta p_{\perp,lab} &lt; 0.12 \text{ GeV/c (loopers)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>loopers ((\Delta p_{\perp,lab} &lt; 0.25 \text{ GeV}))</td>
<td>(\Delta p_{\perp,lab} &lt; 0.15 \text{ GeV/c (ghost)})</td>
</tr>
<tr>
<td>((</td>
<td>\cos \theta</td>
</tr>
<tr>
<td>ghosts ((p_{\perp} &lt; 0.35 \text{ GeV}))</td>
<td>Opposite sign: (</td>
</tr>
<tr>
<td>(N_{DCH}^1 &lt; 45 - N_{DCH}^2)</td>
<td>(</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of track selection cuts as adopted from [52]. \(N_{DCH}^1\) and \(N_{DCH}^2\) are the numbers of DCH hits for ghost candidate tracks. Track 1, the one with most DCH is selected.

3.2 Particle Identification

3.2.1 Electron Identification

The track selection criteria are tightened for electrons selection to suppress background and to ensure a reliable momentum measurement and identification efficiency. There are requirements in
addition for transverse momentum \( p_\perp > 0.1 \) GeV/c, and \( N_{DCH} \geq 12 \) for the number of associated drift chamber hits. Furthermore, only electron candidates with a laboratory momentum \( p_{lab} > 0.5 \) GeV/c are considered.

Electrons are identified using a likelihood-based selector [51], which uses a number of discriminating variables:

- \( E_{cal}/p_{lab} \), the ratio of \( E_{cal} \), the energy deposited in the EMC, and \( p_{lab} \) the momentum in the laboratory rest frame measured using the tracking system; \( LAT \), the lateral shape of the calorimeter deposit (defined by Eq. 3.3); \( \Delta \Phi \), the azimuthal distance between the centroid of the EMC cluster and the impact point of the track on the EMC; and \( N_{cry} \), the number of crystals in the EMC cluster;
- \( dE/dx \), the specific energy loss in the DCH;
- the Čerenkov angle \( \theta_C \) and \( N_C \), the number of photons measured in the DRC.

First, muons are rejected on the basis of \( dE/dx \) ratio value and the shower energy relative to the momentum. For the remaining tracks, likelihood functions are computed assuming the particle is an electron, pion, kaon, or proton. These likelihood functions are based on probability density functions that are derived from pure particle data control samples for each of the discriminating variables. For hadrons, we take into account the correlations between energy and shower-shapes.

Using combined likelihood functions

\[
L(\xi) = \frac{P(E/p, LAT, \Delta \Phi, dE/dx, \theta_C|\xi)}{P_{EMC}(E/p, LAT, \Delta \Phi|\xi)} \cdot \frac{P_{DCH}(dE/dx|\xi)}{P_{DRC}(\theta_C|\xi)}
\]

for the hypotheses \( \xi \in \{e, \pi, K, p\} \), the fraction

\[
F_e = \frac{\int L(\nu)}{\sum \int L(\xi)}
\]

is defined, where, for the relative particle fractions, \( f_e : f_\pi : f_K : f_p = 1 : 5 : 1 : 0.1 \) is assumed. A track is identified as an electron if \( F_e > 0.95 \).

The electron identification efficiency has been measured using radiative \( \text{Bhabha events} \), as function of laboratory momentum \( p_{lab} \) and polar angle \( \theta_{lab} \). The misidentification rates for pions, kaons, and protons are extracted from selected data samples. Pure pions are obtained from kinematically selected \( K^0_S \rightarrow \pi^+ \pi^- \) decays and three prong \( \tau^\pm \) decays. Two-body \( \Lambda \) and \( D^0 \) decays provide pure samples of protons and charged kaons.

The performance of the likelihood-based electron identification algorithm is summarized in Fig. 3.2, in terms of the electron identification efficiency and the per track probability that an hadron is misidentified as an electron.
3.2 Particle Identification

Figure 3.2: Electron identification and hadron misidentification probability for the likelihood-based electron selector as a function of momentum (left) and polar angle (right). Note the different scales for identification and misidentification on the left and right ordinates, respectively.

The average hadron fake rates per track are determined separately for positive and negative particles, taking into account the relative abundance from Monte Carlo simulation of $B\bar{B}$ events, with relative systematic uncertainties of 3.5%, 15% and 20% for pions, kaons, and protons, respectively. The resulting average fake rate per hadron track of $p_{lab} > 1.0$ GeV/c, is of the order of 0.05% for pions and 0.2% for kaons.

3.2.2 Muon Identification

The identification of muons relies mostly on the performance of the IFR. A set of simple cut based selectors has been developed for the selection of muon tracks at the beginning of the experiment. However, due to the non-optimal quantity of iron affecting the original design of the IFR and the fast degradation of the performance of RPCs, the development of more sophisticated algorithms proved to be necessary.

In our analysis, muons are selected by using the $\text{NNtightMuonSelection}$ selector, which is based on the use of the Neural Network (NN) technique. The variables used in the selection are (in the order as they appear on the input layer of the NN):

- $\Delta \lambda = \lambda_{exp} - \lambda_{meas}$: the difference between the expected and the measured number of interaction length traversed by the track in the muon hypothesis;
- $\chi^2_{hit} = \chi^2 / d.o.f$ of the IFR hit strips in the cluster with respect to the track extrapolation;
- $\sigma_m$: the standard deviation of the average multiplicity of hit strips per layer;
- $T_C$: the continuity of the track in the IFR;
Event Reconstruction

- $E_{cal}$: the energy deposited in the EMC;
- $\lambda_{meas}$: the number of interaction length traversed by the track;
- $\chi^2_{hit} = \chi^2/d.o.f$ of the IFR hit strips with respect to a third order polynomial fit of the cluster;
- $\bar{m}$: the average multiplicity of hit strips per layer.

The NN implemented uses one input layer accepting the 8 variables listed above, one hidden layer with 16 nodes and one output layer with one node. Due to the different performance of the chambers in the different sections of the IFR (old and new RPC’s, LST’s) and the decrease with time of RPC’s performance, the training sample for the Neural Network has been split into several subsamples.

![Graphs showing efficiency for different angular ranges](image)

Figure 3.3: Performance of the NNTightMuonSelection: the selection efficiency for separately $\mu^+$ and $\mu^-$ as a function of momentum is shown in three different ranges of the polar angle.

Figures 3.3 and 3.4 show the efficiency and the pion mis-identification rate for the selector we use in our analysis, averaged over Run1-Run6 data-taking periods.

### 3.2.3 Charged Kaon Identification

A standard selector, based only on track candidates with an associated momentum above 300 GeV/c and exploiting variables based on information from the DRC, the DCH and the SVT, is used to identify charged kaons. Likelihood functions are computed separately for charged particles, as products of three terms, one for each detector subsystem and then combined, similarly to the electron algorithm previously described. Fig. 3.5 shows a comparison of the charged kaon efficiency versus the charged pion misidentification.
Figure 3.4: Performance of the \textit{NNTightMuonSelection}: the probability of $\pi^+$ and $\pi^-$ to pass the selection as a function of momentum is shown in three different ranges of the polar angle.

Figure 3.5: Charged kaon identification and pion misidentification probability for the tight kaon micro selector as a function of momentum (left) and polar angle (right). The solid markers indicate the efficiency for positive particles, the empty markers the efficiency for negative particles. Note the different scales for identification and misidentification on the left and right ordinates, respectively.
Neutral Particles Reconstruction

Neutral particles (photons, $\pi^0$, neutral hadrons) are detected in the EMC as clusters of close crystals where energy has been deposited. They are required not to be matched to any charged track extrapolated from the tracking volume to the inner surface of the EMC. For this analysis a neutral particle is selected by its local maximum energy depositions in the EMC. These energy clusters originate mostly from photons, thus momenta and angles are assigned to be consistent with photons originating from the interaction region. The list of neutrals is also used to reconstruct the neutral pions. In Sec. 3.4.1 is described the selection of the $\pi^0$ candidates used in the $B_{\text{reco}}$ reconstruction.

Photon candidates are required to have an energy $E_\gamma > 30$ MeV in order to reduce the impact of the sizeable beam-related background of low energy photons. Some additional backgrounds, due to hadronic interactions, either by $K_L$ or neutrons, can be reduced by applying requests on the shape of the calorimeter clusters.

The variable $\text{LAT}$, used to discriminate between electromagnetic and hadronic showers in the EMC, is defined as

$$\text{LAT} = \frac{\sum_{i=3}^{N} E_i r_i^2}{\sum_{i=3}^{N} E_i r_i^2 + E_1 r_0^2 + E_2 r_0^2},$$

where $N$ is the number of crystals associated with the electromagnetic shower, $r_0$ is the average distance between two crystals, which is approximately 5 cm for the BaBar calorimeter, $E_i$ is the energy deposited in the i-th crystal, numbering them such that $E_1 > E_2 > \ldots > E_N$ and $r_i, \phi_i$ are the polar coordinates in the plane perpendicular to the line pointing from the interaction point to the shower center centered in the cluster centroid. Considering that the summations start from $i = 3$, they omit the two crystals containing the highest amounts of energy. Since electrons and photons deposit most of their energy in two or three crystals, the value of LAT is small for electromagnetic showers. Multiplying the energies by the squared distances enhances the effect for hadronic showers, compared with electromagnetic ones.

Another useful shape variable is the so-called S9S25, that is the ratio of the energy deposited in the 9 closest crystals from the cluster centroid over the energy deposited in the 25 closest clusters. Are assigned to the neutral particles all the local energy maxima, not matching with charged tracks, and stored in a list the relative parameters, determined by assuming that the particle is a photon.

Clusters, which are considered as neutral candidates, although they are close to tracks, need to be rejected. These unmatched clusters are due to inefficiencies in the matching algorithms and lead to double counting of their energies. In order to study them, the distance in $\phi(d\phi)$ and $\theta(d\theta)$ with respect to all the tracks which do not have a matched cluster, were considered. In this analysis, a cut on the 3-D angle

$$\alpha = \cos^{-1} [\cos \theta_{cl} \cos \theta_{tr} + \sin \theta_{cl} \sin \theta_{tr} \cos (\phi_{cl} - \phi_{tr})]$$

(3.4)
was considered, where $\theta_{c,t,v}$ and $\phi_{c,t,v}$ are the polar coordinates for clusters and tracks respectively. Clusters which satisfy $\alpha < 0.08$ with the closest track that is not matched to another EMC cluster are considered unassociated clusters and not used in the further analysis [52].

Tab. 3.2 shows a detailed summary of the selection criteria applied on the photon candidates.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Selection criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of crystals $N_c$</td>
<td>$N_c &gt; 2$</td>
</tr>
<tr>
<td>Cluster energy $E_{clus}$</td>
<td>$E_{clus} &gt; 50$ MeV</td>
</tr>
<tr>
<td>LAT</td>
<td>$LAT &lt; 0.6$</td>
</tr>
<tr>
<td>Geometrical acceptance $\theta_{clus}$</td>
<td>$0.32 &lt; \theta_{clus} &lt; 2.44$</td>
</tr>
<tr>
<td>unmatched clusters</td>
<td>$\alpha &gt; 0.08$</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of the photon requirements.

### 3.4 Meson Reconstruction

This section describes the reconstruction of the mesons used in the full reconstruction of the $B$. A number of control samples have been employed to perform all the relevant studies; in most cases, the optimizations have been obtained by using only part of the available data, and it has been assumed that the results are valid for the entire sample as well.

#### 3.4.1 $\pi^0$ Reconstruction

A wide energy spectrum of $\pi^0$'s ranging from particles almost at rest up to several GeV is needed in this analysis. For instance, lowest energy $\pi^0$'s are used to reconstruct the $D^{*0} \rightarrow D^0\pi^0$ decays (usefull in this analysis for the $B \rightarrow D^{*}\ell\nu$ partial reconstruction method) while the decay products in the $B \rightarrow D\pi\pi^0$ channel have quite large momentum.

The $\pi^0$'s are reconstructed using pairs of neutral clusters with a lower energy at 30 MeV and applying a cut on the LAT variable. The $\pi^0$ candidate has to have an energy below 450 MeV. A mass windows of 110-155 MeV$/c^2$, corresponding to $(-4\sigma, +3\sigma)$, is required. In Figure 3.6 the invariant masses distributions for simulated events and data are shown.

#### 3.4.2 $K^0_S$ Reconstruction

$K^0_S$ are reconstructed in the channel $K^0_S \rightarrow \pi^+\pi^-$ by pairing all possible tracks of opposite sign and looking for the 3D point (vertex) which is more likely to be common to the two tracks. The
algorithm is based on a $\chi^2$ minimization and uses as starting point for the vertex finding the point of closest approach of the two tracks in 3D. No constraint is applied on the invariant mass of the pair, but a $\pm 2\sigma$ cut around the nominal value is imposed: $0.490 < m_{\pi^+\pi^-} < 0.505$ GeV/$c^2$. The invariant mass distribution of the $\pi^+\pi^-$ obtained from data is shown in Fig. 3.7. A comparison between data and Monte Carlo for the $K_S^0$ momentum and polar angle is shown in Fig. 3.8. The channel $K_S^0 \rightarrow \pi^0\pi^0$ is not reconstructed in this analysis.

3.4.3 $D$ Reconstruction

The reconstruction of the $B$ mesons in hadronic modes utilizes charmed $D$ mesons decaying in a variety of channels. These channels and their branching fractions are summarized in Tab. 3.3.

The $D^0$ is reconstructed in the modes $D^0 \rightarrow K\pi$, $D^0 \rightarrow K3\pi$, $D^0 \rightarrow K\pi\pi^0$ and $D^0 \rightarrow K_S^0\pi\pi$. The charged tracks originating from a $D$ meson are required to have a minimum momentum of 200 MeV/$c$ for the channel $D^0 \rightarrow K\pi$ and 150 MeV/$c$ for the remaining three modes. The $D^0$ candidates are required to lie within $\pm 3\sigma$ of the nominal $D^0$ mass. All $D^0$ candidates must have momentum greater than 1.3 GeV/$c$ and lower than 2.5 GeV/$c$ in the $T(4S)$ frame. The lower bound is needed to reduce combinatorial, the upper one is the kinematic endpoint of the $D^0$ coming from a $B \rightarrow D^0X$ decay or $B \rightarrow D^{*+}X$ with $D^{**+} \rightarrow D^0\pi^+$. A vertex fit is performed and a $\chi^2$ probability greater than 0.1% is required. The selection criteria are summarized in Tab. 3.4.

$D^+$ candidates are reconstructed in the modes $D^+ \rightarrow K^-\pi^+\pi^+$, $D^+ \rightarrow K^-\pi^+\pi^+\pi^0$, $D^+ \rightarrow K_S^0\pi^+\pi^+$, $D^+ \rightarrow K_S^0\pi^+\pi^+\pi^0$. We require that the kaon used in the $K^-\pi^+\pi^+$ and $K^-\pi^+\pi^+\pi^0$ modes have a minimum momentum of 200 MeV/$c$ while the pions are required to have
<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching Fraction(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^* \rightarrow D^0 \pi^0; D^0 \rightarrow K\pi$</td>
<td>2.55 ± 0.06</td>
</tr>
<tr>
<td>$D^* \rightarrow D^0 \pi^0; D^0 \rightarrow K3\pi$</td>
<td>5.0 ± 0.2</td>
</tr>
<tr>
<td>$D^* \rightarrow D^0 \pi^0; D^0 \rightarrow K\pi\pi^0$</td>
<td>8.8 ± 0.6</td>
</tr>
<tr>
<td>$D^* \rightarrow D^0 \pi^0; D^0 \rightarrow K_S^0\pi\pi (K_S^0 \rightarrow \pi^+\pi^-)$</td>
<td>1.35 ± 0.08</td>
</tr>
<tr>
<td>$D^+ \rightarrow K\pi\pi$</td>
<td>9.5 ± 0.3</td>
</tr>
<tr>
<td>$D^+ \rightarrow K_S^0\pi^0 (K_S^0 \rightarrow \pi^+\pi^-)$</td>
<td>0.94 ± 0.06</td>
</tr>
<tr>
<td>$D^+ \rightarrow K\pi\pi\pi^0$</td>
<td>5.5 ± 2.7</td>
</tr>
<tr>
<td>$D^+ \rightarrow K_S^0\pi\pi^0 (K_S^0 \rightarrow \pi^+\pi^-)$</td>
<td>2.38 ± 0.31</td>
</tr>
<tr>
<td>$D^+ \rightarrow K_S^0\pi\pi^0 (K_S^0 \rightarrow \pi^+\pi^-)$</td>
<td>3.5 ± 1.0</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\pi^0; D^0 \rightarrow K\pi$</td>
<td>2.35 ± 0.12</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\pi^0; D^0 \rightarrow K3\pi$</td>
<td>4.6 ± 0.3</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\pi^0; D^0 \rightarrow K\pi\pi^0$</td>
<td>8.1 ± 0.7</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\pi^0; D^0 \rightarrow K_S^0\pi\pi (K_S^0 \rightarrow \pi^+\pi^-)$</td>
<td>1.2 ± 0.1</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\gamma; D^0 \rightarrow K\pi$</td>
<td>1.44 ± 0.19</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\gamma; D^0 \rightarrow K3\pi$</td>
<td>2.82 ± 0.18</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\gamma; D^0 \rightarrow K\pi\pi^0$</td>
<td>5.0 ± 0.4</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^{0}\gamma; D^0 \rightarrow K_S^0\pi\pi (K_S^0 \rightarrow \pi^+\pi^-)$</td>
<td>0.7 ± 0.1</td>
</tr>
<tr>
<td>$D^0 \rightarrow K\pi$</td>
<td>3.80 ± 0.07</td>
</tr>
<tr>
<td>$D^0 \rightarrow K3\pi$</td>
<td>7.72 ± 0.28</td>
</tr>
<tr>
<td>$D^0 \rightarrow K\pi\pi^0$</td>
<td>14.1 ± 0.5</td>
</tr>
<tr>
<td>$D^0 \rightarrow K_S^0\pi\pi$</td>
<td>2.03 ± 0.12</td>
</tr>
</tbody>
</table>

Table 3.3: $D$ mesons decays used in the Semi-exclusive $B$ reconstruction.
Figure 3.7: Mass distributions for $K_S^0 \rightarrow \pi^+\pi^-$ on data. The distribution is fitted with a sum of a double Gaussian and a first order polynomial function.

Figure 3.8: $K_S^0$ momentum (left) and polar angle (right) distributions in data (solid markers) and Monte Carlo simulation (hatched histogram), normalized to the same area.

momentum greater than 150 MeV/c. For the $K_S^0\pi^+X$ modes, the minimum charged track momentum is required to be 200 MeV/c. $D^+$ candidates are required to have an invariant mass within ±3σ, calculated on an event-by-event basis, of the nominal $D^+$ mass. The $D^+$ candidates must have momentum greater than 1.0 GeV/$c$ in the $\Upsilon(4S)$ frame for the three cleanest modes ($D^+ \rightarrow K^-\pi^+\pi^+$, $D^+ \rightarrow K_S^0\pi^+$ and $D^+ \rightarrow K_S^0\pi^+\pi^0$) and greater than 1.6 GeV/$c$ for the two remaining ones ($D^+ \rightarrow K^-\pi^+\pi^0\pi^0$ and $D^+ \rightarrow K_S^0\pi^+\pi^+\pi^0$). Moreover, all $D^+$ candidates must have momentum lower than 2.5 GeV/$c$ in the $\Upsilon(4S)$ frame, as the $D^0$ case. A vertex fit is performed and a $\chi^2$ probability greater than 0.1% is
required. The selection criteria are summarized in Tab. 3.5.

<table>
<thead>
<tr>
<th></th>
<th>(D^0 \to K\pi)</th>
<th>(D^0 \to K\pi\pi)</th>
<th>(D^0 \to K\pi\pi\pi)</th>
<th>(D^0 \to K\pi\pi\pi\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mass window</td>
<td>± 15 MeV/c²</td>
<td>± 25 MeV/c²</td>
<td>± 15 MeV/c²</td>
<td>± 20 MeV/c²</td>
</tr>
<tr>
<td>Charged Tracks: lower (p^*) cut</td>
<td>&gt; 200 MeV/c</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D^0) upper (p^*) cut</td>
<td></td>
<td>&lt; 2.5 GeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D^0) lower (p^*) cut</td>
<td></td>
<td>&gt; 1.3 GeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex Fit</td>
<td></td>
<td>(\chi^2 &gt; 0.01)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Summary of criteria applied for the \(D^0\) selection.

<table>
<thead>
<tr>
<th></th>
<th>(D^+ \to K\pi\pi)</th>
<th>(D^+ \to K\pi\pi\pi)</th>
<th>(D^+ \to K\pi\pi\pi\pi)</th>
<th>(D^+ \to K\pi\pi\pi\pi\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mass window</td>
<td>± 20 MeV/c²</td>
<td>± 20 MeV/c²</td>
<td>± 30 MeV/c²</td>
<td>± 30 MeV/c²</td>
</tr>
<tr>
<td>(D^+) lower (p^*) cut</td>
<td>&gt; 1.0 GeV/c</td>
<td></td>
<td>&gt; 1.6 GeV/c</td>
<td></td>
</tr>
<tr>
<td>(D^+) upper (p^*) cut</td>
<td></td>
<td>&lt; 2.5 GeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charged Tracks: lower (p^*) cut</td>
<td></td>
<td>&gt; 200 MeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex fit</td>
<td></td>
<td>(\chi^2 &gt; 0.01)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: Summary of criteria applied for the \(D^+\) selection.

\(D^{*+}\) candidates are formed by combining a \(D^0\) with a pion which has momentum greater than 70 MeV/c. Due to the limited phase space available from the \(D^+ - D^0\) mass difference \(\Delta m\), the pion coming from the \(D^*\) has a low momentum, below 450 MeV/c, and is referred to as soft pion (see Fig. 3.9). Only the reconstruction of the \(D^{*+} \to D^0\pi^+\) channel is discussed here, since \(D^{*+} \to D^+\pi^0\) events enter in the \(B \to D^+X\) category of the Semi-exclusive reconstruction, as explained in Section 3.5. A vertex fit for the \(D^{*+}\) is performed using a constraint to the beam spot to improve the angular resolution for the soft pion. A fixed \(\sigma_y = 30\) \(\mu m\) is used to model the beam spot spread in the vertical direction, to avoid bias in the \(D^{*+}\) vertex fit. The fit is required to converge, but no cut is applied on the probability of \(\chi^2\). After fitting, selected \(D^{*+}\) candidates are required to have \(\Delta m\) within ±3σ of the measured nominal value (see Fig. 3.9). \(\Delta m\) distribution is fitted with a double Gaussian distribution. The width is taken to be a weighted average of the core and broad Gaussian distributions. The selection criteria are summarized in Tab. 3.6.

\(D^{*0}\) candidates are reconstructed by combining a selected \(D^0\) with a either a \(\pi^0\) or a photon
Figure 3.9: Distribution of soft pion momentum in the \( T(4S) \) frame (left) and \( m(D^{*+}\pi^-) - m(D^0) \) mass distribution for \( D^{*+} \) candidates in the \( B \to D^{*+}\pi^- \), \( D^0 \to K\pi \) mode. Units in both plots are GeV. Data and Signal MC (true \( D^* \)) haven’t the same luminosity because only the shapes are important to define the cuts. Vertical lines indicate the signal windows used in the selection.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^{*+} \to D^0\pi^+ )</td>
<td></td>
</tr>
<tr>
<td>Vertexing and ( \chi^2 )</td>
<td>beam spot constraint (( \sigma_y = 30 ) ( \mu m )), convergence ( \pm 3\sigma ) MeV/c^2</td>
</tr>
<tr>
<td>( m(D\pi^+) - m(D^0) )</td>
<td></td>
</tr>
<tr>
<td>( p^* (\pi^+) )</td>
<td>[70, 450] MeV/c</td>
</tr>
</tbody>
</table>

Table 3.6: Summary of criteria applied for the \( D^{*+} \) selection.

Having a momentum less than 450 MeV/c in the \( T(4S) \) frame. The minimum momentum for the \( \pi^0 \) corresponds to 70 MeV while the photons are required to have an energy greater than 100 MeV. For \( D^{*0} \to D^0\pi^0 \) decay, selected \( D^{*0} \) candidates are required to have \( \Delta m \) within 4 MeV/c^2 of the nominal value while a wider window, 127 MeV/c^2 < \( \Delta m < 157 \) MeV/c^2, is used for \( D^{*0} \to D^0\gamma \). The selection criteria are summarized in Tab. 3.7.
3.5 Semi-exclusive Reconstruction Method

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{*0} \rightarrow D^0\pi^0$</td>
<td></td>
</tr>
<tr>
<td>$m(D^0\pi^0) - m(D^0)$</td>
<td>$\pm 4$ MeV/$c^2$</td>
</tr>
<tr>
<td>$p^*(\pi^0)$</td>
<td>$[70, 450]$ MeV/$c$</td>
</tr>
<tr>
<td>$p^*(D^{*0})$</td>
<td>$1.3 &lt; p^* &lt; 2.5$ GeV/$c$</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^0\gamma$</td>
<td></td>
</tr>
<tr>
<td>$m(D^0\gamma) - m(D^0)$</td>
<td>$[127, 157]$ MeV/$c^2$</td>
</tr>
<tr>
<td>$E^*(\gamma)$</td>
<td>$[100, 450]$ MeV</td>
</tr>
<tr>
<td>$p^*(D^{*0})$</td>
<td>$1.3 &lt; p^* &lt; 2.5$ GeV/$c$</td>
</tr>
</tbody>
</table>

Table 3.7: Summary of criteria applied for the $D^{*0}$ selection.

3.5 Semi-exclusive Reconstruction Method

The aim of the Semi-exclusive reconstruction is to get as many as possible $\bar{B}$ mesons in fully hadronic modes in order to study the properties of the recoiling $B$ meson in $\Upsilon(4S) \rightarrow B\bar{B}$ transitions.

The sum of a few, very pure exclusive modes ensures very high purity but low efficiency. On the other hand a fully inclusive approach with high multiplicities is not feasible since the level of combinatorial background would be too high. A compromise has been set up, where only favored modes are considered and an algorithm which combines the final state particles neglecting the intermediate states as inclusive as possible is used.

The strategy of the semi exclusive reconstruction is designed to accumulate very large samples of decay of the type $B \rightarrow D^0\Upsilon$, in order to perform studies on their recoil. Here “D” refers to a charm meson, and $\Upsilon$ represents a collection of hadron of charge $\pm$ and composed of $n_1\pi + n_2K + n_3K^0 + n_4\pi^0$. The charm mesons serve as first “seed” for the selection of decays and we use four different seeds: $D^\pm$ and $D^{*\pm}$ for $B^0$ and $D^0$ and $D^{*0}$ for $B^{\pm}$. Reconstruction on high multiplicity modes relies on an appropriate reduction of combinatorial. The technical aspects of the algorithm can be found in [49]. The first analysis step implemented by the algorithm is selection of clean list of charged particles ( pions and kaons: $\pi K$ list ) that can be associated to the seed in order to form a $B$ candidate. To do that the following criteria are used:

- tracks in the GoodTracksVeryLoos list are considered [53]
- tracks that are compatible with $\lambda \rightarrow p\pi$, gamma conversion, electrons or muons ( with the selectors defined before ) are discarded.

All the tracks that are identified as Kaons are assigned the kaon mass while the others are treated...
as pions. With the available charged particle list (πK) it is possible to construct pairs of opposite charge hadrons (V^0 = h^+h^-) or quartets of hadrons (W^0 = h^+h^-h^+h^-), where h stands for π or K. Each combination of hadrons in the πK list is considered as (V^0) or W^0. A special treatment is reserved to the case where both the daughters of a K^0 are among the tracks used for V^0 or a W^0. In this case the two tracks are, in fact, replaced by the K^0; a V^0 would become a K^0 and a W^0 either a K^0π or a K^0 K^0. Having in hand all the possible combinations of particles for each event is then possible to go to the last step in the reconstruction chain: the creation of **B candidate list**. Starting from an user provided seed (e.g., a D*0 candidate) one element of the πK list is added. This is driven by the requirement that the newly formed seed have the same charge as the B meson: after that only objects, V^0, W^0 or π^0 can be added. The B candidates obtained are classified accordingly to the values of two kinematics variables, the beam energy substituted mass m_{ES} = √s/2 - p_B^2 and the energy difference ΔE = E_B - √s/2 is the total energy in the T(4S) center of mass frame, and p_B and E_B denote the momentum and energy of the B_{reco} candidate in the same frame (in the next section are available more detailed informations for these two variables). Those two variables are commonly used in BAbAr to select B candidate and to study and reject combinatorial background. Four cases are given, depending if candidates fall in the ΔE, m_{ES} plane regions (figure 3.10) marked by:

![Diagram](image)

Figure 3.10: Sketch of the definition of the m_{ES}-ΔE plane regions, depending on where the B candidate fall and it can be: (A) used only as candidate, (B) used both as candidate and as seed, (C) used as seed but not as candidate and (D) not used

A. The B candidate has a ΔE too high and adding one more pion would push it out. These candidates
are considered but not as seed;

B The $B$ candidates is considered as signal, is selected and can serve as a new seed to which additional pion pairs can be added by iterating the procedure;

C The $B$ candidate is out of the signal region, but can still be used as a new seed;

D The $B$ candidate is not considered further (discarded)

Usually the semi exclusive method provides a number of possible $B$ candidates larger than one per event and for this reason we have performed a specific algorithm for the **Best $B$** selection. It’s based on a $\chi^2$ probability maximization of discriminating variables that describe how well the event has been reconstructed (In section 3.5.3 is available a complete description of the bestB algorithm).

### 3.5.1 Definition of $\Delta E$ and $m_{ES}$

Two main variables are used to select $B$ candidates, to extract the yields and to define a sideband region to study the background: $\Delta E$ and $m_{ES}$.

The energy difference $\Delta E$ is defined as:

$$\Delta E = E_B^* - \sqrt{s}/2,$$

where $E_B^*$ is the energy of the $B$ candidate in the $T(4S)$ rest frame (CM) and $\sqrt{s}$ is the total energy of the $e^+e^-$ system in the CM rest frame. The resolution of this variable is affected by the detector momentum resolution and by the particle identification since a wrong mass assignment results in a shift in $\Delta E$. Due to the energy conservation, signal events are Gaussian distributed according to a Gaussian function in $\Delta E$ around zero. Continuum and part of the $b\bar{b}$ background have a $\Delta E$ distribution that can be modeled with a polynomial distribution. Instead, some other $b\bar{b}$ background, due to misidentification, gives shifted Gaussian peaks. The resolution of this variable depends essentially on the reconstructed $B$ mode and $\pi^0$ multiplicity and it can vary from 20 to 40 MeV.

The beam energy-substituted mass $m_{ES}$ is defined as

$$m_{ES} = \sqrt{\left(\sqrt{s}/2\right)^2 - p_B^2},$$

where $p_B^*$ is the $B$ candidate momentum in the CM rest frame. Since $|p_B^*| \ll \sqrt{s}/2$, the experimental resolution on $m_{ES}$ is dominated by beam energy fluctuations. To an excellent approximation, the shapes of the $m_{ES}$ distributions for $B$ meson reconstructed in a final states with charged tracks only are Gaussian and practically identical. Otherwise the presence of neutrals in the final states, in case their showers are not fully contained in the calorimeter, can introduce tails.
Figure 3.11: Left: fit of the Argus function (Eq. 3.7) to the $m_{ES}$ distributions for candidates in the continuum background ($udsc$).

It is important to notice that, since the sources of experimental smearing are uncorrelated (beams energy for $m_{ES}$ and detector momentum resolution for $\Delta E$), $m_{ES}$ and $\Delta E$ also are basically uncorrelated. The background shape in $m_{ES}$ is parametrized using the Argus function [55]:

$$\frac{dN}{dm_{ES}} = N \cdot m_{ES} \cdot \sqrt{1 - x^2} \cdot \exp \left( -\chi \cdot (1 - x^2) \right)$$

(3.7)

where $x = m_{ES}/m_{max}$ and $\chi$ is a free parameter determined from the fit. The $m_{max}$ represents the endpoint of the Argus distribution. The Argus function provides a good parametrization of the continuum ($uuddc\bar{s}s$), as Fig. 3.11 shows.

The signal component is fitted using a modified Gaussian function [56], which will be described in detail in Chapter 5. The total fit (Argus and signal function) to the data sample is shown in Fig. 3.12. The radiative tail of this function can take into account cases where the energy of the neutral candidates is not fully deposited in the EMC crystals. The left tail of the distribution depends on the reconstructed $B$ mode and in particular on the number of $\pi^0$. The maximum total number of floating parameters in the $m_{ES}$ fits is 7. Two of them refer to the Argus shape, while the remaining five parameters belong to the signal function.

In the following the number of signal and background events (indicated as $S$ and $B$ in the plots) are estimated as the area of the signal and the Argus functions integrated for $m_{ES} > 5.27$ GeV.
3.5 Semi-exclusive Reconstruction Method

Figure 3.12: The $m_{ES}$ distribution for data (points with statistical errors) is shown together with the results of the fit (solid line) for selected semileptonic decays from $B^+B^-$ events (left) and $B^0\bar{B}^0$ events (right). The dashed line shows the contribution from combinatorial and continuum background.

3.5.2 Study of the $Y$ System

The choice of the submodes is crucial in the reconstruction method. The identification of the clean modes allows to set up the most efficient and pure selection among the multiple candidates in different modes. A detailed study of the $Y$ system, looking for resonances in the signal and background shape is performed.

As an example, the invariant mass of the $Y = \pi\pi^0$ system in the $B \rightarrow D^*\pi\pi^0$ mode is shown in Fig. 3.13. There is a large contribution below 1.5 GeV/$c^2$ due to the $\rho$ resonance, but there is also a small amount of signal at $\sim 2.4-2.6$ GeV/$c^2$, but not clear enough for a specific selection. Therefore two sub-modes are defined depending on whether $m_{\pi\pi^0}$ is smaller than 1.5 GeV/$c^2$ or greater than 1.5 GeV/$c^2$, without requiring the sub-mode belonging to a precise resonance structure. In this way the clean $B \rightarrow D^*\pi\pi^0$ sub-mode $(m_{\pi\pi^0} < 1.5 \text{ GeV}/c^2)$ has been separated from the low purity ones $(m_{\pi\pi^0} > 1.5 \text{ GeV}/c^2)$.

Finally, the total number of the reconstructed $B$ decay modes are 52 and 53 for the $D^0$ and $D^+$ seeds, respectively. The total number of the decay modes is 1007. A summary is shown in Tab. 3.8.

3.5.3 Best $B$ Selection and comparison with the priority purity method

The semi-exclusive algorithm [49] generates a list of possible $B$ candidates following the method described in section 3.5. In figure 3.14 we can observe the multiplicity of different $B$ candidates per event present in a subset of simulated data. (the tail can reach up to 36).
<table>
<thead>
<tr>
<th>Channel</th>
<th>pre-seed mode</th>
<th>number of $B$ modes</th>
<th>total number of modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to D^0 Y$</td>
<td>$D^0 \to K \pi$</td>
<td>52</td>
<td>208</td>
</tr>
<tr>
<td></td>
<td>$D^0 \to K \pi \pi^0$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^0 \to K_S^0 \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^0 \to K \pi \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>$B \to D^+ Y$</td>
<td>$D^+ \to K \pi \pi$</td>
<td>53</td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>$D^+ \to K \pi \pi \pi^0$</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^+ \to K_S^0 \pi$</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^+ \to K_S^0 \pi \pi^0$</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>$B^\pm \to D^{*0}$</td>
<td>$D^{*0} \to D^0 \pi^0, D^0 \to K \pi$</td>
<td>52</td>
<td>416</td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \pi^0, D^0 \to K \pi \pi^0$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \pi^0, D^0 \to K_S^0 \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \pi^0, D^0 \to K \pi \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \gamma, D^0 \to K \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \gamma, D^0 \to K \pi \pi^0$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \gamma, D^0 \to K_S^0 \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*0} \to D^0 \gamma, D^0 \to K \pi \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>$B^0 \to D^{*+} Y$</td>
<td>$D^{*+} \to D^0 \pi, D^0 \to K \pi$</td>
<td>52</td>
<td>208</td>
</tr>
<tr>
<td></td>
<td>$D^{*+} \to D^0 \pi, D^0 \to K \pi \pi^0$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*+} \to D^0 \pi, D^0 \to K_S^0 \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D^{*+} \to D^0 \pi, D^0 \to K \pi \pi \pi$</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1097</td>
</tr>
</tbody>
</table>

Table 3.8: Summary of the number of Semi-exclusive modes.
Figure 3.13: Mass distribution of the $Y$ system $\pi\pi^0$ in data for the $B \to D^*\pi\pi^0$ decay mode. The normalized background, as evaluated from sidebands, is overlaid.

For that reason is important to develop a criteria to choose the best-B candidate. In the bsemieclusive original version (used in all the previous inclusive $|V_{ub}|$ measurements with hadronic tags) a tuned $\Delta E$ cut was applied for each decay mode in order to decrease the combinatorial background and after that if more than one $'B$ candidates was presents the one with highest a priority purity was chosen as best B. We define a priority purity the ratio $S/S+B$ where S and B are the number of signal and background events respectively determined by fitting the $m_{ES}$ distribution. This kind of method as bestB selection relies on $\Delta E$ cuts and $m_{ES}$ fits performed in 2002 and therefore in principle it needs of frequent updates. For that reason we have decided to perform a new algorithm based on a selection that in principle doesn’t need updates. This alternative selection, which is employed in this thesis, is described below:

The best B selection comprises the following steps:

• First of all, the new algorithm doesn’t use 10 of the 53 $B$ modes implemented in the $B$ semi exclusive reconstruction because we consider them too dirty ($S/\sqrt{S+B} < 0.02$ for both $B^0\bar{B}^0$ and $B^+B^-$ MC events). In table 3.9 are reported these modes discarded from the bestB selection. The number of signal and background events is computed using a truth-matching algorithm described in Chapter 5 and rejecting events with $m_{ES} < 5.27$;

• The B candidates are fitted using a "TreeFitter" algorithm where none mass constraints on the daughter is imposed. The algorithm fits the complete B decay chain;
Figure 3.14: Number of multiple possible B candidates per event after the bsemi-exclusive reconstruction.

<table>
<thead>
<tr>
<th>B mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow D\pi\pi\pi\pi K_s$</td>
</tr>
<tr>
<td>$B \rightarrow DK\pi\pi\pi(&gt;2.7\text{GeV})$</td>
</tr>
<tr>
<td>$B \rightarrow DK\pi\pi(&gt;1.8-2.2\text{GeV})$</td>
</tr>
<tr>
<td>$B \rightarrow DK\pi\pi(&gt;2.7\text{GeV})$</td>
</tr>
<tr>
<td>$B \rightarrow DKK\pi\pi&lt;2.7\text{GeV}$</td>
</tr>
<tr>
<td>$B \rightarrow DKK\pi\pi(&gt;2.7\text{GeV})$</td>
</tr>
<tr>
<td>$B \rightarrow D\pi\pi\pi K_S \text{ no } D^*$</td>
</tr>
</tbody>
</table>

Table 3.9: B-modes discarded from the bestB selection because present a ratio $S\sqrt{S+B} < 0.02$. We have used $B^0\bar{B}^0$ and $B^+B^-\text{ MC samples and a truth }$ matching algorithm to distinguish between signal and background events.

- The algorithm discriminates the different B candidates using a $\chi^2$ probability from the following variables:
  1. $M_{First}$
     - It’s the reconstructed mass value of the first B daughter and it can be only one of the
3.5 Semi-exclusive Reconstruction Method

following particles: $D^0$, $D^\pm$, $D^{*0}$ and $D^{*\pm}$;

2. $\Delta E$

It’s a kinematic variable and it’s equal to $E_B - \sqrt{s}/2$ where $\sqrt{s}$ refers to the total energy in the $T(4S)$ center of mass frame while $E_B$ is the $B_{Reco}$ energy in the same frame;

3. $\chi^2_{VTX}$

It’s the $\chi^2$ of the the B decay chain fit.

4. With these variables we create a $\chi^2$ and we extrapolate the $\chi^2$ probability using the degrees of freedom. We choose as best B the one with the highest $\chi^2$ probability ( $P(\chi^2 d.o.f)$ )

$$\chi^2_{Total} = \chi^2_{Vertex} + \left(\frac{M_{D_{Reco}} - M_{D_{Nominal}}}{\sigma_{D_{Reco}}}\right)^2 + \left(\frac{\Delta E}{\sigma_{\Delta E}}\right)^2,$$  \hspace{1cm} (3.8)

$$d.o.f_{Total} = d.o.f_{Vertex} + 1 + 1,$$ \hspace{1cm} (3.9)

In tables 3.10 and 3.11 are reported the total number of $V_{cb}$ and Other events (our background) obtained at various selection stages using the two different algorithms on Generic Monte Carlo (see Chapter 4). Are also reported the same kind of events in the region $m_X < 1.55 \text{ GeV/c}$ where the events $B \rightarrow X_u \ell \nu$ are enriched. In the tables 3.12 and 3.13 are reported the total number of $|V_{ub}|$ events (our signal) obtained at various selection stages using the two different algorithms on resonant and non-resonant Monte Carlo events (see Chapter 4). Are also reported the same kind of events in the region $m_X < 1.55 \text{ GeV/c}$ where the events $B \rightarrow X_u \ell \nu$ are enriched. All the events counts in Tables 3.10 to 3.13 are obtained removing the combinatorial background using the truth-matching described in Chapter 6. Finally comparing these tables, the new bestB algorithm increases the number of $|V_{ub}|$ events by about 4% while decreases the background events ($V_{cb+Other}$) by about 3%. 


<table>
<thead>
<tr>
<th></th>
<th>Vcb</th>
<th>Other</th>
<th>Vcb  $m_X&lt;1.55$</th>
<th>Other  $m_X&lt;1.55$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semileptonic Cut</td>
<td>659475</td>
<td>65013</td>
<td>316761</td>
<td>15456</td>
</tr>
<tr>
<td>Total Event Charge Cut</td>
<td>408543</td>
<td>36882</td>
<td>172382</td>
<td>8571</td>
</tr>
<tr>
<td>Miss Neutrino mass square Cut</td>
<td>172129</td>
<td>15501</td>
<td>10253</td>
<td>681</td>
</tr>
<tr>
<td>D* Veto Cut</td>
<td>149626</td>
<td>14318</td>
<td>8721</td>
<td>602</td>
</tr>
<tr>
<td>D* Veto with $\pi^0$ Cut</td>
<td>92995</td>
<td>8486</td>
<td>5941</td>
<td>501</td>
</tr>
</tbody>
</table>

Table 3.10: Total number of Vcb and Other events at various selection stages using the old best B selection on Generic Monte Carlo. Are also reported The number of events in the region $m_X<1.55$ Gev/c where the events $B \to X_u \ell \nu$ are enriched are also reported.

<table>
<thead>
<tr>
<th></th>
<th>Vcb</th>
<th>Other</th>
<th>Vcb  $m_X&lt;1.55$</th>
<th>Other  $m_X&lt;1.55$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semileptonic Cut</td>
<td>648738</td>
<td>61407</td>
<td>312946</td>
<td>14696</td>
</tr>
<tr>
<td>Total Event Charge Cut</td>
<td>401874</td>
<td>34073</td>
<td>169877</td>
<td>7850</td>
</tr>
<tr>
<td>Miss Neutrino mass square Cut</td>
<td>168443</td>
<td>13986</td>
<td>9855</td>
<td>633</td>
</tr>
<tr>
<td>D* Veto Cut</td>
<td>147704</td>
<td>12567</td>
<td>8483</td>
<td>578</td>
</tr>
<tr>
<td>D* Veto with $\pi^0$ Cut</td>
<td>91760</td>
<td>7755</td>
<td>5786</td>
<td>470</td>
</tr>
</tbody>
</table>

Table 3.11: Total number of Vcb and Other events at various selection stages using the new best B selection on Generic Monte Carlo. The number of events in the region $m_X<1.55$ Gev/c where the events $B \to X_u \ell \nu$ are enriched are also reported.
## 3.5 Semi-exclusive Reconstruction Method

| $|V_{ub}|$ with old Best B selection | Not Res. | Res. | Not Res. $(m_X<1.55)$ | Res. $(m_X<1.55)$ |
|-----------------------------------|----------|------|----------------------|------------------|
| Semileptonic Cut                  | 20023    | 17991| 14301                | 15903            |
| Total Event Charge Cut            | 13998    | 13399| 9730                 | 11769            |
| Miss Neutrino mass square Cut     | 10011    | 10295| 6299                 | 8720             |
| D* Veto Cut                       | 9691     | 9991 | 6072                 | 8467             |
| D* Veto with $\pi^0$ Cut          | 8252     | 9001 | 5469                 | 7780             |

Table 3.12: Total number of $|V_{ub}|$ events at various selection stages using the old best B selection on Resonant and not Resonant $|V_{ub}|$ Monte Carlo. The number of events in the region $m_X<1.55$ Gev/c where the events $B \rightarrow X_u \ell \nu$ are enriched are also reported.

| $|V_{ub}|$ with old New B selection | Not Res. | Res. | Not Res. $(m_X<1.55)$ | Res. $(m_X<1.55)$ |
|-----------------------------------|----------|------|----------------------|------------------|
| Semileptonic Cut                  | 20325    | 18186| 14479                | 16004            |
| Total Event Charge Cut            | 14492    | 14082| 10054                | 12269            |
| Miss Neutrino mass square Cut     | 10382    | 10685| 6513                 | 8933             |
| D* Veto Cut                       | 9984     | 10434| 6274                 | 8732             |
| D* Veto with $\pi^0$ Cut          | 8513     | 9392 | 5636                 | 8016             |

Table 3.13: Total number of $|V_{ub}|$ events at various selection stages using the new B selection on Resonant and not Resonant $|V_{ub}|$ Monte Carlo. The number of events in the region $m_X<1.55$ Gev/c where the events $B \rightarrow X_u \ell \nu$ are enriched are also reported.
Chapter 4

Data and Monte Carlo Samples

The event samples used in this thesis, consisting of both real and simulated data, are detailed in this Chapter.

4.1 Data

The total dataset used in this analysis correspond to an integrated on-peak luminosity of 425.8 fb$^{-1}$, recorded by BaBar in the years 1999-2007. They correspond to about 467.6 million of $B\bar{B}$ pairs. Table 4.1 summarizes data event samples divided by run period.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>integrated luminosity on peak</th>
<th>$N_{B\bar{B}}(10^6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>20.4 fb$^{-1}$</td>
<td>22.4</td>
</tr>
<tr>
<td>Run 2</td>
<td>61.1 fb$^{-1}$</td>
<td>67.5</td>
</tr>
<tr>
<td>Run 3</td>
<td>32.3 fb$^{-1}$</td>
<td>35.6</td>
</tr>
<tr>
<td>Run 4</td>
<td>100.3 fb$^{-1}$</td>
<td>110.5</td>
</tr>
<tr>
<td>Run 5</td>
<td>133.3 fb$^{-1}$</td>
<td>147.2</td>
</tr>
<tr>
<td>Run 6</td>
<td>78.4 fb$^{-1}$</td>
<td>84.2</td>
</tr>
</tbody>
</table>

Table 4.1: Data event samples.

4.2 Monte Carlo Samples

The Monte Carlo samples used in this analysis are summarized in Table 4.2.
<table>
<thead>
<tr>
<th>Data Set</th>
<th>1$^\ast$ $B$ mode</th>
<th>2$^\ast$ $B$ mode</th>
<th>equiv. lumin.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$ generic</td>
<td>Generic</td>
<td>Generic</td>
<td>1265 fb$^{-1}$</td>
</tr>
<tr>
<td>$B^\pm$ generic</td>
<td>Generic</td>
<td>Generic</td>
<td>1268 fb$^{-1}$</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$ pure res. generic</td>
<td>$b \to u\ell \bar{\nu}$ exclusive</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$ pure non-res. generic</td>
<td>$b \to u\ell \bar{\nu}$ inclusive</td>
</tr>
</tbody>
</table>

Table 4.2: Monte Carlo event samples used in this analysis.

![Graphs](image)

Figure 4.1: $M_X$ distributions at generator-level for pure resonant (left) and pure non-resonant (right) $b \to u\ell \bar{\nu}$ Monte Carlo.

### 4.2.1 Generic $b\bar{b}$ Monte Carlo

Generic $b\bar{b}$ Monte Carlo represents the full simulation of all possible decays of the $B$ meson and it should represent the data and an unbiased event sample. This sample is actually used to model the data.

### 4.2.2 Resonant Models for $\bar{B} \to X_u \ell \bar{\nu}$

Exclusive charmless semileptonic $\bar{B} \to X_u \ell \bar{\nu}$ decays are simulated as a combination of three-body decays to narrow resonances, $X_u = \pi, \eta, \rho, \omega, \eta'$, etc.

These decays are simulated using the ISGW2 model [57]. To be consistent with the latest measurements, generation values of branching ratios (detailed in Table 4.3) have been adjusted in a reweighting procedure to match the current PDG values [54] (see also Tab. 4.4). The hadronic invariant mass spectrum at the generator level for these decays is shown in Fig. 4.1 (left).
4.2 Monte Carlo Samples

<table>
<thead>
<tr>
<th>mode</th>
<th>BR</th>
<th>hadron mass [GeV]</th>
<th>mode</th>
<th>BR</th>
<th>hadron mass [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to \pi^- \ell^+ \nu$</td>
<td>$180 \cdot 10^{-6}$</td>
<td>0.1396</td>
<td>$B^+ \to \pi^0 \ell^+ \nu$</td>
<td>$90 \cdot 10^{-6}$</td>
<td>0.135</td>
</tr>
<tr>
<td>$B^0 \to \rho^- \ell^+ \nu$</td>
<td>$260 \cdot 10^{-6}$</td>
<td>0.7669</td>
<td>$B^+ \to \rho^0 \ell^+ \nu$</td>
<td>$130 \cdot 10^{-6}$</td>
<td>0.77</td>
</tr>
<tr>
<td>$B^0 \to a_0^- \ell^+ \nu$</td>
<td>$14 \cdot 10^{-6}$</td>
<td>0.983</td>
<td>$B^+ \to a_0^0 \ell^+ \nu$</td>
<td>$7 \cdot 10^{-6}$</td>
<td>0.983</td>
</tr>
<tr>
<td>$B^0 \to a_1^- \ell^+ \nu$</td>
<td>$165 \cdot 10^{-6}$</td>
<td>1.26</td>
<td>$B^+ \to a_1^0 \ell^+ \nu$</td>
<td>$82 \cdot 10^{-6}$</td>
<td>1.26</td>
</tr>
<tr>
<td>$B^0 \to a_2^- \ell^+ \nu$</td>
<td>$14 \cdot 10^{-6}$</td>
<td>1.318</td>
<td>$B^+ \to a_2^0 \ell^+ \nu$</td>
<td>$7 \cdot 10^{-6}$</td>
<td>1.318</td>
</tr>
<tr>
<td>$B^0 \to b_1^- \ell^+ \nu$</td>
<td>$102 \cdot 10^{-6}$</td>
<td>1.233</td>
<td>$B^+ \to B_1^0 \ell^+ \nu$</td>
<td>$48 \cdot 10^{-6}$</td>
<td>1.233</td>
</tr>
<tr>
<td>exclusive</td>
<td>$735 \cdot 10^{-6}$</td>
<td></td>
<td>exclusive</td>
<td>$730 \cdot 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>inclusive</td>
<td>$1365 \cdot 10^{-6}$</td>
<td></td>
<td>inclusive</td>
<td>$1365 \cdot 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>$2100 \cdot 10^{-6}$</td>
<td></td>
<td>total</td>
<td>$2095 \cdot 10^{-6}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Branching fractions and hadron masses used in the generator for $b \to u \ell \nu$ decay.

4.2.3 Non-Resonant Model for $\bar{B} \to X_u \ell \bar{\nu}$

The BABAR simulation code models the inclusive charmless semileptonic $B$ decays into hadronic states with masses larger than $2m_\nu$ using a prescription by De Fazio and Neubert (DFN) [58]; other theoretical models, like the ones described in Sec. 1.3.3 are not yet implemented. However, predictions from the DFN model in this analysis are used only to calculate the detector reconstruction efficiency for events in restricted phase-space regions. In this respect, differences with the more detailed theoretical models shown in Chapter 1 are negligible and indeed, as it will be shown in Chapter 7, the systematic uncertainties due to the limited knowledge of the parameters involved in the definition of the signal model, are among the smallest ones. The most recent theoretical models are nevertheless used to
compute the acceptances needed to relate the partial charmless semileptonic branching fraction to \(|V_{ub}|\).

The DFN model calculates the triple-differential decay rate, \(d^3\Gamma / (dq^2 dE_t ds_H) (s_H = M_B^2)\), up to \(\mathcal{O}(\alpha_s)\) corrections. The non-perturbative corrections to this observable (the \(S\) function in Eq. 1.35) can be associated with the motion of \(b\) quark inside the \(B\) meson, and are commonly referred as “Fermi motion”.

Fermi motion effects are included in the heavy-quark expansion by re-summing an infinite set of leading-twist correction into a shape function \(F(k_+),\) which governs the light-cone momentum distribution of the heavy quark inside the \(B\) meson [20, 19]. The physical decay distributions are obtained from a convolution of parton model spectra with this function.

As already mentioned in Chapter1, the shape function is a universal characteristic of the \(B\) meson governing inclusive decay spectra in processes with mass-less partons in the final state, such as \(\bar{B} \rightarrow X_u \ell \bar{\nu}\) and \(\bar{B} \rightarrow X_u \gamma\). The convolution of the parton spectra with this function is such that in the perturbative formulae for the decay distributions the \(b\)-quark mass \(m_b\) is replaced by the momentum dependent mass \(m_b + k_+\) and similarly the parameter \(\Lambda = M_B - m_b\) is replaced by \(\Lambda - k_+\). Here \(k_+\) takes values between \(-m_b\) and \(\Lambda\), with a distribution centered around \(k_+ = 0\) and with a characteristic width of \(\mathcal{O}(\Lambda)\).

Several functional forms for the shape function have been suggested in the literature but they are all subject to constraints on the moments of this function, \(A_n = \langle k_+^n \rangle\), which are related to the forward matrix elements of local operator on the light cone [20]. The first three moments must satisfy

\[
A_0 = \int F(k_+) dk_+ = 1
\]

\[
A_1 = \int k_+ F(k_+) dk_+ = 0
\]

\[
A_2 = \int k_+^2 F(k_+) dk_+ = \frac{\mu_2^2}{3}
\]

where \(\mu_2^2\) is the average momentum squared of the \(b\) quark inside the \(B\) meson [59]. The form of the shape function is unknown and usually is adopted the simple form [60]

\[
F(k_+) = N(1 - x)^a e^{(1+a)x}; \quad x = \frac{k_+}{\Lambda} \leq 1
\]

which is such that \(A_1 = 0\) by construction (neglecting exponentially small terms in \(m_b/\Lambda\)), whereas the condition \(A_0 = 1\) fixes the normalization \(N\). The parameter \(a\) can be related to the second moment, yielding \(A_2 = \mu_2^2/3 = \Lambda^2/(1+a)\). Thus the \(b\) quark mass (or \(\Lambda\)) and the quantity \(\mu_2^2\) are the two parameters of the function.

In the simulation the hadron \(X_u\) is produced with a non-resonant and continuous invariant mass spectrum according to the DFN model. Finally, the fragmentation of the \(X_u\) system into final state
hadrons is performed by JETSET [61]. Invariant hadronic mass spectrum for pure non-resonant charmless semileptonic $B$ decays at the generator level is shown in Fig. 4.1 (right).

A reweighting of the Fermi motion distribution is used to obtain distributions for different values of $\lambda$ and $\mu^2$. Several experimental measurements of these heavy quark parameters are available. In this thesis, the determination from the $B \to X_s \gamma$ spectrum as measured by Belle, CLEO and $BaBar$ and from $b \to c \ell \bar{\nu}$ moments measured by several experiments and extracted by Flächer and Buchmüller [10]

$$m_b = 4.590 \pm 0.039 \text{ GeV}$$

$$\mu^2 = 0.401 \pm 0.040 \text{ GeV}^2,$$

and also comprehensively presented by the HFAG group [62], has been used. In Figure 4.2 a fit to the heavy quark parameters using constraints from different measurements are displayed.

4.2.4 $BaBar$ Hybrid Model for $\bar{B} \to X_s \ell \bar{\nu}$

Neither the resonant, or the non-resonant models alone are adequate for a proper simulation of charmless semileptonic decays. The non-resonant generator for instance is not able to produce hadronic final states with masses below $2m_\pi$ and it does not produce any resonant structure in the hadron mass. Therefore the resonant and non-resonant components are combined such that the total
branching fraction is consistent with the measured value \[63, 64\] and that the fraction of events below a given threshold in \(M_X\) is similar to the non-resonant case (except local discontinuity due to the resonant structure). This requirement is imposed in order to minimize theoretical uncertainties related to the hadronization in the charmless decay and to ensure that the OPE is valid. This reweighting procedure is common among the other \(|V_{ub}|\) analyses performed in \(BaBar\).

### 4.2.5 Reweighting Hybrid Model for \(\bar{B} \rightarrow X_u\ell\bar{\nu}\)

The hybrid model described in the previous paragraph considers the non-resonant model only above 1.264 GeV/\(c^2\), while it is in principle possible to go down to the allowed kinematic limit. So the non-resonant events have been reweighted in the hybrid model in order to have a better agreement between the model and the measured fraction of resonant and non-resonant events.

The \((M_X, q^2, E_\ell)\) phase space is divided into an \(8 \times 8 \times 8\) grid; the number of non-resonant (resonant) events in each bin is denoted as \(N_{nr}^i\) (\(N_r^i\), then the weights \(w\) are determined in the following way:

- the ratio between the number of non-resonant and resonant events after reweighting is equal to the imposed one

\[
\frac{\sum_i w_i \cdot N_{nr}^i}{N_r} = R = \frac{B(b \rightarrow u) - B(X_u\ell\nu \text{ resonant})}{B(X_u\ell\nu \text{ resonant})} 
\]  
(4.6)

- the fraction of hybrid events in a given bin \(H_yb^i/H_yb\) is the same as the fraction of non-resonant events form the reference sample in that bin \(Ref^i/Ref\):

\[
\frac{H_yb^i}{H_yb} = \frac{w_i \cdot N_{nr}^i + N_r^i}{N_r + \sum_i w_i \cdot N_{nr}^i} = \frac{N_{nr}^i}{N_{nr}} = \frac{Ref^i}{Ref} 
\]  
(4.7)

where \(H_yb = N_r + N_{nr} = N_r + \sum_i w_i \cdot N_{nr}^i\).

Solving these two equations the weights are

\[
w_i = \frac{Ref^i - N_r}{N_{nr}^i} 
\]  
(4.8)

- if the \(w_i\) is negative, is set to zero. To preserve the overall normalization between the reference sample and the hybrid, a global weight is applied to the inclusive component of the hybrid corresponding to and the weights for the inclusive is recomputed. The correct weight \(corr_w^i\) it then

\[
corr_w^i = w_i \cdot \frac{GlobalW \cdot T - E}{Y} 
\]  
(4.9)

where \(GlobalW\) is the ratio \(Reference \text{ Inclusive/sum of Hybrid}\), \(T\) is the branching ratio of the total hybrid, \(E (Y)\) is the branching ratio of exclusive (inclusive) part of the hybrid model. This preserves not only the normalization between the reference inclusive and the hybrid but also the exclusive/inclusive BRs.
4.2 Monte Carlo Samples

Since the generation values are not updated to the latest measurements, inclusive and exclusive branching ratios are reweighted to the measured values as reported in Table 4.4. The experimental errors on these measurements are one of the sources of the systematic uncertainties.

All branching fractions and theory parameters involved in this reweighting technique are varied within their errors in the evaluation of the associated uncertainty. The uncertainty due to the specific parametrization given in Equation 4.4 has also been evaluated by using other two empirical parametrization (see Sec. 7.3).

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>generation value ($B^0$)</th>
<th>new value with error ($B^0$)</th>
<th>generation value ($B^+$)</th>
<th>new value with error ($B^+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow \pi \ell \nu$</td>
<td>1.80</td>
<td>1.36 $\pm$ 0.09</td>
<td>0.90</td>
<td>0.77 $\pm$ 0.12</td>
</tr>
<tr>
<td>$B \rightarrow \eta \ell \nu$</td>
<td></td>
<td></td>
<td>0.30</td>
<td>0.64 $\pm$ 0.20</td>
</tr>
<tr>
<td>$B \rightarrow \rho \ell \nu$</td>
<td>2.60</td>
<td>2.47 $\pm$ 0.33</td>
<td>1.30</td>
<td>1.28 $\pm$ 0.18</td>
</tr>
<tr>
<td>$B \rightarrow \omega \ell \nu$</td>
<td></td>
<td></td>
<td>1.30</td>
<td>1.30 $\pm$ 0.60</td>
</tr>
<tr>
<td>$B \rightarrow \eta' \ell \nu$</td>
<td></td>
<td></td>
<td>0.60</td>
<td>0.04 $\pm$ 0.04</td>
</tr>
<tr>
<td>other resonant</td>
<td>2.95</td>
<td>0.00 (see note *)</td>
<td>2.90</td>
<td>0.00 (see note *)</td>
</tr>
<tr>
<td>non-resonant</td>
<td>13.65</td>
<td>18.70 (see note **)</td>
<td>13.65</td>
<td>20.09 (see note **)</td>
</tr>
<tr>
<td>Total sample</td>
<td>21.00</td>
<td>22.53</td>
<td>20.95</td>
<td>24.12</td>
</tr>
</tbody>
</table>

Table 4.4: Comparison between the BABAR generated branching fraction and the latest measurements with corresponding errors (in $10^{-4}$) to which they are rescaled. *: Since there are no measurements for other resonances ($a,b,f,\ldots$), these BR are set to 0. **: The fraction for inclusive events is adjusted to preserve the correct overall normalization of the hybrid sample.

Figure 4.3 shows the comparison, for charged and neutral $B$, between pure non-resonant signal Monte Carlo and its rescaled component in the hybrid model along with the rescaled resonant component. The hybrid model is represented by the black line.
Figure 4.3: $M_X$ distribution for the resonant (“exclusive”), the pure non-resonant (“inclusive”) $b \to u\ell\bar{\nu}$ MC and for the reweighted combination of exclusive and non-resonant MC (“hybrid”) for $B^0$ (top) and $B^+$ (bottom) decays.
Chapter 5

Signal Selection

In this thesis, the $\Upsilon(4S) \rightarrow BB$ process is used to study semileptonic decays of the $B$ meson recoiling against a $\bar{B}$ meson whose decay into hadronic final states is fully reconstructed. Compared with untagged analyses, where little information on the companion $\bar{B}$ is used, this tagging technique offers many advantages:

- the $B_{\text{reco}}$ decay products consist of particles that are not used to reconstruct the $B_{\text{reco}}$ meson; in other terms, it is possible to assign particles to the $B_{\text{reco}}$ in an inclusive and unambiguous way, and compute the relevant hadronic variables;
- it is possible to require charge conservation in the event and impose the missing mass of the event (due to the undetected neutrino) to be compatible with zero;
- the momentum of the recoiling $B$ is known and therefore it is possible to apply a Lorentz transformation and compute the relevant kinematic variables in the rest frame of the decaying $B$;
- charge and flavor of the $B$ mesons are known, and the correlation between the charge of the lepton and flavor of the $B_{\text{reco}}$ can be exploited to reject $B \rightarrow DX \rightarrow \ell Y$ background events.

The only disadvantage of this technique is a low reconstruction efficiency, typically in the $0.1 - 0.4\%$ range.

The idea to isolate $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays in regions where the $b \rightarrow c$ transitions are forbidden is conceptually simple (see Fig. 5.1 left plots), but the measurement of kinematic variables is not trivial. Undetected particles and reconstruction errors distort the measured distribution (see Fig. 5.1 right plots) and lead to a large background from the dominant $b \rightarrow c \ell \bar{\nu}$ transition. The data samples are divided in two event sub-samples, one that is enriched in $b \rightarrow u$ transitions by a veto on the presence of kaons in the recoil system, and one that is enriched in $b \rightarrow c$ transitions by requiring the detection
of at least one charged or neutral kaon. The latter can be used as a control sample to check the agreement between data and Monte Carlo for background, so that the remaining background in the $b \rightarrow u$ enriched sample in the phase space region under study can be obtained by extrapolation from the $b \rightarrow c$ dominated region.

5.1 Reconstruction of the Recoil System

Events containing a $B$ meson decaying in a fully reconstructed hadronic final state are selected by the Semi-exclusive reconstruction technique (see Sec. 3.5).

Then we require the presence of at least one lepton (e or $\mu$) in the recoil of the $B_{\text{reco}}$. According with the nature of the lepton we group the events in different categories:

- $B \rightarrow X_s \ell \nu$:
  
  In this category lepton comes from Cabibbo suppressed semileptonic decays and it's represent our signal.
5.1 Reconstruction of the Recoil System

- \( B \to X_c \ell \nu \):
  In this category lepton comes from Cabibbo favored semileptonic decays and it’s represent our main background. The \( X_c \) hadronic state is the sum of many different components \( X_c = D, D^*, D^{**} \).

- other:
  All leptons that don’t come from \( B \to X_u \ell \nu \) and \( B \to X_c \ell \nu \) events are grouped in this category, called other. This category includes leptons that comes from secondary \( B_{\text{reco}} \) decays and hadrons incorrectly identified as leptons:
    - \( B \to D, D_s \to \ell \)
    - \( B \to J/\psi \to \ell \)
    - \( B \to \tau \to \ell \)
    - \( B \to \) fake \( \ell \)

The hadronic system \( X \) is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the \( B_{\text{reco}} \) candidate or the identified lepton. The criteria used for the charged particles selection and neutral clusters selection are described in section 3.1 and 3.3 respectively.

The measured four-momentum \( p_X^\text{reco} \) of the \( X \) system can be written as

\[
p_X^m = \sum_{i=1}^{N_h} p_{X}^{ch} + \sum_{j=1}^{N_{\gamma}} p_{X}^{\gamma}
\]  

(5.1)

where \( p \) are four-momenta and the indices \( ch \) and \( \gamma \) refer to the selected number of charged tracks, and photons. Care is taken to eliminate fake charged tracks (see Section 3.1), as well as low-energy beam-generated photons and energy depositions in the calorimeter (Section 3.3) due to charged particles. The reconstruction of \( K_S^0 \) mesons is used for veto purposes only, without applying any mass constraints.

The neutrino four-momentum \( p_\nu \) is estimated from the missing momentum four-vector

\[
p_{\text{miss}}^m = p_{\gamma(4S)}^m - p_{B_{\text{reco}}}^m - p_X^m - p_\ell^m = Q_{CM} - p_{\text{reco}}^m - p_X^m - p_\ell^m
\]  

(5.2)

where all momenta are measured in the laboratory frame, \( p_{\gamma(4S)} \) refers to the \( \Upsilon(4S) \) meson and \( Q_{CM} \) is the four-momenta of the colliding beams. The measured invariant mass squared, \( m^2_{\text{miss}} = p_{\text{miss}}^2 \), is an important estimator of the quality of the reconstruction of the total recoil system. Undetected particles and measurement uncertainties affect the determination of the four-momenta of the \( X \) system and neutrino, and lead to a large leakage of \( \bar{B} \to X_c \ell \nu \) background from the high \( M_X \) into the low \( M_X \) region and, similarly, in other kinematic regions which would otherwise be background-free. Likewise any sizable energy loss of the leptons via bremsstrahlung or internal radiation will impact the measurement of these two quantities. However, the effect of initial state radiation is small, due to the fact that the width of the \( \Upsilon(4S) \) resonance is rather small.
5.2 Selection of Semileptonic Decays

The following requirements have been optimized to select a sample of $B \rightarrow X \ell \nu$ decays ($N_{sl}^{\text{meas}}$):

- **cut on the angular acceptance for the lepton list** The angular acceptance for tracks associated to leptons (only electron or muons are considered as leptons in this analysis) is $0.450 < \theta < 2.473$ in order to exclude regions where lepton identification efficiency are not well known from the data control samples.

- **Lepton with a momentum in the $B$ rest frame** $p_{\ell}^* > 1$ GeV/c. Semileptonic $B$ decays are identified by the presence of a high momentum electron or muon. A minimum lepton momentum, in the $B$ rest frame, is required to reduce background leptons from secondary charm or $\tau^\pm$ decays, and fake leptons. It is possible to boost the lepton momentum to the rest frame of the recoiling $B$ since the momenta of the $\Upsilon(4S)$ and the reconstructed $B$ are known. The cut $p_{\ell}^* > 1$ GeV/c removes about 10% of the fraction of signal events, as shown in Figure 5.2.

![Figure 5.2: The momentum $p_{\ell}^*$ of the lepton in the recoiling $B$ rest frame after all analysis cuts (left) and after semileptonic cuts (red). Blue plot shows the spectrum for $b \rightarrow u\ell\bar{\nu}$ transitions while the red for $b \rightarrow c\ell\bar{\nu}$.](image)

- **Lepton Charge and $B$ Flavor Correlation.** In semileptonic decays of $B$ mesons the lepton charge is correlated with the $B$ flavor. This leads to the relation $Q_{B_{\text{recoil}}}Q_{\ell} > 0$ for primary leptons and $Q_{B_{\text{recoil}}}Q_{\ell} < 0$ for secondary leptons (here $Q_{B_{\text{recoil}}}$ refers to the $b$ quark charge and $Q_{\ell}$ to the lepton charge in the semileptonic decay). The former condition is imposed on
charged $B$ decays. No requirements are made on neutral $B$ decays, due to flavor oscillations; this implies a small mixing correction to be applied on the results (see Sec. 6.1).

### 5.2.1 Selection of Charmless Semileptonic Decays

Further requirements refine the previously selected sample, enrich it with charmless semileptonic decays, and reject as much charm background as possible. The relevant variables involved are:

- **Number of Leptons**

In $b \rightarrow c\ell\bar{v}$ transitions it is possible to find out a second lepton originated in cascade decays of the charm particles, whereas secondary leptons coming from $b \rightarrow u\ell\bar{v}$ decays are very rare. Therefore, charmless decays can be isolated by requesting one and only one lepton with $p_t^\ell > 1$ GeV/$c$ in the event. On the other hand, additional leptons are accepted when measuring $N_{l}^{\text{meas}}$, which is a quantity dominated by $b \rightarrow c\ell\bar{v}$ transitions. The number of detected leptons per event is shown in Fig. 5.3. A cut at 1 GeV/$c$ offers a reasonable compromise between the uncertainties due to available statistics, lepton identification, background estimate and theoretical models. We require a number of leptons equal to 1.

![Graph showing number of leptons](image)

**Figure 5.3:** Number of identified leptons per event with $p_t^\ell > 1$ GeV/$c$ after all analysis cuts (left) and after semileptonic cuts (right). Blue plot shows the distribution for $b \rightarrow u\ell\bar{v}$ transitions while the red for $b \rightarrow c\ell\bar{v}$.

- **Total Charge of the event**

The reconstructed kinematic variables of the recoil system are distorted if one or more charged particles are lost, therefore charge conservation $Q_{\text{tot}} = Q_{B\text{reco}} + Q_{B\text{recoil}} = 0$ is imposed. This
requirement rejects not only events with missing reconstructed charged particles, but also those with an additional charged particle due to $\gamma \to e^+e^-$ conversions or tracking errors. This cut is also important to reject $\bar{B} \to X_c\ell\bar{\nu}$ events, that are more prone to inefficiencies in particle detection, due to their higher charged multiplicity. As an example, a sizeable fraction of decays with low momentum pions coming from $D^*$ decays are rejected by a requirement on the total charge of the event. Figure 5.4 shows the distributions of $Q_{\text{tot}}$. We require a total charge of the event equal to 0.

![Graph showing total event charge distributions](image)

**Figure 5.4:** Total charge of events selected after all analysis cuts (left) and after semileptonic cuts (right). Blue plot shows the distribution for $b \to u\ell\bar{\nu}$ transitions while the red for $b \to c\ell\bar{\nu}$.

- **Missing mass squared**, $m_{\text{miss}}^2 < 0.5 \text{ GeV}^2/c^4$. The only undetected particle in a semileptonic $B$ decay should be the neutrino. Therefore a cut on the missing mass of the recoil is a powerful tool to reject events in which one or more particles are undetected or poorly measured. As illustrated in Fig. 5.5, the $m_{\text{miss}}^2$ distribution is much wider and extends to higher values for $b \to c\ell\bar{\nu}$ decays. A cut in this variable results in a valuable background suppression, due to higher multiplicities and/or the presence of an additional neutrino or $K_L$ in charm decays. The optimal requirement $m_{\text{miss}}^2 < 0.5 \text{ GeV}^2/c^4$ is chosen because a tighter cut introduces large systematic uncertainties due to differences in $m_{\text{miss}}^2$ resolution in data and Monte Carlo simulations, and a looser cut results in poor signal-to-background ratio and thus unacceptable statistical and systematic errors due to background subtraction.

- **Partially-Reconstructed Missing Mass Squared**
  One of the dominant backgrounds ($\sim 50\%$ of the entire $\bar{B} \to X_c\ell\bar{\nu}$) decays is due to $B \to D^*\ell\bar{\nu}$ decays. In particular the $B^0 \to D^{(*)}\ell\bar{\nu}$ decay, with $D^* \to D^0\pi$ can be identified exploiting the
5.2 Selection of Semileptonic Decays

![Missing Mass Squared plots](image)

Figure 5.5: Missing mass squared of events selected after all analysis cuts (left) and after semileptonic cuts (right). Blue plot shows the distribution for $b \rightarrow u \nu \ell$ transitions while the red for $b \rightarrow c \bar{\nu} \ell$.

The fact that the mass difference between the $D^*$ and the $D^0$ is close to the pion mass and therefore the pion produced in the $D^*$ decay is soft and basically collinear with the $D^*$. Using the approximation that the direction of flight of the pion is the same as the $D^*$ one, and taking into account that the soft pion energy in the $D^*$ rest frame is fixed ($E^\pi_\nu = 145.0$ MeV), the pion energy in the laboratory frame is

$$E^\pi_\nu = \gamma (E'^\pi_\nu - \beta P^\nu_\pi)$$

where $\beta$ and $\gamma$ refer to the $D^*$ boost and $P^\nu_\pi = 39.0$ MeV/c is the soft pion momentum in the $D^*$ frame. The $D^*$ energy in the laboratory frame can be computed, by neglecting the second term in equation 5.3, as

$$E_{D^*} = \gamma M_{D^*} = E^\pi_\nu \frac{M_{D^*}}{E^\pi_\nu}$$

Given that the 4-momentum $P_{D^*}$ of the $D^*$ is now known, the missing invariant mass can be computed as

$$m^2_{\text{miss,PR}} = |P_{\text{recoil}} - P_{D^*} - P_{\text{lepton}}|^2.$$  

This variable is computed for each $\ell - \pi$ pair with opposite charge and pion momentum less then 250 MeV/c. The $m^2_{\text{miss,PR}}$ distribution peaks at zero for background and varies smoothly for signal, as it can be seen in Figure 5.6. We veto events with $m^2_{\text{miss,PR}} < 3$ GeV$^2$/c$^4$.

- Partially-Reconstructed using $\pi^0_{\text{soft}}$ Missing Mass Squared.

A partial reconstruction of the $B \rightarrow D^* \ell \nu$ decays similar to the one described above can be
performed using soft $\pi^0$ that comes from the decay chain $B^- \rightarrow D^{*0} \ell^- \nu_\ell$, $D^{*0} \rightarrow D^0 \pi^0$ and $B^0 \rightarrow D^{*+} \ell^- \nu_\ell$, $D^{*+} \rightarrow D^+ \pi^-$. In this case, both charged and neutral $B$ will contribute to the veto. Table 5.1 shows the $D^*$ decay modes and branching fraction. The missing neutrino invariant mass square, $m_{\text{miss,PR}}^2$, is computed as described above for $D^{*+} \rightarrow D^0 \pi$ decay mode and its distribution for signal and background can be seen in figure 5.7. We veto events with $m_{\text{miss,PR}}^2 < -2 \text{GeV}^2/c^4$.

<table>
<thead>
<tr>
<th>$D^*$ Decay</th>
<th>$D^*$ Branching Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{*+} \rightarrow D^0 \pi^+$</td>
<td>68.3</td>
</tr>
<tr>
<td>$D^{*+} \rightarrow D^+ \pi^0$</td>
<td>30.6</td>
</tr>
<tr>
<td>$D^{*+} \rightarrow D^+ \gamma$</td>
<td>1.1</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^0 \pi^0$</td>
<td>61.9</td>
</tr>
<tr>
<td>$D^{*0} \rightarrow D^0 \gamma$</td>
<td>38.1</td>
</tr>
</tbody>
</table>

Table 5.1: The $D^*$ decay modes and Branching Fraction.

- **Number of Kaons** Since kaons are produced in nearly all charm semileptonic decays, whereas they are highly suppressed in $\bar{B} \rightarrow X_u \ell^+ \nu$ decays, rejecting events where a kaon has been detected in the recoil system reduces the background from $\bar{B} \rightarrow X_u \ell^+ \nu$ decays. Both the number of identified charged kaons and the number of detected $K_S$ are therefore required to be zero.

![Figure 5.6](image1.png)

![Figure 5.6](image2.png)

Figure 5.6: $m_{\text{miss,PR}}^2$ after all analysis cuts (left) and after semileptonic cuts for neutral $B$ meson decays with a positively identified soft pion. Blue plot shows the distribution for $b \rightarrow u \ell \nu$ transitions while the red for $b \rightarrow c \ell \nu$.
(see Figure 5.8). Studies performed show that EMC and IFR information does not permit the identification of $K_L$ and $K_S \rightarrow \pi^0\pi^0$ with a sufficient degree of purity. Charged kaons are identified with an efficiency varying between 60% at the highest and almost 100% at the lowest momenta. The $K_S \rightarrow \pi^+\pi^-$ decays are reconstructed with an efficiency of 80% from pairs of oppositely charged tracks with an invariant mass between 486 and 510 MeV/c$^2$. We require a number of kaons equal to 0.

The criteria for the selection of semileptonic events, after having found a $B_{\text{reco}}$ candidate, and of the final sample enriched in $b \rightarrow u\bar{\nu}$ signal events are summarized in Table 5.2.

In tables 5.3, 5.4, 5.5, 5.6 are reported the number of $b \rightarrow u\bar{\nu}$ events, $b \rightarrow c\ell\nu$ events, Other events and the signal to noise ratio ($S/B$) at various selection stages exploiting different phase space regions. The efficiency of the different cuts on $m_X$ and $q^2$ are also shown in figures 5.9, 5.10. After all cuts requirements the $m_X$, $q^2$ and $P_+\nu$ resolutions can be fitted with the sum of two gaussians and the RMS are about 325 MeV/c$^2$, 1.14 GeV$^2$/c$^4$ and 220 MeV/c respectively, as can been seen in figure 5.11. All the data achieved in these tables are computing on Run1-Rum6 Generic MC and the combinatorial background has been removed using the truth matching algorithm described in 5.3.3.

Figure 5.7: $m^2_{\text{miss,PR}}$ after all analysis cuts (left) and after semileptonic cuts (right) for $B$ meson decays with a neutral identified soft pion. Blue plot shows the distribution for $b \rightarrow u\bar{\nu}$ transitions while the red for $b \rightarrow c\ell\bar{\nu}$. 
Figure 5.8: Number of identified charged plus neutral kaons after all analysis cuts (left) and after semileptonic cuts (right). Blue plot shows the distribution for $b \to u\bar{c}\bar{\nu}$ transitions while the red for $b \to c\bar{d}\bar{\nu}$.

| Semileptonic selection | at least one lepton  
$p^*_L > 1 \text{ GeV}/c$  

| Final selection | only one lepton  

$m_{\text{miss}}^2 < 0.5 \text{ GeV}^2/c^4$  

Sum of all charged particles equal to zero,$Q_{\text{tot}}=0$  

Reject events with kaons in $B_{\text{recoil}}$, $N_{K^\pm} = N_{K_S} = 0$  

Reject events with partially reconstructed $D^*\ell\nu$  

- $m_{\text{miss},PR}^2 < -3 \text{ GeV}^2/c^4$  

- $m_{\text{miss},PR_{\text{soft}}}^2 < -2 \text{ GeV}^2/c^4$

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| $b \to u\bar{c}\bar{\nu}$ | $b \to c\bar{d}\bar{\nu}$  

Table 5.2: Selection criteria for semileptonic events and final selection.
5.2 Selection of Semileptonic Decays

<table>
<thead>
<tr>
<th>Cut Applied</th>
<th>$N_{b\rightarrow u}$</th>
<th>$N_{b\rightarrow c}$</th>
<th>$N_{other}$</th>
<th>$S/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semileptonic Cut</td>
<td>7989</td>
<td>356859</td>
<td>1060000</td>
<td>0.0173</td>
</tr>
<tr>
<td>Total Event Charge Cut</td>
<td>4577</td>
<td>179645</td>
<td>47255</td>
<td>0.0202</td>
</tr>
<tr>
<td>Miss Neutrino mass square Cut</td>
<td>2415</td>
<td>54034</td>
<td>17133</td>
<td>0.0335</td>
</tr>
<tr>
<td>D* Veto Cut</td>
<td>2112</td>
<td>36532</td>
<td>13320</td>
<td>0.0424</td>
</tr>
<tr>
<td>D* Veto with $\pi^0$ Cut</td>
<td>1685</td>
<td>22257</td>
<td>7402</td>
<td>0.0568</td>
</tr>
<tr>
<td>Kaons Veto</td>
<td>1539</td>
<td>10832</td>
<td>3903</td>
<td>0.1044</td>
</tr>
</tbody>
</table>

Table 5.3: Total number of $b \rightarrow u\ell\nu$ ($N_{b\rightarrow u}$), $b \rightarrow c\ell\nu$ ($N_{b\rightarrow c}$), other ($N_{other}$) events and signal to noise ratio ($S/B$) at various selection stages using all the phase space on Generic Monte Carlo.

<table>
<thead>
<tr>
<th>Cut Applied ($m_X &lt; 1.55 \text{ GeV}/c^2$)</th>
<th>$N_{b\rightarrow u}$</th>
<th>$N_{b\rightarrow c}$</th>
<th>$N_{other}$</th>
<th>$S/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semileptonic Cut</td>
<td>4833</td>
<td>189056</td>
<td>23081</td>
<td>0.0226</td>
</tr>
<tr>
<td>Total Event Charge Cut</td>
<td>2680</td>
<td>88606</td>
<td>10460</td>
<td>0.0271</td>
</tr>
<tr>
<td>Miss Neutrino mass square Cut</td>
<td>1107</td>
<td>4102</td>
<td>518</td>
<td>0.2396</td>
</tr>
<tr>
<td>D* Veto Cut</td>
<td>1003</td>
<td>2716</td>
<td>427</td>
<td>0.3191</td>
</tr>
<tr>
<td>D* Veto with $\pi^0$ Cut</td>
<td>891</td>
<td>1901</td>
<td>326</td>
<td>0.4001</td>
</tr>
<tr>
<td>Kaons Veto</td>
<td>854</td>
<td>1402</td>
<td>266</td>
<td>0.5120</td>
</tr>
</tbody>
</table>

Table 5.4: Total number of $b \rightarrow u\ell\nu$ ($N_{b\rightarrow u}$), $b \rightarrow c\ell\nu$ ($N_{b\rightarrow c}$), other ($N_{other}$) events and signal to noise ratio ($S/B$) at various selection stages in a phase space region where the cut $m_X < 1.55 \text{ GeV}/c^2$ is applied on Generic Monte Carlo.

<table>
<thead>
<tr>
<th>Cut Applied ($P_+ &lt; 0.66 \text{ GeV}/c$)</th>
<th>$N_{b\rightarrow u}$</th>
<th>$N_{b\rightarrow c}$</th>
<th>$N_{other}$</th>
<th>$S/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semileptonic Cut</td>
<td>4145</td>
<td>143343</td>
<td>16910</td>
<td>0.0259</td>
</tr>
<tr>
<td>Total Event Charge Cut</td>
<td>2361</td>
<td>68157</td>
<td>7779</td>
<td>0.0311</td>
</tr>
<tr>
<td>Miss Neutrino mass square Cut</td>
<td>1055</td>
<td>4644</td>
<td>684</td>
<td>0.1980</td>
</tr>
<tr>
<td>D* Veto Cut</td>
<td>958</td>
<td>3153</td>
<td>561</td>
<td>0.2579</td>
</tr>
<tr>
<td>D* Veto with $\pi^0$ Cut</td>
<td>846</td>
<td>2216</td>
<td>408</td>
<td>0.2224</td>
</tr>
<tr>
<td>Kaons Veto</td>
<td>810</td>
<td>1525</td>
<td>330</td>
<td>0.4366</td>
</tr>
</tbody>
</table>

Table 5.5: Total number of $b \rightarrow u\ell\nu$ ($N_{b\rightarrow u}$), $b \rightarrow c\ell\nu$ ($N_{b\rightarrow c}$), other ($N_{other}$) events and signal to noise ratio ($S/B$) at various selection stages in a phase space region where the cut $P_+ < 0.66 \text{ GeV}/c$ is applied on Generic Monte Carlo.
Signal Selection

<table>
<thead>
<tr>
<th>Cut Applied</th>
<th>(N_{b \rightarrow u})</th>
<th>(N_{b \rightarrow c})</th>
<th>(N_{other})</th>
<th>(S/B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semileptonic Cut</td>
<td>3602</td>
<td>143857</td>
<td>12969</td>
<td>0.0230</td>
</tr>
<tr>
<td>Total Event Charge Cut</td>
<td>1965</td>
<td>66313</td>
<td>5455</td>
<td>0.0274</td>
</tr>
<tr>
<td>Miss Neutrino mass square Cut</td>
<td>766</td>
<td>2351</td>
<td>176</td>
<td>0.3031</td>
</tr>
<tr>
<td>D* Veto Cut</td>
<td>699</td>
<td>1544</td>
<td>136</td>
<td>0.4161</td>
</tr>
<tr>
<td>D* Veto with (\pi^0) Cut</td>
<td>631</td>
<td>1068</td>
<td>107</td>
<td>0.5370</td>
</tr>
<tr>
<td>Kaons Veto</td>
<td>608</td>
<td>829</td>
<td>85</td>
<td>0.6652</td>
</tr>
</tbody>
</table>

Table 5.6: Total number of \(b \rightarrow u\ell\nu\) \(N_{b \rightarrow u}\), \(b \rightarrow c\ell\nu\) \(N_{b \rightarrow c}\), other \(N_{other}\) events and signal to noise ratio \(S/B\) at various selection stages in a phase space region where the cut \(m_X < 1.7 \text{ GeV}/c^2\) and \(q^2 < 8 \text{ GeV}^2/c^4\) is applied on Generic Monte Carlo.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figures.png}
  \caption{\(m_X\) efficiency for \(b \rightarrow u\ell\nu\) events (left) and \(b \rightarrow c\ell\nu\) events (right) at various selection stage.}
\end{figure}

5.3 \(m_{ES}\) Fit

In this thesis We estimate the number of signal events using a modified Crystal Ball function peaked at the \(B\) meson mass, inspired by the work from Thorsten Brandt [66]. In our default approach the amount of combinatorial background, originating from \(e^+e^- \rightarrow q\bar{q} (\ q = u, d, s, c\) continuum and \(BB\) events, is described by an Argus function [55] and is subtracted by performing a likelihood fit to the \(m_{ES}\) distribution. An alternative approach was considered, where a peaking component coming
Figure 5.10: $q^2$ efficiency for $b \to u\ell\nu$ events (left) and $b \to c\ell\nu$ events (right) at various selection stage.

from the $B\bar{B}$ background whose $m_{ES}$ distribution has a similar shape as the signal is estimated using a truth–matching algorithm and subtracted. The two approaches give consistent results, but the truth–matching based method was used to perform cross checks and evaluate systematic uncertainties due to the $m_{ES}$ fits.

5.3.1 Signal

The region around the peak of the $m_{ES}$ distribution shows an almost Gaussian shape with a slight tail to the left. Usually such a distribution is modeled by a Crystal Ball function, which is not working in our case. Following the approach from reference [66] we build a three region function, starting from a Crystal Ball function. The right side of the function is described by the sum of the derivates of $\tanh(x)$

$$f_{\text{tanh}}(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

and a Gaussian function:

$$f_{\text{sigR}}(x) = N \frac{r}{\sigma_{R1}} f_{\text{tanh}}\left(\frac{x - x_c}{\sigma_{R1}}\right) + N \frac{1 - r}{\sigma_{R2}} f_{\text{Gauss}}\left(\frac{x - x_c}{\sigma_{R2}}\right)$$

The left side of the function is described by a modified Crystal Ball function, where the Gaussian
part has been substituted by $f_{\text{tanh}}(x)$:

$$f_{\text{sig}}(x) = \begin{cases} 
N_b \exp(\frac{-x}{\sigma_L}) & x \geq x_c - \alpha_{\text{sig}}\sigma_L \\
N_b \exp(-\alpha_{\text{sig}}x_c - x) & x < x_c - \alpha_{\text{sig}}\sigma_L 
\end{cases}$$  \hspace{1cm} (5.8)

with

$$A = -2\frac{\alpha_{\text{sig}}\sigma_L}{1 - \exp(\alpha_{\text{sig}}x_c)}, \quad B = \frac{\exp(\alpha_{\text{sig}}x_c)}{(1 + \exp(\alpha_{\text{sig}}x_c))^2}.$$

Both functions $f_{\text{sigR}}(x)$ and $f_{\text{sigL}}(x)$ are combined into the final signal function:

$$f_{\text{sig}}(x) = N \times \begin{cases} 
C f_{\text{sigL}}(m_{\text{ES}}, x_c, \sigma_L, \alpha_{\text{sig}}, n_{\text{sig}}) & x \leq x_c \\
f_{\text{sigR}}(m_{\text{ES}}, x_c, r, \sigma_{R1}, \sigma_{R2}) & x > x_c 
\end{cases}$$ \hspace{1cm} (5.10)

with

$$C = f_{\text{sigR}}(x_c)/f_{\text{sigL}}(x_c).$$ \hspace{1cm} (5.11)

Figure 5.12-bottom shows the result of fitting the signal function $f_{\text{sig}}(x)$ to the $m_{\text{ES}}$ distribution using only signal candidates. Figure 5.13 shows an example of the result of fitting the signal and combinatorial background to the $m_{\text{ES}}$ distribution for a generic MC sample. Studies showed that, to allow the convergences of the fit, the parameters $\alpha, n, \sigma_{R2}$ and $r$ of the signal function $f_{\text{sig}}(x)$ need to be fixed. The function fits well in all three regions (left tail, peak, right tail). Table 5.7 shows the results of the fit. These values are used as start values for all subsequent fits, and their errors in the evaluation of the systematic uncertainties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_c$ - 5279 MeV</td>
<td>0.70 ± 0.03</td>
</tr>
<tr>
<td>$\sigma_L$ [MeV]</td>
<td>2.12 ± 0.01</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>3.40 ± 0.01</td>
</tr>
<tr>
<td>$n$</td>
<td>2.05 ± 0.02</td>
</tr>
<tr>
<td>$\sigma_{R1}$ [MeV]</td>
<td>0.78 ± 0.03</td>
</tr>
<tr>
<td>$\sigma_{R2}$ [MeV]</td>
<td>3.09 ± 0.01</td>
</tr>
<tr>
<td>$r$</td>
<td>0.40 ± 0.01</td>
</tr>
<tr>
<td>$\chi$</td>
<td>−47.44 ± 0.21</td>
</tr>
<tr>
<td>$x_{\text{max}}$ - 5289 MeV [MeV]</td>
<td>−0.13 ± 0.01</td>
</tr>
</tbody>
</table>

Table 5.7: Results from fitting the $m_{\text{ES}}$ function to $B\bar{B}$ MC. The parameters $\alpha, n, \sigma_{R1}$ are kept fixed in the fit, but their uncertainties are used in the evaluation of the systematic uncertainties due to the fitting technique.
5.3.2 Combinatorial background

In principle, a $B$ meson can be reconstructed in a different decay mode (defined in the semi-exclusive reconstruction algorithm) is actually decaying into. Daughters of the two $B$ mesons can be lost or assigned to the wrong $B$ in the reconstruction. The $m_{ES}$ distribution of these events exhibits a broad peaking component, which is referred to as "peaking background". A study of this effect on simulated $b \rightarrow ul\nu$ events shows that there exists a set of configurations for which not all particles are correctly associated to their parent $B$ mesons, but provide good resolution of the reconstructed kinematic variables $m_X, q^2$ and $P_\tau$ (see a comparison of the $m_X$ resolution achieved by selecting well reconstructed $B$ mesons only, and by including some peaking background in Figure 5.14) Furthermore, rejecting these peaking background events can introduce a bias in the high $m_X$ end of the spectrum, where the larger particle multiplicity makes it more likely for particles to be swapped between the $B$ mesons. In reference [67] are been considered two possible approaches. In the first, a 2-PDF fit has been performed to subtract the combinatoric background that is estimated by the Argus function: in this case part of the peaking background was taken into account by Argus function, the rest was considered as signal. An alternative approach involved a looser definition of well-reconstructed $B_{reco}$ decays, which could be adopted as basis of pseudo-truth-matching algorithm; in such a case, the peaking background component can be estimated and subtracted from the signal. In this thesis we use the first approach to obtain the result of our analyses. The second approach is used to check our code and fitting technique, and as tool for the calculation of systematic uncertainties coming from $m_{ES}$ fits.

<table>
<thead>
<tr>
<th>CODE</th>
<th>$m^{neu}$</th>
<th>$l^{neu}$</th>
<th>$m^{chg}$</th>
<th>$l^{chg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>&lt; 3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>&lt; 3</td>
<td>&lt; 2</td>
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</tr>
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<td>7</td>
<td>&lt; 2</td>
<td>&lt; 2</td>
<td>&lt; 2</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>8</td>
<td>$m^{chg} + m^{neu} &lt; 2$</td>
<td>$l^{chg} + l^{neu} &lt; 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>reconstructed $B_{reco}$ mode equals generated $B_{reco}$ mode</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8: Truth-matching algorithms.
5.3.3 Truth-matching

We define the signal sample based on the shape of the resulting $m_{ES}$ distribution. In this study we use the number of generated $n_{gen}$ and reconstructed $n_{reco}$ daughter particles of the $B_{reco}$ subdivided according to their charge ($n_{gen}^{ch}$, $n_{gen}^{neu}$), and the number of reconstructed daughter particles that are truth-matched, $n_{tm}^{ch}$ and $n_{tm}^{neu}$ according to their charge, i.e. which are coming from the $B_{reco}$ according to simulation information. We consider $m_{ch} = n_{reco}^{ch} - n_{tm}^{ch}$; $m_{neu} = n_{reco}^{neu} - n_{tm}^{neu}$. Different choices for $m_{neu}$, $m_{ch}$ lead to differences in the identification of the signal component. An example is shown in Figure 5.15 and 5.16, where, using generic MC samples, the truth-matched signal and background components are shown for different truth-matching algorithms (see Table 5.8), and the result of the fit to the full $m_{ES}$ distribution in Figure 5.17 is overlaid. One can see that no truth-matching algorithm actually reproduces the fit results, but it is possible to estimate the bias that is introduced and correct for it. We choose $m_{neu} < 3$, $m_{ch} > 3$ and $m_{ch} = 0$ giving the best agreement for the resulting “signal” $m_{ES}$ distribution with the modified Crystal Ball function.

5.3.4 Data

To describe the $m_{ES}$ distribution in data we use the same sum of Argus and modified Crystal Ball functions that we introduced for our MC samples, keeping fixed the same parameters. Figure 5.18 shows the $m_{ES}$ distribution in data, after the semi leptonic selection. The fitted function describes the distribution well over the full range. The result of the fit is shown in Table 5.9 As in the MC case, these values are used as start values for all subsequent fits, and their errors in the evaluation of the systematic uncertainties.

5.3.5 Binned vs. Unbinned Fits

Due to the large number of events that survive the semileptonic selection, it is not very practical to perform every fit as an unbinned maximum likelihood fit. On the other side, the unbinned fit is needed when fitting low statistics samples. These are either single bins in $M_X$, $P_+$ and $(M_X, q^2)$ or small data samples, e.g. the $b \to u \ell \nu$ Monte Carlo sample. Datasets with less than $N_t = 20000$ events are fitted using an unbinned maximum likelihood fit, all other datasets are fitted using a binned one. With this threshold most of the fits are unbinned. The $N_t$ threshold has been varied from 10000 up to 100000 and removed it entirely. No appreciable differences in the fitted yields and the ratio of partial branching ratios have been found.
### 5.4 Data/Monte Carlo Comparison

A good description of the relevant variables by the Monte Carlo simulation is important for this inclusive analysis. Distributions showing data and Monte Carlo agreement for Run1-Run6 are shown in Figures 5.19 to 5.28, according to event type (signal-enriched or -depleted) for the full running period. The distributions for other run periods are very similar.

Since \( b \rightarrow u\bar{u}\bar{d} \) events are basically free of kaons, the total number of kaons (neutral or charged) in the recoil system is a powerful tool to discriminate between signal and background. On the basis of this kaon veto, two data samples are defined:

1. the **signal-enriched** sample (events with \( N_{K^\pm} = N_{K^0} = 0 \)), and

2. the **signal-depleted** sample with \( N_{K^\pm} > 0 \) or \( N_{K^0} > 0 \).

The plots for each variable were produced using data and generic Monte Carlo samples. All the selection cuts were applied, except the one on the plotted variable. All spectra were background-subtracted with the appropriate \( m_{ES} \) sideband distribution after having performed a binned \( m_{ES} \) fit for each bin of the observed variable. Error on data points is the fit error on yields. Data and Monte Carlo were normalized to equal area; each pair of histograms was then tested for compatibility by calculating the \( \chi^2 / \text{d.o.f.} \).

The overall data-Monte Carlo agreement is good; differences between data and Monte Carlo considered as systematic uncertainties are discussed in Chapter 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit</th>
</tr>
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<tbody>
<tr>
<td>( x_c - 5279 \text{ MeV} ) [MeV]</td>
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<tr>
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<tr>
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<tr>
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</tr>
<tr>
<td>( x_{max} - 5289 \text{ MeV} ) [MeV]</td>
<td>−0.13 ± 0.01</td>
</tr>
</tbody>
</table>

Table 5.9: Results from fitting the \( m_{ES} \) function to data. The parameters \( \alpha, n, \sigma_{R1} \) are kept fixed in the fit, but their uncertainties are used in the evaluation of the systematic uncertainties due to the fitting technique.
Figure 5.11: $m_X$ (top), $q^2$ (middle) and $P_+$ (bottom) resolutions for $b \to u\ell\nu$ events with all cuts applied.
Figure 5.12: Top: fit of a Crystal Ball function to the $m_{ES}$ distribution of $B_{reco}$ candidates with correct truth-matching from $B\bar{B}$ Monte Carlo for run period Run1-4. Bottom: fit of Eq. 5.10 to the $m_{ES}$ distribution of the same $B_{reco}$ candidates but for Run1-6 data period. We see that the fit of the 3-wise function has a better agreement with the $m_{ES}$ distribution resulting in better $\chi^2$. The semileptonic selection criteria of Table 5.2 have been applied on the recoiling $B$. 
Figure 5.13: Example of a fit of our 2-pfd model to $m_{ES}$ distribution of $B \rightarrow X_u \ell \nu$ candidates from $B \bar{B}$ MC.
Figure 5.14: Comparison between the $m_X$ resolution obtained by selecting only well-reconstructed $B$ mesons (black points) and by allowing some peaking background contamination. The contamination is defined by the truth-matching algorithms described in Table 5.8, where the points relative to codes 1 to 4 are, respectively, in red, green blue and yellow.
Figure 5.15: Truth-matched distributions for the signal component in Run4 B\bar{B} Monte Carlo, after all analysis cuts have been applied, according to the various truth-matching algorithms described in Table 5.8. The signal PDF, as determined from the $m_{ES}$ fit in Figure 5.17 has been overlaid in each sub-plot.
Figure 5.16: Truth-matched distributions for the background component in Run4 \( B\bar{B} \) Monte Carlo, after all analysis cuts have been applied, according to the various truth-matching algorithms described in Table 5.8. The background PDF, as determined from the \( m_{ES} \) fit in Figure 5.17 has been overlaid in each sub-plot.

Figure 5.17: Fit to \( m_{ES} \) distribution for Run4 \( B\bar{B} \) Monte Carlo, after all analysis cuts have been applied. The two PDF functions for signal and background that have been determined on this plot are overlaid to the truth-matched signal and background events in Figure 5.15 and 5.16.
Figure 5.18: Fit of the $m_{ES}$ function to $m_{ES}$ distribution from data, after the semileptonic selection.
Figure 5.19: Charged track multiplicity (side-band subtracted) in data and generic Monte Carlo for $b \rightarrow u\bar{u}\nu$ enhanced (top row) and depleted (bottom row) event samples.

Figure 5.20: Photon multiplicity (side-band subtracted) in data and generic Monte Carlo for $b \rightarrow u\bar{u}\nu$ enhanced (top row) and depleted (bottom row) event samples.
Figure 5.21: Charged lepton momentum distribution in the CM frame (side-band subtracted) in data and generic Monte Carlo for $b \rightarrow u\ell\bar{\nu}$ enriched (top row) and depleted (bottom row) event samples.

Figure 5.22: Missing mass squared distribution (side-band subtracted) in data and generic Monte Carlo for $b \rightarrow u\ell\bar{\nu}$ enhanced (top row) and depleted (bottom row) event samples.
Figure 5.23: Total charge distribution (side-band subtracted) in data and generic Monte Carlo for $b \to u\bar{u}\nu$ enhanced (top row) and depleted (bottom row) event samples.

Figure 5.24: Sum of charged and neutral kaons (side-band subtracted) in data and generic Monte Carlo for $b \to u\bar{u}\nu$ enhanced (top row) and depleted (bottom row) event samples. Please note that this quantity is used to define the enriched/depleted sample hence the enriched sample shows zeros only.
Figure 5.25: Missing mass squared distribution for the partial $D^{*+}$ reconstruction (side-band subtracted) in data and generic Monte Carlo for $b \to u \bar{v}$ enhanced (top row) and depleted (bottom row) event samples selecting neutral $B$s only.

Figure 5.26: Hadronic recoil invariant mass spectrum (side-band subtracted) in data and generic Monte Carlo for $b \to u \bar{v}$ enhanced (top row) and depleted (bottom row) event samples.
Figure 5.27: $q^2$ distribution (side-band subtracted) in data and generic Monte Carlo for $b \to u\bar{u}\bar{v}$ enhanced (top row) and depleted (bottom row) event samples.

Figure 5.28: $P_+$ distribution in the CM frame (side-band subtracted) in data and generic Monte Carlo for $b \to u\bar{u}\bar{v}$ enhanced (top row) and depleted (bottom row) event samples.
Chapter 6

Partial and Total Charmless Branching Fraction Measurements

In this Chapter are reported the measurements of partial branching fraction for inclusive charmless decays in region of restricted phase space. Also a measurement of the total branching fraction is shown. Then the measurement technique consistency is checked using different fit validation tests.

6.1 Measurement Technique

Inclusive charmless semileptonic branching ratios are determined in phase space regions defined by the $M_X$, $P_t$ and $(M_X, q^2)$ kinematic variables. Rather than perform direct measurements of the $\Delta B(\bar{B} \to X_u \ell \bar{\nu})$, a measurements of the total or partial branching ratios relative to the inclusive semileptonic one $\mathcal{B}(\bar{B} \to X \ell \bar{\nu})$ allows to cancel out relevant contributions to the systematic uncertainties.

To derive total or partial charmless semileptonic branching ratios, the observed number of events, corrected for background and efficiency, is normalized to the total number of semileptonic decays $b \to q \ell \bar{\nu}$ (here $q$ stands for $c$ or $u$) in the $B_{\text{reco}}$ event sample. The measurement of a ratio of branching ratios offers at least three experimental advantages:

1. the efficiency of the Semi-exclusive reconstruction is not needed. This is very important because the Semi-exclusive reconstruction efficiency is affected by large uncertainties due to the fact that many of the reconstructed modes are not well described in Monte Carlo;

2. most of the systematics, due to charged lepton identification, are removed since they are present in both numerator and denominator of the ratio, and they vanish;
3. the normalization to the number of semileptonic events, that is extracted from a fit to the $m_{ES}$
distribution, is less affected by biases, which affect in almost the same way the numerator and
the denominator of this ratio.

The number of observed $B_{\text{recoil}}$ events which survive the semileptonic event selection (Table 5.2) is
denoted as $N_{sl}^{\text{meas}}$. It can be related to the true number of semileptonic decays, $N_{sl}^{\text{true}} = N_{sl}^{\text{true}} + N_{u}^{\text{true}}$, and the remaining background $BG_{sl}$ which is to be subtracted,

$$N_{sl}^{\text{meas}} = \epsilon_{l}^{s} \epsilon_{c}^{c} N_{c}^{\text{true}} + \epsilon_{l}^{u} \epsilon_{l}^{u} N_{u}^{\text{true}} + BG_{sl} = \epsilon_{l}^{s} \epsilon_{l}^{s} N_{sl}^{\text{true}} + BG_{sl},$$ (6.1)

thus

$$N_{sl}^{\text{true}} = \frac{N_{sl}^{\text{meas}} - BG_{sl}}{\epsilon_{l}^{s} \epsilon_{l}^{s}} = \frac{N_{sl}^{\text{meas}} - BG_{sl}}{\epsilon_{l}^{s} \epsilon_{l}^{s}}.$$ (6.2)

Here $\epsilon_{l}^{s}$ refers to the efficiency of the semileptonic selection on a semileptonic $B$ decay in an
event tagged with efficiency $\epsilon_{l}^{l}$. As already discussed, additional selection criteria are imposed to
select $b \rightarrow u\ell\bar{\nu}$ decays. If we denote as $N_{u}^{\text{meas}}$ the number of events fitted in the sample after all
requirements, with $BG_{u}$ the background coming from semileptonic decays other than the signal, the measured
number of events can be expressed in terms of $N_{u}^{\text{true}}$, the true number of signal events, as

$$N_{u} = N_{u}^{\text{meas}} - BG_{u} = \epsilon_{u}^{\text{sel}} \epsilon_{\text{kin}}^{u} \epsilon_{l}^{u} N_{u}^{\text{true}}$$ (6.3)

where $\epsilon_{u}^{\text{sel}}$ is the efficiency for detecting $B \rightarrow X_{u} \ell\bar{\nu}$ decays after applying all cuts but the one on
the kinematic variables, relative to the semileptonic event selection, and $\epsilon_{\text{kin}}^{u}$ is the efficiency for
$B \rightarrow X_{u} \ell\bar{\nu}$ decays when cutting on the kinematic variables $M_{X}$, $P_{+}$ or $(M_{X}, q^{2})$.

To determine $N_{u}$, the background $(BG_{u})$ is subtracted by performing a $\chi^{2}$ fit on the $P_{+}$, $M_{X}$
or $(M_{X}, q^{2})$ distributions, where the background shape is computed from Monte Carlo simulation,
and its normalization is floating. An example of such a $\chi^{2}$ fit is shown in Fig. 6.1. The $P_{+}$, $M_{X}$ or
$(M_{X}, q^{2})$ distributions in data and Monte Carlo are determined by $m_{ES}$ fits in individual $P_{+}$, $M_{X}$ or
$(M_{X}, q^{2})$ bins. For charged $B$ mesons, the charge of the direct lepton from a semileptonic decay is
exactly correlated with the charge of the flavor of the $b$ quark. For neutral $B$ mesons, the effect of
$B^{0} - \bar{B}^{0}$ mixing needs to be taken into account.

If the sample were made only of direct cascade leptons from neutral $B$ decays, the right ($rs$) and
wrong ($ws$) sign events would be related to the direct ($B \rightarrow X\ell\bar{\nu}$) and cascade ($D \rightarrow X\ell\nu$) decays by

$$N_{rs} = (1 - \chi_{d}) N_{B} + \chi_{d} N_{D}$$ (6.4)

$$N_{ws} = \chi_{d} N_{B} + (1 - \chi_{d}) N_{D}$$ (6.5)

where $\chi_{d} = 0.188 \pm 0.003$ [54] is the neutral $B_{d}$ mixing parameter. The contribution from cascade
would be subtracted in an exact way by computing

$$N_{B} = \frac{1 - \chi_{d} N_{rs} - \frac{\chi_{d}}{1 - 2\chi_{d}} N_{ws}}{1 - 2\chi_{d}}.$$ (6.6)
6.1 Measurement Technique

Figure 6.1: $\chi^2$ fit to the $M_X$ distribution. Left: fit result with "other" background (red), $b \rightarrow c\ell \bar{\nu}$ (yellow) and signal (green) shapes superimposed. Middle: same as in the left plot with finer binning. Right: $M_X$ distribution subtracted of the backgrounds (binning as in the middle plot).

Actually there are events which do not contain any leptons and events that contain two $D$ mesons and can therefore have a right sign lepton even if it is not direct. These components are very small and neglected.

In order to have Monte Carlo distributions as much similar as possible to data, the ratio of the charged to neutral $B$ yields in Monte Carlo is reweighted to be the same observed in data.

Monte Carlo and data distribution are divided into 3 flavor subsamples, corresponding to charged $B$ mesons, right-sign and wrong-sign neutral $B$ mesons. The $m_{ES}$ distribution of events surviving the semileptonic selection is fitted for each subsample in order to determine the number of semileptonic $B$ decays and subtract the combinatorial backgrounds. To get convergence, the parameters $\alpha$, $n$, $r$ and $\sigma_{R2}$ of the signal function are fixed in these fits. The mixing correction of Eq. 6.6 is applied to data and Monte Carlo subsamples, charge reweighting is applied to Monte Carlo only.

The distributions of the kinematic variables ($M_X$, $P_\perp$, $(M_X, q^2)$) are divided into bins and for each bin a $m_{ES}$ fit is performed on events surviving the signal selection, again for each of the 3 flavor subsamples. In these fits, all parameters of the signal function are fixed to the values fitted in the semileptonic selection samples; only the shape of the Argus function and the signal and background normalizations are free to float. The number of events in each bin of the kinematic variable under study is obtained after applying the mixing correction and charge reweighting. The remaining background is due to $b \rightarrow c\ell \bar{\nu}$ transitions and to "other" components such cascade decays and non semileptonic decays that survive the selection criteria; this background is subtracted with a $\chi^2$ fit described further on.

The ratio between the partial branching fraction for the signal decays in a given phase space region
and $B \rightarrow X \ell \bar{\nu}$ decays is given by

$$
\Delta R_{u/sl} = \frac{B(\text{signal})}{B(B \rightarrow X \ell \bar{\nu})} = \frac{N_{u/X}^{\text{true}}}{N_{u/X}^{\text{sl}}} = \frac{(N_{u}^{\text{meas}} - BG_{u})/(\varepsilon_{u}^{\text{sl}} \varepsilon_{\text{kin}})}{(N_{u}^{\text{meas}} - BG_{sl})} \times \frac{\varepsilon_{i}^{\ell} \varepsilon_{i}^{\ell}}{\varepsilon_{i}^{\ell} \varepsilon_{i}^{\ell}}.
$$

(6.7)

The efficiency ratio (last term of Eq. 6.7) is expected to be close to, but not equal to, unity. Due to the difference in multiplicity and the different lepton momentum spectra, the tag efficiency $\varepsilon_{i}$ and lepton efficiency $\varepsilon_{i}$ are expected to be different for the two classes of events.

Partial branching fractions for charmless decays in the regions of interest are then obtained from $\Delta R_{u/sl}$ using the semileptonic branching ratio determined by the HFAG group by averaging over several experiments [62]

$$
B(B \rightarrow X \ell \bar{\nu}) = (10.75 \pm 0.16)\%.
$$

(6.8)

In order to extract the partial charmless semileptonic branching ratio $\Delta B(B \rightarrow X \ell \bar{\nu})$ in a given region of the $M_{X}$, $(M_{X}, q^{2})$ or $P_{+}$ distributions, signal events are defined as the ones with true values of the kinematic variables in the chosen region. The $b \rightarrow u \ell \bar{\nu}$ events with true values of the kinematic variables outside the signal region are treated as background. This means that in applying Eq. 6.7, the $b \rightarrow u \ell \bar{\nu}$ events outside the signal region are included in $BG_{u}$ and the quoted efficiencies refer only to events with true values of the kinematic variables in the chosen kinematic region. These efficiencies are computed on Monte Carlo, and therefore are based on the DFN model. The associated theoretical uncertainty on the final result is small compared to the extrapolation error to the full phase space.

The binned kinematic distribution obtained is fitted with a $\chi^2$ minimization which extracts $N_{u}$ and $BG_{u}$ as defined by Eq. 6.7 using signal and background shapes taken from simulation, and by determining their relative normalizations with respect to the experimental distribution. The distribution of the kinematic variable(s) on data ($N_{i}^{u/\text{meas}}$) is fitted to the sum of three distributions, the $b \rightarrow u \ell \bar{\nu}$ events inside the signal region ($N_{i}^{u/\text{in}}$), the $b \rightarrow u \ell \bar{\nu}$ events outside the signal region ($N_{i}^{u/\text{out}}$), and the sum of $b \rightarrow e \ell \bar{\nu}$ and “other” backgrounds ($N_{i}^{\text{bkg}}$)

$$
\mu_{i} = C_{i} N_{i}^{u/\text{in}} + C_{out} N_{i}^{u/\text{out}} + C_{b} N_{i}^{\text{bkg}}.
$$

(6.9)

The $\chi^2$ function is:

$$
\chi^2(C_{i}, C_{\text{out}}, C_{\text{bkg}}) = -\sum_{i} \left( \frac{N_{i}^{u/\text{meas}} - \mu_{i}}{\sqrt{\delta N_{i}^{u/\text{meas}} + \delta N_{i}^{u/\text{MC}}}} \right)^2
$$

(6.10)

where the sum runs over the bins, $N_{i}^{u/\text{meas}}$ is the number of observed events in bin $i$, $\delta N_{i}^{u/\text{meas}}$ and $\delta N_{i}^{u/\text{MC}}$ are the corresponding statistical errors coming from the $m_{ES}$ fits for data and Monte Carlo respectively. $C_{i}$, $C_{\text{out}}$ and $C_{\text{bkg}}$ are the normalizations of the three components which are free parameters of the fit. The constraint $C_{i} = C_{\text{out}}$ is applied. Only the full phase space analysis uses a different $\chi^2$ minimization strategy. In fact in this case the distribution of the kinematic variables on
6.2 One Dimensional $m_X$ Fit and Results

data is fitted to the sum of three distribution, the $b \rightarrow u \ell \nu$ events ($N_{b \rightarrow u}$), the $b \rightarrow c \ell \nu$ events ($N_{b \rightarrow c}$), and the other events ($N_{other}$).

$$\mu_i = C_u N_{i \mu}^{MC} + C_c N_{i c}^{MC} + C_{oth} N_{i \mu}^{MC}. \quad (6.11)$$

None constraint is applied on the normalizations of the tree components ($C_u, C_c, C_{oth}$).

6.2 One Dimensional $m_X$ Fit and Results

The $m_X$ data distribution, obtained by following the procedure outlined in Section 6.1, is fitted by using the $m_{ES}$ fit approach outlined in Section 5.3.1 and $N_u$ and $BG_u$, as defined by Eq. 6.7, are determined.

Fits to the $m_X$ distribution have been performed for phase space region defined by $m_X$ smaller than 1.55 GeV/$c^2$ on data for different run periods, and for the entire sample.

Results are reported in Table 6.1 while in Figure 6.2 is shown the $\chi^2$ fit on the entire dataset (the other configurations are shown in Appendix A).

The value for $\Delta R_{u/sl}$ obtained by using the full Run1-Run6 dataset and $m_X < 1.55$ GeV/$c^2$ is

$$\Delta R_{u/sl}(m_X < 1.55 \text{ GeV}/c^2) = (109 \pm 8 \pm 5) \times 10^{-4} \quad (6.12)$$

where the first error is statistical, the second is systematic.

Using the above result and the measurement of the inclusive semileptonic branching fraction (Eq. 6.8), the partial branching fraction for charmless semileptonic $B$ decays is measured to be

$$\Delta B(\bar{B} \rightarrow X_u \ell \bar{\nu}, m_X < 1.55 \text{ GeV}/c^2) = (1.17 \pm 0.08_{stat} \pm 0.06_{sys}) \times 10^{-3} \quad (6.13)$$

where the first error is statistical and the second is systematic.

Systematic uncertainties will be discussed in Chapter 7.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Run1-Run2</th>
<th>Run3</th>
<th>Run4</th>
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<td>71056 ± 325</td>
<td>40806 ± 759</td>
<td>235997 ± 835</td>
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<tr>
<td>$BG_{sl}$</td>
<td>2720 ± 36</td>
<td>1225 ± 17</td>
<td>3609 ± 26</td>
<td>5319 ± 24</td>
<td>2240 ± 42</td>
<td>14402 ± 51</td>
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<td>$N_{sl} - BG_{sl}$</td>
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<td>52878 ± 372</td>
<td>65736 ± 301</td>
<td>38566 ± 718</td>
<td>221595 ± 784</td>
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<td>$N_{in}^{u}$</td>
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<td>298 ± 13</td>
<td>196 ± 11</td>
<td>1007 ± 23</td>
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<td>$\epsilon_{sel}$</td>
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<td>$\epsilon_{kin}$</td>
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<td>0.891</td>
<td>0.885</td>
<td>0.875</td>
<td>0.889</td>
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<tr>
<td>$(\epsilon_{sel}^u/\epsilon_{kin}^u)$</td>
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<td>1.21 ± 0.05</td>
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<td>1.20 ± 0.04</td>
</tr>
<tr>
<td>$\Delta R_{u/sl}(10^{-4})$</td>
<td>118 ± 16 ± 3</td>
<td>123 ± 27 ± 6</td>
<td>118 ± 15 ± 3</td>
<td>109 ± 14 ± 3</td>
<td>68 ± 20 ± 2</td>
<td>109 ± 8 ± 1</td>
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</tbody>
</table>

Table 6.1: Summary of the fit to the $m_X$ distribution and results for $m_X < 1.55\, \text{GeV}/c^2$ for the full sample and for various subsamples. The fit to all data and MC $m_{ES}$ distributions approach was used. $N_{in}^{u}$ is the number of signal events in the signal region ($m_X < 1.55\, \text{GeV}/c^2$). The first error on $\Delta R_{u/sl}(m_X < 1.55\, \text{GeV}/c^2)$ is statistical, the second is due to MC statistics. The non-resonant component in the Monte-Carlo has been reweighted according to the shape function parameters from the HFAG combination using $\bar{B} \to X_c \ell \bar{\nu}$ and $\bar{B} \to X_s \gamma$ decays as described in Section 4.2.5.
6.3 One Dimensional $P_+$ Fit and Results

The $P_+$ data distribution, obtained by following the procedure outlined in Section 6.1, is fitted by using the $m_{ES}$ fit approaches outlined in Section 5.3.1 and $N_u$ and $BG_u$, as defined by Eq. 6.7, are determined.

Reference [30] suggests that the region defined by $P_+ < 0.66 \text{ GeV}/c$ is optimal for a reliable calculation of the theoretical phase space acceptance. Fits to the $P_+$ distribution have been performed for different run periods, and on the full data sample. Results are shown in Table 6.2 while in Figure 6.3 is shown the $\chi^2$ fit on the entire dataset (the other configurations are shown in Appendix B).

The value for $\Delta R_{u/sl}$ obtained by using the full dataset is

$$\Delta R_{u/sl}(P_+ < 0.66 \text{ GeV}/c) = (99 \pm 7 \pm 6) \times 10^{-4} \quad (6.14)$$

where the first error is statistical, the second is systematic.

Using the result of the fit, the partial branching fraction for charmless semileptonic $B$ decays, in
the phase space region $P_+ < 0.66 \text{ GeV}/c^2$, is measured to be

$$\Delta B(B \to X_u \ell \bar{\nu}, P_+ < 0.66 \text{ GeV}/c^2) = (1.06 \pm 0.07_{\text{stat.}} \pm 0.07_{\text{sys.}}) \times 10^{-3}$$

(6.15)

where the measurement of the inclusive semileptonic branching fraction reported in Eq. 6.8 has been used. The first error is statistical, the second is systematic.

Figure 6.3: One-dimensional $P_+$ analysis: 2-parameter $\chi^2$ fit to the $P_+$ distribution for Run1-Run6 data (fit to all data and MC $m_{ES}$ distributions approach). Left: Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from $b \to ul\bar{\nu}$ events with true $P_+ < 0.66 \text{ GeV}/c$, the rest of the $b \to ul\bar{\nu}$ events, and background events. The signal box is defined by $P_+ < 0.66 \text{ GeV}/c$. Right: $P_+$ distribution subtracted of the backgrounds. $\chi^2$ per degree of freedom = 10.0/10.
<table>
<thead>
<tr>
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<tr>
<td>$BG_{sl}$</td>
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<td>$N_{sl} - BG_{sl}$</td>
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<td>398 ± 16</td>
<td>240 ± 13</td>
<td>1309 ± 29</td>
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<td>0.408</td>
<td>0.415</td>
<td>0.414</td>
<td>0.407</td>
<td>0.414</td>
</tr>
<tr>
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<td>0.917</td>
<td>0.905</td>
<td>0.887</td>
<td>0.908</td>
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<tr>
<td>$(\epsilon_{\text{sel}}^{\text{in}})/(\epsilon_{\text{in}})$</td>
<td>1.10 ± 0.06</td>
<td>1.03 ± 0.08</td>
<td>1.10 ± 0.08</td>
<td>1.10 ± 0.04</td>
<td>1.12 ± 0.06</td>
<td>1.09 ± 0.03</td>
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<tr>
<td>$\Delta R_{u/sl}(10^{-4})$</td>
<td>108 ± 14 ± 3</td>
<td>99 ± 21 ± 4</td>
<td>112 ± 14 ± 2</td>
<td>96 ± 12 ± 2</td>
<td>59 ± 16 ± 2</td>
<td>99 ± 7 ± 1</td>
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</tbody>
</table>

Table 6.2: Summary of the fit to the $P_+$ distribution and results for $P_+ < 0.66$ GeV/c for the full sample and for various subsamples. The fit to all data and MC $m_{ES}$ distributions approach was used. $N_{in}^{\text{in}}$ is the number of signal events in the signal region ($P_+ < 0.66$ GeV/c). The first error on $\Delta R_{u/sl}(P_+ < 0.66$ GeV/c) is statistical, the second is due to MC statistics. The non-resonant component in the Monte-Carlo has been reweighted according to the shape function parameters from the HFAG combination using $B \to X_c \ell \nu$ and $B \to X_s \gamma$ decays as described in Section 4.2.5.
6.4 Two Dimensional \(M_X, q^2\) Fit and Results

The \((M_X, q^2)\) two-dimensional distribution for data, obtained by following the procedure outlined in Section 6.1, is fitted by using the two \(m_{ES}\) fit approaches outlined in Section 5.3.1 and \(N_u\) and \(BG_u\), as defined by Eq. 6.7, are determined.

Previous studies [68] have shown that the phase space region defined by \(M_X < 1.7 \text{ GeV}/c^2\) and \(q^2 > 8 \text{ GeV}^2/c^4\) is optimal for a good measurement of the partial branching ratio.

Fits to the \((M_X, q^2)\) distribution have been performed for different run periods, and on the full data sample. Results are shown in Table 6.3 while in Figure 6.4 is shown the \(\chi^2\) fit on the entire dataset (the other configurations are shown in Appendix C).

In the signal region defined by \(M_X < 1.7 \text{ GeV}/c^2\) and \(q^2 > 8 \text{ GeV}^2/c^4\), the value for \(\Delta R_{u/sl}\) obtained by using the full dataset is

\[
\Delta R_{u/sl}(M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^4) = (67 \pm 5 \pm 4) \times 10^{-4}
\]

where the first error is statistical, the second is systematic.

Using the result of the fit, the partial branching fraction for charmless semileptonic \(B\) decays, for the phase space region defined by \((M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^4)\), is measured to be

\[
\Delta B\ (\bar{B} \rightarrow X_u \ell \bar{\nu}, M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^4) = \\
= (0.72 \pm 0.05_{\text{stat.}} \pm 0.05_{\text{sys.}}) \times 10^{-3}
\]

where the measurement of the inclusive semileptonic branching fraction reported in Eq. 6.8 has been used. The first error is statistical and the second is systematical.
### 6.4 Two Dimensional $(m_X, q^2)$ Fit and Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Run1-Run2</th>
<th>Run3</th>
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<td>38566 ± 718</td>
<td>221594 ± 784</td>
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<td>$N^m_u$</td>
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<td>$N_{bkg}$</td>
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<td>0.937</td>
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<tr>
<td>$(c^u_{sel})/(c^u_{kin})$</td>
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<td>$\Delta R_{u/s}(10^{-4})$</td>
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<td>54 ± 12 ± 2</td>
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Table 6.3: Summary of the fit to the $m_X - q^2$ distributions and results for $m_X < 1.7$ GeV/$c^2$ and $q^2 > 8$ GeV$^2$/c$^4$ for the full sample and for various subsamples. The fit to all data and MC $m_{ES}$ distributions approach was used. $N^m_u$ is the number of signal events in the signal region ($m_X < 1.7$ GeV/$c^2$, $q^2 > 8$ GeV$^2$/c$^4$). The first error on $\Delta R_{u/s}(m_X < 1.7$ GeV/$c^2$, $q^2 > 8$ GeV$^2$/c$^4$) is statistical, the second is due to MC statistics. The non-resonant component in the Monte-Carlo has been reweighted according to the shape function parameters from the HFAG combination using $B \to X_c \ell \nu$ and $B \to X s \gamma$ decays as described in Section 4.2.5.
Figure 6.4: Two-dimensional $m_X - q^2$ analysis: 2-parameter $\chi^2$ fit to the $m_X - q^2$ distribution for Run1-Run6 data (fit to all data and MC $m_{ES}$ distributions approach). The plots represent $q^2$ projections in 4 $m_X$ bins. Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from $b \to u \ell \nu$ events with true $m_X < 1.7 \text{ GeV}/c^2$, $q^2 > 8 \text{ GeV}^2/c^4$, the rest of the $b \to u \ell \nu$ events, and background events. The signal box is defined by $m_X < 1.7 \text{ GeV}/c^2$, $q^2 > 8 \text{ GeV}^2/c^4$. $\chi^2$ per degree of freedom = 36.7/28.
6.5 Two-dimensional $m_X - q^2$ fits: cut on high $q^2$

6.5.1 Introduction

One of the effects that is not included in the current theoretical calculations of the partial decay rate, is weak annihilation (WA) [19], which is expected to contribute at the level of a few percent [30, 34]. Simply speaking, WA refers to the annihilation of the $b - \bar{\pi}$ pair to a virtual $W$ boson, and results in an enhancement of the decay rate near the endpoint of the $q^2$ spectrum. This means that WA is included in every cut that include the $q^2$ endpoint. It has been proposed [30, 34], that a cut $q^2 < q^2_{\text{max}}$, together with a cut on $m_X$ or $P_\pi$, would mitigate the uncertainty due to the WA effect resulting in a lower total theoretical uncertainty. From the experimental side, cutting away the region with high $q^2$, that is the region with favorable signal/noise ratio, results in a reduced efficiency and a reduced sensitivity due to the larger background subtraction. In the following we show the results of a fit in the combined variables $m_X - q^2$, with different values for $q^2_{\text{max}}$ applied together with the cut $m_X < 1.55$ GeV.

6.5.2 Results

In order to extract the partial partial branching ratio in a given region of the $m_X - q^2_{\text{cut}}$ distribution, we follow the same procedure used for the standard two-dimensional $m_X - q^2$ fit. We determine the partial branching ratio imposing $m_X < 1.55$ GeV/c$^2$ together with the cut $q^2 < q^2_{\text{max}}$, where $q^2_{\text{max}} = 8$ GeV$^2$/c$^4$, 10 GeV$^2$/c$^4$, 12 GeV$^2$/c$^4$ and 14 GeV$^2$/c$^4$. Results are shown in Table 6.4 and Figure 6.5 shows the fit result for the phase space region with $q^2 < 12$ GeV$^2$/c$^4$. The other plots are shown in Appendix D.

Using the results of the 2-parameter fit the partial branching ratio for the charmless semileptonic B decays are:

$$
\Delta B(B \to X_u \ell \nu)(q^2 < 8 \text{ GeV}^2/c^4) = (0.51 \pm 0.06_{\text{stat.}} \pm 0.03_{\text{sys.}}) \times 10^{-3} \quad (6.18)
$$

$$
\Delta B(B \to X_u \ell \nu)(q^2 < 10 \text{ GeV}^2/c^4) = (0.69 \pm 0.07_{\text{stat.}} \pm 0.04_{\text{sys.}}) \times 10^{-3} \quad (6.19)
$$

$$
\Delta B(B \to X_u \ell \nu)(q^2 < 12 \text{ GeV}^2/c^4) = (0.78 \pm 0.07_{\text{stat.}} \pm 0.05_{\text{sys.}}) \times 10^{-3} \quad (6.20)
$$

$$
\Delta B(B \to X_u \ell \nu)(q^2 < 14 \text{ GeV}^2/c^4) = (0.96 \pm 0.08_{\text{stat.}} \pm 0.06_{\text{sys.}}) \times 10^{-3} \quad (6.21)
$$

(6.22)

where the measurement of the inclusive semileptonic branching fraction reported in Eq. 6.8 has been used. The first error is statistical and the second is systematical. In figure 6.6 are shown the different measurements as function of the $q^2$ cut.
### Table 6.4: Summary of the fit to the $m_X, q^2$ distributions and results for $m_X < 1.55$ GeV/$c^2$ and $q^2 < 8, 10, 12, 14$ GeV/$c^2$ for the full sample. The fit to all data and MC $m_{ES}$ distributions approach was used. $N_{in}$ is the number of signal events in the signal region. The first error on $\Delta R_{u/sl}$ is statistical, the second is due to MC statistics. The non-resonant component in the Monte-Carlo has been reweighted according to the shape function parameters from the HFAG combination using $B \to X_s \ell \nu$ and $B \to X_s \gamma$ decays as described in Section 4.2.5.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$q^2 &lt; 8$ GeV/$c^2$</th>
<th>$q^2 &lt; 10$ GeV/$c^2$</th>
<th>$q^2 &lt; 12$ GeV/$c^2$</th>
<th>$q^2 &lt; 14$ GeV/$c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sl}$</td>
<td>$235997 \pm 835$</td>
<td>$235997 \pm 835$</td>
<td>$235997 \pm 835$</td>
<td>$235997 \pm 835$</td>
</tr>
<tr>
<td>$BG_{sl}$</td>
<td>$14403 \pm 51$</td>
<td>$14404 \pm 51$</td>
<td>$14404 \pm 51$</td>
<td>$14404 \pm 51$</td>
</tr>
<tr>
<td>$N_{sl} - BG_{sl}$</td>
<td>$221594 \pm 784$</td>
<td>$221593 \pm 784$</td>
<td>$221593 \pm 784$</td>
<td>$221593 \pm 784$</td>
</tr>
<tr>
<td>$N_{in}$</td>
<td>$343 \pm 43$</td>
<td>$503 \pm 51$</td>
<td>$607 \pm 58$</td>
<td>$775 \pm 63$</td>
</tr>
<tr>
<td>$N_{out}$</td>
<td>$36 \pm 3$</td>
<td>$36 \pm 3$</td>
<td>$52 \pm 5$</td>
<td>$49 \pm 6$</td>
</tr>
<tr>
<td>$N_{bg}$</td>
<td>$429 \pm 10$</td>
<td>$561 \pm 13$</td>
<td>$703 \pm 16$</td>
<td>$835 \pm 20$</td>
</tr>
<tr>
<td>$\epsilon_{sel}^u$</td>
<td>$0.390$</td>
<td>$0.392$</td>
<td>$0.397$</td>
<td>$0.402$</td>
</tr>
<tr>
<td>$\epsilon_{in}^u$</td>
<td>$0.747$</td>
<td>$0.779$</td>
<td>$0.804$</td>
<td>$0.823$</td>
</tr>
<tr>
<td>$(\epsilon_{sel}^u/\epsilon_{in}^u)$</td>
<td>$1.12 \pm 0.04$</td>
<td>$1.17 \pm 0.03$</td>
<td>$1.18 \pm 0.04$</td>
<td>$1.19 \pm 0.04$</td>
</tr>
<tr>
<td>$\Delta R_{u/sl}(10^{-4})$</td>
<td>$47 \pm 6 \pm 1$</td>
<td>$64 \pm 6 \pm 1$</td>
<td>$73 \pm 7 \pm 1$</td>
<td>$89 \pm 7 \pm 1$</td>
</tr>
</tbody>
</table>

6.6 Two Dimensional $(m_X, q^2)$ Fit without kinematic cuts

We have extracted the full charmless semileptonic branching ratio by adapting the analysis investigating the phase space in $m_X - q^2$ described in section 6.4.

As in the $m_X - q^2$ analysis, a binned distribution in the $m_X - q^2$ variables is obtained by taking into account the combinatorial background bin-by-bin with the approach described in Section 6.1. This $m_X - q^2$ distribution is then fitted with a $\chi^2$ minimization which extracts $N_u$ and $BG_u$ as defined by Eq. 6.7 using signal and background shapes taken from simulation, and by determining their relative normalizations with respect to the experimental distribution, as described in section 6.1.

We have performed 2-parameter fits to the $m_X - q^2$ distribution on data for Run1 to Run6. Results are shown in Table 6.5 and in Figure 6.7.

The value for $R_{u/sl}$ obtained with a 2-parameter fit by using the full Run1-Run6 dataset is

$$R_{u/sl} = (208 \pm 15 \pm 21) \times 10^{-4}$$ (6.23)

where the first error is statistical, the second is systematic. Systematic uncertainties will be discussed in next chapter.

Using the result of the 2-parameter fit the branching fraction for charmless semileptonic $B$ decays
6.6 Two Dimensional \((m_X,q^2)\) Fit without kinematic cuts

![Graphs showing two-dimensional \((m_X,q^2)\) analysis](image)

Figure 6.5: Two-dimensional \(m_X - q^2\) analysis: 2-parameter \(\chi^2\) fit to the \(m_X - q^2\) distribution for Run1-Run6 data (fit to all data and MC \(m_{ES}\) distributions approach). The plots represent \(q^2\) projections in 4 \(m_X\) bins. Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from \(b \to u\ell\nu\) events with true \(m_X < 1.55 \text{ GeV}^2/c^4\), \(q^2 < 12 \text{ GeV}^2/c^4\), the rest of the \(b \to u\ell\nu\) events, and background events. The signal box is defined by \(m_X < 1.55 \text{ GeV}/c^2\), \(q^2 < 12 \text{ GeV}^2/c^4\). \(\chi^2\) per degree of freedom 45.0/28.

is

\[
B(B \to X_u\ell\nu) = (2.23 \pm 0.16_{\text{stat.}} \pm 0.16_{\text{sys.}}) \times 10^{-3} \quad (6.24)
\]

where we have used the measurement of the inclusive semileptonic branching fraction reported in Eq. 6.8. The first error is statistical and the second is systematical.
Figure 6.6: Partial branching fractions of \( B \rightarrow X_u \ell \nu \) as function of the \( q^2 \) cuts \((8,10,12,14 \text{ GeV}^2/\text{c}^4)\) in a phase space region with \( m_X < 1.55 \text{ GeV}/\text{c}^2 \). These measurements are obtained using a two dimensional fit on \((m_X - q^2)\) as described in section 6.5. Is also reported the one dimensional fit on \( m_X \) described in section 6.2. In the figure the first error is statistic and the second error is systematic.

6.7 Fit Validation

The measurement technique described in this chapter has been checked using different tests reported in this section.

6.7.1 MC vs. MC Fits

We performed a Monte Carlo versus Monte Carlo analysis to verify that the fit is able to reproduce the \( B \rightarrow X_u \ell \nu \) total and partial branching ratio for each kinematic cut. We split the generic \( B \bar{B} \) and our Monte Carlo into two sets. We treat the first set \((1/3 \text{ of the total sample})\) as data, the second as Monte Carlo events. We apply all the usual weights \((B \rightarrow D^*, \text{ signal Hybrids etc.})\) to the Monte Carlo that we consider as data. Subsequently, all the steps of the analysis are performed as on real data. The result for the various kinematic cuts are reported in Table 6.6 (for each analysis is reported
Figure 6.7: Two-dimensional $m_X - q^2$ analysis: 2-parameter $\chi^2$ fit to the $m_X - q^2$ distribution for Run1-Run6 data (fit to all data and MC $m_{ES}$ distributions approach). The plots represent $q^2$ projections in 4 $m_X$ bins. Points are data, the blue, yellow and gray histograms represent respectively the fitted contributions from $b \rightarrow u\ell\nu$ events, charmed background and other background events. $\chi^2$ per degree of freedom = 23.2/24.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Run1-Run6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sl}$</td>
<td>$23597 \pm 835$</td>
</tr>
<tr>
<td>$BG_{sl}$</td>
<td>$8687 \pm 31$</td>
</tr>
<tr>
<td>$N_{sl} - BG_{sl}$</td>
<td>$227310 \pm 804$</td>
</tr>
<tr>
<td>$N_{b\rightarrow u}$</td>
<td>$1614 \pm 119$</td>
</tr>
<tr>
<td>$N_{\text{other}}$</td>
<td>$223 \pm 137$</td>
</tr>
<tr>
<td>$N_{b\rightarrow c}$</td>
<td>$7494 \pm 196$</td>
</tr>
<tr>
<td>$\epsilon_{u}^{sel}$</td>
<td>$0.342$</td>
</tr>
<tr>
<td>$\epsilon_{c}^{sel}$</td>
<td>$1.000$</td>
</tr>
<tr>
<td>$(\epsilon_{u}^{sel})/\epsilon_{c}^{sel}$</td>
<td>$0.9981 \pm 0.0118$</td>
</tr>
<tr>
<td>$R_{u/sl}(10^{-4})$</td>
<td>$208 \pm 15 \pm 3$</td>
</tr>
</tbody>
</table>

Table 6.5: Summary of the fit to the $m_{X} - q^{2}$ distributions and results obtained without applying kinematic cuts. The fit to all data and MC $m_{ES}$ distributions approach was used. $N_{u}$ is the number of signal events. The first error on $R_{u/sl}$ is statistical, the second is due to MC statistics. The non-resonant component in the Monte-Carlo has been reweighted according to the shape function parameters from the HFAG combination using $B \rightarrow X \ell \nu$ and $B \rightarrow X \gamma$ decays as described in Section 4.2.5.

the total branching fraction $B(B \rightarrow X_u \ell \nu)$ extrapolated considering the particular region explored and in figures 6.8, 6.9.6.10.6.11.

<table>
<thead>
<tr>
<th>Kinematic cut</th>
<th>$B(B \rightarrow X_u \ell \nu) \times 10^{-3}$</th>
<th>SP8 expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{X} &lt; 1.55$</td>
<td>$2.28 \pm 0.21$</td>
<td>$2.33$</td>
</tr>
<tr>
<td>$P_{+} &lt; 0.66$</td>
<td>$2.21 \pm 0.17$</td>
<td>$2.33$</td>
</tr>
<tr>
<td>$m_{x} &lt; 1.7$ and $q^{2} &gt; 8$</td>
<td>$2.28 \pm 0.22$</td>
<td>$2.33$</td>
</tr>
<tr>
<td>$m_{x} - q^{2}$ no kinematic cuts</td>
<td>$2.38 \pm 0.17$</td>
<td>$2.33$</td>
</tr>
</tbody>
</table>

Table 6.6: Summary of the results analysing $1/3$ of the $BB$ generic MC like real data. The results of the partial branching ratio are compared with the expectation in SP8 MC. The results are in good agreement.
6.7 Fit Validation

**Figure 6.8:** One-dimensional $m_X < 1.55$ GeV/c$^2$ analysis: 2-parameter $\chi^2$ fit to the $m_X$ distribution on MC. Left: Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from $b \to u\nu$ events with true $m_X < 1.55$ GeV/c$^2$, the rest of the $b \to u\nu$ events, and background events. The signal box is defined by $m_X < 1.55$ GeV/c$^2$. Right: $m_X$ distribution subtracted of the backgrounds.

**Figure 6.9:** One-dimensional $P_+ < 0.66$ GeV/c analysis: 2-parameter $\chi^2$ fit to the $m_X$ distribution on MC. Left: Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from $b \to u\nu$ events with true $P_+ < 0.66$ GeV/c, the rest of the $b \to u\nu$ events, and background events. The signal box is defined by $P_+ < 0.66$ GeV/c. Right: $P_+$ distribution subtracted of the backgrounds.

### 6.7.2 Toy MC

In order to check that the statistical error is properly estimated in the $\chi^2$ fit, a Toy MC procedure has been setup:

- each bin of the kinematic variable analyzed is randomized in a Poisson way around the original value in the bin
- for each bin the part of error due to $m_{ES}$ fit procedure is considered constant and fixed to its
Figure 6.10: Two-dimensional \( m_X - q^2 \) analysis: 2-parameter \( \chi^2 \) fit to the \( m_X, q^2 \) distribution for MC. The plots represent \( q^2 \) projections in 4 \( m_X \) bins. Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from \( b \to u\bar{c} \nu \) events with true \( m_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^4 \), the rest of the \( b \to u\bar{c} \nu \) events, and background events. The signal box is defined by \( m_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^4 \).

- the whole fitting procedure is iterated

This procedure does not tell anything about analysis biases introduced before the selection. Using the Toy MC procedure we have studied the statistic error associated to the MC components of the following analysis:

- \( m_X < 1.55 \text{ GeV}/c^2 \)
- \( P_+ < 0.66 \text{ GeV}/c \)
- \( m_X < 1.7 \text{ GeV}/c^2 \cdot q^2 > 8 \text{ GeV}^2/c^4 \)
Figure 6.11: Two-dimensional $m_X - q^2$ analysis: 2-parameter $\chi^2$ fit to the $m_X, q^2$ distribution for MC obtained without applying kinematic cuts. Points are data, the blue, yellow and gray histograms represent respectively the fitted contributions from $b \rightarrow u\ell\nu$ events, charmed background and other background events.

- $m_X - q^2$ without kinematic cuts

Figures 6.12, 6.13, 6.14 and 6.15 show the so-called pull distributions, that are computed as:

$$pull = \frac{(fitted\ value)-(true\ value)}{error\ of\ the\ fitted\ value}$$

and then fitted with a single Gaussian function. The results are reported in table 6.7.

Toy study results show that the errors from the fits are correct at few percent level. The analysis without cuts shows that the errors returned are slightly overestimated by 7%.
### Table 6.7

<table>
<thead>
<tr>
<th>Analysis</th>
<th>$\sigma^{IN}_{b\rightarrow u}$</th>
<th>$\sigma^{OUT}_{b\rightarrow u}$</th>
<th>$\sigma_{b\rightarrow c}$</th>
<th>$\sigma_{other}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_X &lt; 1.55 \text{ GeV/c}^2$</td>
<td>0.98 ± 0.03</td>
<td>0.99 ± 0.03</td>
<td>0.98 ± 0.03</td>
<td>0.94 ± 0.03</td>
</tr>
<tr>
<td>$P_+ &lt; 0.66 \text{ GeV/c}$</td>
<td>0.95 ± 0.02</td>
<td>0.95 ± 0.02</td>
<td>1.05 ± 0.003</td>
<td>1.01 ± 0.003</td>
</tr>
<tr>
<td>$m_X - q^2$ no kinematic cuts</td>
<td>0.93 ± 0.02</td>
<td>0.94 ± 0.02</td>
<td>0.90 ± 0.02</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 6.12**: Toy MC results for the one-dimensional $m_X < 1.55 \text{ GeV/c}^2$ analysis: The plots in the top row represent the distribution of the number of $b \rightarrow u\nu$ events with true $m_X < 1.55 \text{ GeV/c}^2$, background events and the rest of the $b \rightarrow u\nu$ events respectively. In the bottom row are shown the respective pull.

### 6.7.3 Fit to the signal depleted sample

A useful cross check can be performed on the $b \rightarrow u$ depleted sample. In fact in principle if the data and MC are in good agreement the background normalization factor obtained by $\chi^2$ minimization procedure should be independent from the particular cuts applied in order to reduce the $b \rightarrow c$ decays. A background control sample with quite low S/B ratio has been produced requiring at least one neutral or charged kaon or alternately an event with missing neutrino mass square from $D^*$ partial reconstruction consistent with 0 GeV$^2$/c$^4$. In table 6.8 are reported the background normalization factors obtained fitting our analyses with the signal depleted sample and with the default one. The plot 6.16 show a good Data/MC agreement and this is a check that the resolution and the background shape are well modelled.
6.7 Fit Validation

Figure 6.13: Toy MC results for the one-dimensional $P_+ < 0.66$ GeV/c analysis: The plots on the top row represent the distribution of the number of $b \to u\ell\nu$ events with true $P_+ < 0.66$ GeV/c, background events and the rest of the $b \to u\ell\nu$ events respectively. In the bottom row are shown the respective pull.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Depleted sample</th>
<th>Default sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_X &lt; 1.55$ GeV/c$^2$</td>
<td>$0.312 \pm 0.003$</td>
<td>$0.331 \pm 0.008$</td>
</tr>
<tr>
<td>$P_+ &lt; 0.66$ GeV/c</td>
<td>$0.311 \pm 0.007$</td>
<td>$0.333 \pm 0.007$</td>
</tr>
<tr>
<td>$m_X - q^2$</td>
<td>$0.309 \pm 0.003$</td>
<td>$0.330 \pm 0.008$</td>
</tr>
<tr>
<td>$m_X - q^2$ no kinematic cuts</td>
<td>$0.239 \pm 0.002$</td>
<td>$0.251 \pm 0.007$</td>
</tr>
</tbody>
</table>

Table 6.8: Background normalization factors obtained fitting depleted and default samples in different configurations.

6.7.4 Electron and Muon Fits

In principle the partial branching ratio $\Delta B(B \to Xu\ell\nu)$ in a particular phase space region is independent from the kind of lepton identified in the semileptonic decay. For that reason performing our analyses for electron and muon separately can be a good Cross check. In tables 6.9 are reported the different partial branching ratios for the different analyses used in this thesis.
Figure 6.14: Toy MC results for the two-dimensional $m_X - q^2$ analysis: The plots on the top row represent the distribution of the number of $b \rightarrow u\ell\nu$ events with true $m_X < 1.7$ GeV/$c^2$, $q^2 > 8$ GeV$^2/c^2$ background events and the rest of the $b \rightarrow u\ell\nu$ events respectively. In the bottom row are shown the respective pull.

Figure 6.15: Toy MC results for the two-dimensional $m_X - q^2$ without kinematic cuts analysis: The plots on the top row represent the distribution of the number of $b \rightarrow u\ell\nu$ events, $b \rightarrow c\ell\nu$ events and other events respectively. In the bottom row are shown the respective pull.
Figure 6.16: One-dimensional $m_X < 1.55$ GeV/c$^2$ analysis using a $b \rightarrow ul\nu$ depleted sample: 2-parameter $\chi^2$ fit to the $m_X$ distribution for Run1-Run6 data (fit to all data and MC $m_{ES}$ distributions approach). Left: Points are data, the blue, magenta and yellow histograms represent respectively the fitted contributions from $b \rightarrow ul\nu$ events with true $m_X < 1.55$ GeV/c$^2$, the rest of the $b \rightarrow ul\nu$ events, and background events. The signal box is defined by $m_X < 1.55$ GeV/c$^2$. Right: $m_X$ distribution subtracted of the backgrounds.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>$\Delta R_{u/sl}^{\pi\mu} (10^{-4})$</th>
<th>$\Delta R_{u/sl}^{e} (10^{-4})$</th>
<th>$\Delta R_{u/sl}^{\mu} (10^{-4})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_X &lt; 1.55$ GeV/c$^2$</td>
<td>109 ± 8 ± 1</td>
<td>104 ± 9 ± 2</td>
<td>112 ± 12 ± 2</td>
</tr>
<tr>
<td>$P_+ &lt; 0.66$ GeV/c</td>
<td>99 ± 7 ± 1</td>
<td>100 ± 8 ± 2</td>
<td>98 ± 11 ± 2</td>
</tr>
<tr>
<td>$m_X - q^2$</td>
<td>67 ± 5 ± 1</td>
<td>62 ± 6 ± 1</td>
<td>74 ± 8 ± 4</td>
</tr>
<tr>
<td>$m_X - q^2$ no kinematic cuts</td>
<td>208 ± 15 ± 3</td>
<td>201 ± 18 ± 3</td>
<td>215 ± 24 ± 3</td>
</tr>
</tbody>
</table>

Table 6.9: Summary of the fit to the $m_X$, $P_+$, $m_X - q^2$ and $m_X - q^2$ without kinematical cuts distributions and results for electrons and muons separately.
Chapter 7

Systematic Uncertainties

Theoretical uncertainties appearing when extrapolating measurements of partial branching fraction for charmless semileptonic decays to the full phase space are taken from theory papers, and shown in the next chapter. The other systematic uncertainties are extremely similar to the one described in previous BABAR inclusive semileptonic branching fraction measurements [69, 70, 71, 37]. Since a ratio of branching fractions is measured, most of them cancel. The sources of systematic uncertainties on the (partial) ratio of branching fraction are summarized in tables 7.5 and 7.6 and can be grouped in various categories:

- Detector related effects
- $m_{\text{GS}}$ fits
- signal knowledge
- background knowledge

7.1 Detector-related Effects

7.1.1 Charged Particle Tracking

Any difference between data and Monte Carlo simulation can potentially lead to a distortion in the distribution of the kinematical variables under study, as well as in the efficiency calculations.

The tracking efficiencies are well reproduced by the Monte Carlo simulations and, as shown in Figure 5.19, the number of charged tracks per events is in good agreement between data and Monte Carlo. A similar agreement has been obtained on other control samples. For high momentum tracks, $e^+e^- \rightarrow \tau^+\tau^-$ events, where one $\tau$ decaying leptonically and the other to three charged hadrons (plus
an arbitrary number of neutrals), are used. They are a good control sample for this purpose because the $e^+e^- \to \tau^+\tau^-$ cross section is 0.94 nb and the branching fraction to $\ell + 3$ hadrons is 11\% so this sample allows high statistics tests. Moreover, the momentum distribution of tracks from $\tau$ decays is similar to the one from $B$ decays. Data and Monte Carlo efficiencies are in good agreement within the statistical errors. To assign a systematic uncertainty on the charged particle tracking, a common prescription within BABAR measurements has been followed and no correction has been applied to Monte Carlo tracks, but a systematic uncertainty per track has been assigned depending on the run period. The Monte Carlo has been reweighted by randomly eliminating tracks with probabilities detailed in Table 7.1, and the difference observed with respect to the default measurements is taken as the systematic uncertainty.

<table>
<thead>
<tr>
<th>Run period</th>
<th>syst. uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>0.67</td>
</tr>
<tr>
<td>Run 2</td>
<td>0.35</td>
</tr>
<tr>
<td>Run 3</td>
<td>0.45</td>
</tr>
<tr>
<td>Run 4</td>
<td>0.66</td>
</tr>
<tr>
<td>Run 5</td>
<td>0.67</td>
</tr>
<tr>
<td>Run 6</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 7.1: Systematic uncertainty per track used in the calculation of tracking systematic uncertainties, divided by run period.

### 7.1.2 Neutral Reconstruction

Differences between data and Monte Carlo simulation in the photon detection efficiency and resolution, as well as additional energy depositions in the EMC, can impact the distributions of the kinematic variables used in this analysis.

Two different control samples are used to check for disagreements between data and Monte Carlo simulation in efficiency and energy resolution. The study is performed using the $\tau$ hadronic decays that represent an abundant source of neutral pions. The $\tau \to e\nu\bar{\nu}$ decay is identified in $e^+e^- \to \tau^+\tau^-$ events. The ratio $R = N(\tau \to h^+\pi^0\nu_\tau)/N(\tau \to h^+\pi^0\pi^0\nu_\tau)$ is computed both for data and Monte Carlo as a function of the $\pi^0$ energy in order to evaluate possible differences in efficiency. The agreement has been found to be good and the ratio is compatible with the unity in the full range. A systematic uncertainty of 1.8\% per photon is assigned, due to uncertainties in the hadronic interactions in the EMC, to the photon background being not perfectly modeled in the Monte Carlo, and to the uncertainty in the $\tau$ branching fractions in $\pi\nu_\tau$ and $\rho\nu_\tau$ final states. The corresponding systematic uncertainty is about 0.1\% for all analyses.
7.1 Detector-related Effects

The resolution has been studied taking $\pi^0$'s from both $\tau \rightarrow h^\pm \pi^0 \nu_\tau$ and $\tau \rightarrow h^\pm \pi^0 \pi^0 \nu_\tau$ decays. The $\pi^0$ mass is fitted in energy bins and the resolution (corresponding to the $\sigma$ of a Gaussian fit) is then compared between data and Monte Carlo. The Monte Carlo resolution is changed by applying a smearing factor such to be identical to data. Similar corrections are applied on Monte Carlo to take into account differences in the energy scale and effects due to energy deposits close to crystal boundaries and to the edges between the barrel and the endcap of the EMC. These factors are determined as well with control samples such as $\mu\mu\gamma$ and $B \rightarrow K^*(K^+\pi^-)\gamma$ decays. All corrections turn out to be small. Same control sample has been used to evaluate the reconstruction of $\pi^0$ down to 100 MeV. The systematic uncertainty due to the reconstruction of neutral particles has been obtained by repeating the analysis without applying the corrections and taking the difference with respect to the default measurements.

7.1.3 $K_L$ Reconstruction

Systematic uncertainties in the simulation of $K_L$ interactions have been estimated according to the results shown in [72]. Several corrections are applied on the Monte Carlo in order to reproduce data. The energy deposition of calorimeter clusters truth-matched to a $K_L$ are corrected by ad-hoc factors. The $K_L$ detection efficiency is corrected by rejecting neutral clusters truth-matched to a $K_L$ with a probability, which is a function of the true $K_L$ momentum.

A correction due to the differences between data and simulation for the $K_L$ production rate is also applied, based on studies detailed in [73]. Given that such a correction can not be accomplished by eliminating neutral clusters, a different approach, also described in [74], has been employed. Clusters truth-matched to $K_L$'s are randomly transformed into "pseudo-photons" and in this way the energy and momentum balance in the event are restored. This is achieved by rescaling the measured energy and momentum of the $K_L$ cluster to the true $K_L$ momentum, assuming zero mass. The probability of the correction depends on the $K_L$ momentum: 22% for momenta between 0 and 0.4 GeV/c, 1% for momenta between 0.4 and 1.4 GeV/c, 9% for momenta larger than 1.4 GeV/c.

The systematic uncertainty due to $K_L$ reconstruction has been determined by repeating the measurements without applying the above corrections in the Monte Carlo, and taking the difference with respect to the default measurements.

7.1.4 Lepton Identification

The systematic uncertainties related to lepton identification efficiencies and misidentification probabilities are derived from control samples. For electron efficiency, radiative Bhabha events are used. Muons with a momentum spectrum covering the range of interest are extracted from the $e^+e^- \rightarrow \mu^+\mu^-\gamma$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ channels. The pions misidentification probabilities are eval-
uated using samples of \( K_S^0 \to \pi^+\pi^- \) and three-prong \( \tau \) decays. Kaons misidentification probabilities are obtained by using samples selecting \( D^{*+} \to D^0\pi^+, \ D^0 \to K\pi \) decays, where only kinematic information is used to identify the kaon.

The statistical and systematic errors from the data-Monte Carlo comparison in bins of momentum and polar angle are used to compute the systematic uncertainties due to particle identification. Each bin is shifted by \( \pm 2\% \) for efficiency and by \( \pm 15\% \) for misidentification and the analysis is repeated. The difference in the results is taken as the systematic uncertainty. The effect of the time dependence (especially in muon identification) has been investigated by using efficiency corrections depending on run periods.

### 7.1.5 Charged Kaon Identification

The systematic uncertainty associated with kaon identification efficiency and misidentification probabilities (shown in Figure 3.5) are obtained with the same technique used for lepton identification. Kaon and pion samples are selected from the \( D^{*+} \to D^0\pi^+, \ D^0 \to K\pi \) decay chain. Each bin (as a function of the momentum and polar angle) is shifted by \( \pm 2\% \) for efficiency and by \( \pm 15\% \) for misidentification.

### 7.2 Uncertainties Related to the \( m_{ES} \) Fits

In the fits to the \( m_{ES} \) distributions, some parameters of the signal function (\( a, n, \sigma, r \)) are kept fixed to values determined from high-statistics data sample or simulation. The systematic uncertainty due to the choice of these parameters has been determined by varying their values within the statistical errors. The small bias introduced in the \( m_{ES} \) fitting approach, where the peaking background is not taken into account, is corrected for and an error of 100\% on this bias is assumed as systematic uncertainty.

### 7.3 Signal Knowledge

Our limited knowledge of the shape function parameters determined from other measurements affects the experimental efficiencies (\( \epsilon_{sel}, \epsilon_{kin} \)). The corresponding uncertainty on the measurement of partial branching fractions is expected to be small. This uncertainty is estimated by varying the heavy quark parameter along the ellipse in the \( (m_b, \mu^2) \) plane corresponding to \( \Delta \chi^2 = 1 \) (Figure 7.1 and Table 7.2) and taking the maximum positive and negative fit deviations from the central point.

Systematic effects due to the form of the shape function have been studied by using alternative parametrization, such as a Gaussian (Eq. 7.1) and a Roman form (Eq.7.2), shown in Figure 7.2, both
7.3 Signal Knowledge

Figure 7.1: $\Delta \chi^2 = 1$ contour plot in the $(m_b, \mu_\pi^2)$ plane in the Kagan-Newbert Scheme [10]. The points around the ellipse are the values used in the evaluation of systematic uncertainties (see also Tab. 7.2).

<table>
<thead>
<tr>
<th>Point</th>
<th>$m_b$ (GeV/$c^2$)</th>
<th>$\mu_\pi^2$ (GeV²/$c^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>4.6586</td>
<td>0.4966</td>
</tr>
<tr>
<td>1</td>
<td>4.6184</td>
<td>0.5132</td>
</tr>
<tr>
<td>2</td>
<td>4.6232</td>
<td>0.4753</td>
</tr>
<tr>
<td>3</td>
<td>4.6359</td>
<td>0.4453</td>
</tr>
<tr>
<td>4</td>
<td>4.6591</td>
<td>0.4257</td>
</tr>
<tr>
<td>5</td>
<td>4.6874</td>
<td>0.4378</td>
</tr>
<tr>
<td>6</td>
<td>4.6983</td>
<td>0.4697</td>
</tr>
<tr>
<td>7</td>
<td>4.6978</td>
<td>0.5095</td>
</tr>
<tr>
<td>8</td>
<td>4.6891</td>
<td>0.5435</td>
</tr>
<tr>
<td>9</td>
<td>4.6732</td>
<td>0.5704</td>
</tr>
<tr>
<td>10</td>
<td>4.644</td>
<td>0.5802</td>
</tr>
<tr>
<td>11</td>
<td>4.6253</td>
<td>0.556</td>
</tr>
</tbody>
</table>

Table 7.2: $m_b$ and $\mu_\pi^2$ values for the $\Delta \chi^2 = 1$ contour plot shown in Fig. 7.1.

of them satisfying the same moments constraints as the original form of Eq. 4.4.

$$F(k_+; c) = N(1 - k_+/\bar{\Lambda})^c e^{-b(1-k_+/\bar{\Lambda})^2}, \quad b = \left(\frac{\Gamma\left(\frac{c+2}{2}\right)}{\Gamma\left(\frac{c+1}{2}\right)}\right)^2$$ (7.1)

$$F(k_+; \rho) = \frac{\kappa}{\sqrt{\pi}} \exp\left( -\frac{1}{4} \left( \frac{\rho}{\kappa 1 - k_+/\bar{\Lambda}} - \kappa (1 - k_+/\bar{\Lambda})^2 \right) \right), \quad \kappa = \frac{\rho}{\sqrt{\pi}} e^{\rho/2} K_1(\rho/2)$$ (7.2)
Other effects due to the modeling of charmless semileptonic decays have been evaluated by varying the branching fractions for the exclusive charmless semileptonic decays within their known uncertainties (Table 4.4).

Signal Monte Carlo contains events where a gluon splits in an $s\bar{s}$ pair, resulting in decays of the heavy $X_u$ states into $K\bar{K}$ pairs. This happens both in the resonant and the non-resonant contributions. In the hybrid model the fractions of events with gluon splitting in $s\bar{s}$ for $B^{+}$ ($B^0$) 12.0% (11.3%) for the non-resonant one. The $s\bar{s}$ contribution is modeled using JETSET. The parameter which sets gluon splitting in $s\bar{s}$ in JETSET is also known as $\gamma_s$ and it is set to $\gamma_s = 0.30$ in Monte Carlo. This parameter has been measured by two experiments at center of mass energies between 12 and 36 GeV as $\gamma_s = 0.35 \pm 0.05$ [75], $\gamma_s = 0.27 \pm 0.06$ [76]. Reference [76] shows how the scaling to lower energies (to 3 GeV, equivalent to the energies involved in the $X_u$ decays for $M_X \sim 1.5$ GeV/$c^2$) works fine compared to [77].

In order to calculate the systematic uncertainty, the fraction of events where a gluon splits in an $s\bar{s}$ pair has been varied by $\pm 30\%$. For non-resonant events this corresponds to taking as 1$\sigma$ interval the sum of the intervals from the two experiment.

### 7.4 Background Knowledge

Exclusive semileptonic branching fractions for $\bar{B} \to X_u\ell\bar{\nu}$ are known with a certain precision. Moreover, the individual branching fractions in the Monte Carlo simulations are known to differ from the current world averages. This difference is corrected by re-weighting simulated events to match the world averages, shown in Table 7.3. Here $D^{**}$ refers to either non-resonant or broad $D^{**}$ states and
the corresponding branching fraction is taken as the difference between the total semileptonic rate and the other measured branching fractions. The systematic uncertainty due to the limited knowledge of the exclusive semileptonic branching fractions is computed by varying them randomly within one standard deviation of their world averages, repeating the measurements, iterating the procedure 100 times, and taking the RMS of the resulting distribution of results.

A similar procedure was followed to estimate the uncertainty due to the branching ratios of charm mesons (Table 7.4) Uncertainties in the form factors in the $B \rightarrow D^* \ell \nu$ decays are also taken into account, by repeating the analyses after varying the form factors within their experimental error [78].

<table>
<thead>
<tr>
<th>$B$ Decay mode</th>
<th>best average (%)</th>
<th>Monte Carlo (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow D^- \ell \nu$</td>
<td>2.13 ±0.14</td>
<td>2.07</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{*-} \ell \nu$</td>
<td>5.53±0.25</td>
<td>5.70</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^*_1 \ell \nu$</td>
<td>0.50±0.08</td>
<td>0.52</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^*_2 \ell \nu$</td>
<td>0.39±0.07</td>
<td>0.23</td>
</tr>
<tr>
<td>$B^0 \rightarrow \chi_{c} \ell \nu$</td>
<td>10.14±0.38</td>
<td>10.2</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^0 l \nu$</td>
<td>2.30±0.16</td>
<td>2.24</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^{*0} l \nu$</td>
<td>5.95±0.24</td>
<td>6.17</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^0_{1} l \nu$</td>
<td>0.54±0.06</td>
<td>0.56</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^0_{2} l \nu$</td>
<td>0.42±0.08</td>
<td>0.30</td>
</tr>
<tr>
<td>$B^+ \rightarrow \chi_{c} l \nu$</td>
<td>10.92± 0.37</td>
<td>11.04</td>
</tr>
</tbody>
</table>

Table 7.3: Branching fractions for $B \rightarrow \chi_{c} \ell \nu$ decays, current best averages and values used in Monte Carlo simulation, plus shift of the results due to the adjustments of the BR. The non resonant $B \rightarrow D l \nu X$ is obtained by difference of the inclusive rate and the other 4 components.

7.5 Summary

Table 7.5 shows the relative uncertainties (in percent) involved in the $m_X$, $P_{\perp}$, $(m_X - q^2)$ and $(m_X - q^2)$ without cuts analyses. In table 7.6 the relative systematic uncertainties for the $(m_X - q^2)$ analysis cutting away different high $q^2$ regions are reported. Statistical errors are slightly larger than the total systematic uncertainties for all analyses. Systematic uncertainties due to detector, fit procedure and signal knowledge contribute roughly equally. The best measurement in terms of overall uncertainty is the one in the phase space region defined by $m_X < 1.55$ GeV/$c^2$. 
<table>
<thead>
<tr>
<th>$D^0$ Decay mode</th>
<th>PDG</th>
<th>MC</th>
<th>$D^+$ Decay mode</th>
<th>PDG</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow K\pi$</td>
<td>0.0389 ± 0.0005</td>
<td>0.0383</td>
<td>$D^+ \rightarrow K^0\pi$</td>
<td>0.0291 ± 0.0006</td>
<td>0.0280</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0\pi^0$</td>
<td>0.0244 ± 0.0012</td>
<td>0.0211</td>
<td>$D^+ \rightarrow K\pi\pi$</td>
<td>0.0922 ± 0.0021</td>
<td>0.0920</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0\pi\pi$</td>
<td>0.0598 ± 0.0034</td>
<td>0.0544</td>
<td>$D^+ \rightarrow K^0\pi\pi^0$</td>
<td>0.1360 ± 0.0100</td>
<td>0.1082</td>
</tr>
<tr>
<td>$D^0 \rightarrow K\pi\pi^0$</td>
<td>0.1390 ± 0.0050</td>
<td>0.1394</td>
<td>$D^+ \rightarrow K^0\pi\pi^0$</td>
<td>0.0600 ± 0.0020</td>
<td>0.0684</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0\pi^0\pi^0$</td>
<td>0.0210 ± 0.0020</td>
<td>0.0163</td>
<td>$D^+ \rightarrow K^0\pi\pi\pi$</td>
<td>0.0604 ± 0.0024</td>
<td>0.0718</td>
</tr>
<tr>
<td>$D^0 \rightarrow K\pi\pi\pi$</td>
<td>0.0810 ± 0.0020</td>
<td>0.0791</td>
<td>$D^+ \rightarrow K\pi\pi\pi\pi$</td>
<td>0.0056 ± 0.0005</td>
<td>0.0075</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0\pi\pi\pi^0$</td>
<td>0.1080 ± 0.0120</td>
<td>0.0977</td>
<td>$D^+ \rightarrow K^0\pi\pi\pi\pi^0$</td>
<td>0.0220 ± 0.0470</td>
<td>0.0084</td>
</tr>
<tr>
<td>$D^0 \rightarrow K\pi\pi\pi\pi^0$</td>
<td>0.1490 ± 0.0500</td>
<td>0.1051</td>
<td>$D^+ \rightarrow K^0\pi\pi\pi\pi^0$</td>
<td>0.0540 ± 0.0300</td>
<td>0.0249</td>
</tr>
<tr>
<td>$D^0 \rightarrow K\pi\pi\pi\pi\pi^0$</td>
<td>0.0420 ± 0.0040</td>
<td>0.0434</td>
<td>$D^+ \rightarrow K^0\pi\pi\pi\pi\pi^0$</td>
<td>0.0008 ± 0.0007</td>
<td>0.0008</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi\pi$</td>
<td>0.001397 ± 0.000027</td>
<td>0.00140</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0020 ± 0.0018</td>
<td>0.0045</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi^0\pi\pi^0$</td>
<td>0.0080 ± 0.00008</td>
<td>0.0008</td>
<td>$D^+ \rightarrow K^0K^0K$</td>
<td>0.0090 ± 0.0042</td>
<td>0.0101</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi\pi\pi\pi^0$</td>
<td>0.0144 ± 0.0006</td>
<td>0.0159</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi^0$</td>
<td>0.00124 ± 0.00007</td>
<td>0.0025</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.00744 ± 0.00021</td>
<td>0.0072</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.00321 ± 0.00019</td>
<td>0.0030</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0042 ± 0.0005</td>
<td>0.0180</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0114 ± 0.0008</td>
<td>0.0118</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0004 ± 0.00012</td>
<td>0.0004</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.00163 ± 0.00016</td>
<td>0.0020</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0\pi\pi\pi\pi\pi^0$</td>
<td>0.0057 ± 0.0006</td>
<td>0.0038</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0029 ± 0.0029</td>
<td>0.0007</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0K^0K$</td>
<td>0.0094 ± 0.0006</td>
<td>0.0093</td>
<td>$D^+ \rightarrow KK\pi^0$</td>
<td>0.0090 ± 0.0042</td>
<td>0.0100</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0K^0\pi^0$</td>
<td>0.0038 ± 0.0006</td>
<td>0.0038</td>
<td>$D^+ \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0046 ± 0.0004</td>
<td>0.0015</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi\pi^0$</td>
<td>0.0100 ± 0.0009</td>
<td>0.0053</td>
<td>$D^+ \rightarrow KK^0$</td>
<td>0.0058 ± 0.0003</td>
<td>0.0073</td>
</tr>
<tr>
<td>$D^0 \rightarrow KK\pi$</td>
<td>0.0096 ± 0.0003</td>
<td>0.0096</td>
<td>$D^+ \rightarrow KK\pi$</td>
<td>0.00080 ± 0.00007</td>
<td>0.0107</td>
</tr>
</tbody>
</table>

Table 7.4: $D$ branching ratios, current best measurements and values used in the Monte Carlo.
### 7.5 Summary

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma_{\Delta B} m_X$</th>
<th>$\sigma_{\Delta B} P_+$</th>
<th>$\sigma_{\Delta B} (m_X, q^2)$</th>
<th>$\sigma_{\Delta B} (m_X, q^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical Error</td>
<td>6.91</td>
<td>6.82</td>
<td>7.42</td>
<td>7.35</td>
</tr>
<tr>
<td>MC Statistics</td>
<td>1.37</td>
<td>1.49</td>
<td>1.60</td>
<td>1.44</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.51</td>
<td>0.47</td>
<td>0.88</td>
<td>1.12</td>
</tr>
<tr>
<td>Neutral efficiency</td>
<td>1.47</td>
<td>2.78</td>
<td>2.07</td>
<td>2.64</td>
</tr>
<tr>
<td>$\pi^0$ efficiency</td>
<td>0.38</td>
<td>0.25</td>
<td>0.11</td>
<td>0.21</td>
</tr>
<tr>
<td>PID eff. &amp; misID</td>
<td>1.86</td>
<td>2.08</td>
<td>2.71</td>
<td>2.47</td>
</tr>
<tr>
<td>$K_L$</td>
<td>0.51</td>
<td>0.61</td>
<td>1.02</td>
<td>1.11</td>
</tr>
</tbody>
</table>

**Fit related:**

| $m_{ES}$ fit parameters      | 2.11                    | 2.41                     | 2.51                          | 2.87                          |
| peaking background           | 1.91                    | 2.12                     | 1.15                          | 1.89                          |

**Signal knowledge:**

| SF parameters                | +0.55                   | +0.72                    | +0.21                         | +1.97                         |
| SF form                      | 0.61                    | 1.21                     | 1.41                          | 1.17                          |
| Exclusive $b \rightarrow u\ell\nu$ | 1.89                    | 2.57                    | 2.31                          | 2.01                          |
| Gluon splitting              | 0.83                    | 0.89                     | 1.32                          | 2.13                          |

**Background knowledge:**

| $K_S$ veto                   | 0.44                    | 0.61                     | 0.48                          | 0.71                          |
| $B$ SL $B$                   | 0.74                    | 1.30                     | 1.09                          | 0.60                          |
| $D$ decays                   | 0.89                    | 1.16                     | 0.63                          | 0.51                          |
| $B \rightarrow D^*\ell\nu$ form factor | 0.36                    | 0.52                     | 0.37                          | 0.10                          |
| Total systematics            | +4.78                   | -6.19                    | +5.89                         | +6.66                         |
|                             | -4.93                   | -6.32                    | -6.08                         | -7.00                         |
| Total error                  | +8.40                   | +9.21                    | +9.47                         | +9.92                         |
|                             | -8.50                   | -9.30                    | -9.59                         | -10.15                        |

Table 7.5: Relative uncertainties in percent for the $m_X$, $P_+$, $(m_X - q^2)$ and the $(m_X - q^2)$ without cuts analyses.
### Table 7.6: Relative uncertainties in percent for the $m_X - q^2$ analysis cutting away for different high $q^2$ values.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma_{\Delta B} (m_X, q^2)$ $q^2 &lt; 8 \text{ GeV}^2/c^4$</th>
<th>$\sigma_{\Delta B} (m_X, q^2)$ $q^2 &lt; 10 \text{ GeV}^2/c^4$</th>
<th>$\sigma_{\Delta B} (m_X, q^2)$ $q^2 &lt; 12 \text{ GeV}^2/c^4$</th>
<th>$\sigma_{\Delta B} (m_X, q^2)$ $q^2 &lt; 14 \text{ GeV}^2/c^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical Error</td>
<td>12.62</td>
<td>10.04</td>
<td>9.45</td>
<td>8.15</td>
</tr>
<tr>
<td>MC Statistics</td>
<td>2.18</td>
<td>1.86</td>
<td>1.67</td>
<td>1.51</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.54</td>
<td>0.61</td>
<td>0.63</td>
<td>0.71</td>
</tr>
<tr>
<td>Neutral efficiency</td>
<td>1.46</td>
<td>1.39</td>
<td>1.40</td>
<td>1.55</td>
</tr>
<tr>
<td>$\pi^0$ efficiency</td>
<td>0.41</td>
<td>0.43</td>
<td>0.43</td>
<td>0.45</td>
</tr>
<tr>
<td>PID eff. &amp; misID</td>
<td>2.76</td>
<td>2.56</td>
<td>2.51</td>
<td>2.73</td>
</tr>
<tr>
<td>$K_L$</td>
<td>0.88</td>
<td>0.95</td>
<td>0.97</td>
<td>0.86</td>
</tr>
<tr>
<td>Fit related:</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$m_{ES}$ fit parameters</td>
<td>2.39</td>
<td>2.45</td>
<td>2.51</td>
<td>2.61</td>
</tr>
<tr>
<td>peaking background</td>
<td>1.49</td>
<td>1.33</td>
<td>1.39</td>
<td>1.25</td>
</tr>
<tr>
<td>Signal knowledge:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SF parameters</td>
<td>+0.94</td>
<td>+0.98</td>
<td>+1.01</td>
<td>+0.96</td>
</tr>
<tr>
<td></td>
<td>-2.58</td>
<td>-2.51</td>
<td>-2.67</td>
<td>-2.47</td>
</tr>
<tr>
<td>SF form</td>
<td>1.54</td>
<td>1.21</td>
<td>1.49</td>
<td>1.78</td>
</tr>
<tr>
<td>Exclusive $b \rightarrow u\ell\nu$</td>
<td>2.52</td>
<td>2.02</td>
<td>2.06</td>
<td>2.77</td>
</tr>
<tr>
<td>Gluon splitting</td>
<td>1.02</td>
<td>1.22</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>Background knowledge:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_S$ veto</td>
<td>0.39</td>
<td>0.45</td>
<td>0.47</td>
<td>0.51</td>
</tr>
<tr>
<td>$B$ SL $B$</td>
<td>1.51</td>
<td>1.10</td>
<td>0.92</td>
<td>0.82</td>
</tr>
<tr>
<td>$D$ decays</td>
<td>1.18</td>
<td>1.03</td>
<td>1.01</td>
<td>0.99</td>
</tr>
<tr>
<td>$B \rightarrow D^*\ell\nu$ form factor</td>
<td>0.47</td>
<td>0.49</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>Total systematics:</td>
<td>+6.18</td>
<td>+5.63</td>
<td>+5.63</td>
<td>+6.07</td>
</tr>
<tr>
<td></td>
<td>-6.64</td>
<td>-6.10</td>
<td>-6.14</td>
<td>-6.84</td>
</tr>
<tr>
<td>Total error:</td>
<td>+14.05</td>
<td>+11.51</td>
<td>+11.00</td>
<td>+10.16</td>
</tr>
<tr>
<td></td>
<td>-14.26</td>
<td>-11.75</td>
<td>-11.77</td>
<td>-10.41</td>
</tr>
</tbody>
</table>
Chapter 8

Measurement of $|V_{ub}|$

The partial branching fractions $\Delta B(\bar{B} \to X_u \ell \bar{\nu})$ determined in Chapter 6 are translated into $|V_{ub}|$ values by means of theoretical calculations. As we discussed in Section 1.3.3, there are many theoretical calculations available in literature. We extract $|V_{ub}|$ using only the BLNP approach [30]. The BLNP calculation gives a complete treatment of the theoretical of different theoretical uncertainties.

8.1 Input Parameters

The BLNP require the knowledge of the Shape Function that cannot be determined by first principles.

The measurements of the non-perturbative HQE obtained from global fits of the $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_c \ell \bar{\nu}$ moments [10], are used to constrain the leading SF moments (first and second moments, which are, at leading order, equal to $\bar{A} = M_B - m_b$ and $\mu_2^2$ respectively), as well as provide accurate values of $m_b$, that is needed to extract $|V_{ub}|$.

The HQE parameters from the global fit has been performed in different $b$-quark mass scheme: the kinetic scheme [10] performed by Flüchter and Buchmüller and the 1S from Bauer et al., Ref.[79]. The BLNP framework require that the HQE parameters are translated in the so called SF scheme . The values of the input parameters used, translated from the fit performed in the kinetic scheme, are reported in Table 8.1.
Table 8.1: Non-perturbative parameters in the SF scheme, translated from the kinetic scheme, used as input to the BLNP calculations.

8.2 $|V_{ub}|$ Extraction from Partial Branching fractions $\Delta B(B \to X_u \ell \bar{\nu})$

BLNP gives results and uncertainties in terms of the reduced decay rate $\tilde{\Gamma}_{\text{th}}$, defined in units of $|V_{ub}|^2$ ps$^{-1}$:

$$\tilde{\Gamma}_{\text{th}} = \frac{\Delta \Gamma_{\text{th}}}{|V_{ub}|^2}$$

where $\Delta \Gamma_{\text{th}}$ is the partial width of the $B \to X_u \ell \bar{\nu}$ decay into the phase space of interest predicted by the theory. $|V_{ub}|$ is related to the measured partial decay rate $\Delta B(B \to X_u \ell \bar{\nu})$ and $\tilde{\Gamma}_{\text{th}}$ through

$$|V_{ub}| = \sqrt{\frac{\Delta B(B \to X_u \ell \bar{\nu})}{\tilde{\Gamma}_{\text{th}} \cdot \tau_B}}$$

where $\tau_B$ is the $B$ meson lifetime. The reduce decay rate $\tilde{\Gamma}_{\text{th}}$ is computed by a Mathematica notebook given by the authors of BLNP. The value of $\tilde{\Gamma}_{\text{th}}$ depends on the analyzed phase space. The $\tilde{\Gamma}_{\text{th}}$ values for the various phase space regions analysis in the current analysis, are shown in Table 8.2 with the corresponding error due to the SF parameters $m_b$ and $\mu^2_{\tau}$ (SF), the error due to the sub-leading shape functions (ssf), the choice of the intermediate scale (scale), and finally the error contribution due to the Weak Annihilation (WA).

8.2.1 $m_X, P_\pi, (m_X - q^2)$ analyses

In summary, in Chapter 6 we have measured the partial branching fractions for $B \to X_u \ell \nu$ decays in three overlapping regions of phase space and the results are shown in table 8.3. Relying on BLNP theoretical predictions (see table 8.2) we extract values for the CKM matrix element $|V_{ub}|$ from our measured $\Delta S$. In table 8.3 are reported the $|V_{ub}|$ values computed for the $m_X, P_\pi$, and $m_X - q^2$ analyses where the first error is statistic, the second is systematic and the third, the dominant, is due to the theoretical uncertainty.

The $|V_{ub}|$ measurements obtained with different kinematic cuts, seems to be in good agreement but to really check the consistency, we need to compute the statistical, systematics, and theoretical correlations. Infact the phase space regions considered are not independent, but highly overlapped.
8.2 $|V_{ub}|$ Extraction from Partial Branching fractions $\Delta B(B \to X_u \ell \bar{\nu})$

<table>
<thead>
<tr>
<th>Phase Space region</th>
<th>$\tilde{\Gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(m_X, q^2)$</td>
<td>$25.0 \pm 3.1_{SF} \pm 0.4_{ssf} \pm 2.3_{scale} \pm 1.3_{WA}$</td>
</tr>
<tr>
<td>$m_X$</td>
<td>$43.4 \pm 5.9_{SF} \pm 1.0_{ssf} \pm 3.1_{scale} \pm 1.3_{WA}$</td>
</tr>
<tr>
<td>$P_+$</td>
<td>$42.3 \pm 6.1_{SF} \pm 1.2_{ssf} \pm 2.7_{scale} \pm 1.3_{WA}$</td>
</tr>
<tr>
<td>$(m_X, q^2) q^2 &lt; 8 \text{ GeV}^2/c^4$</td>
<td>$20.3 \pm 6.5_{SF} \pm 1.1_{ssf} \pm 3.4_{scale} \pm 0.0_{WA}$</td>
</tr>
<tr>
<td>$(m_X, q^2) q^2 &lt; 10 \text{ GeV}^2/c^4$</td>
<td>$26.3 \pm 6.4_{SF} \pm 1.1_{ssf} \pm 3.2_{scale} \pm 0.0_{WA}$</td>
</tr>
<tr>
<td>$(m_X, q^2) q^2 &lt; 12 \text{ GeV}^2/c^4$</td>
<td>$32.1 \pm 6.2_{SF} \pm 1.1_{ssf} \pm 3.1_{scale} \pm 0.0_{WA}$</td>
</tr>
<tr>
<td>$(m_X, q^2) q^2 &lt; 14 \text{ GeV}^2/c^4$</td>
<td>$37.1 \pm 6.1_{SF} \pm 1.1_{ssf} \pm 3.0_{scale} \pm 0.0_{WA}$</td>
</tr>
</tbody>
</table>

Table 8.2: Values of $\tilde{\Gamma}$, from the BLNP method, used in the analyses. The first error is due to SF parameters uncertainties, the second is due to sub-leading shape function error, the third is due to the scale error and the fourth is the due to the weak annihilation contribution error.

At the moment we did not study the correlation among the different analyses but we aim to compute these for the final publication.

Compared with the previous Babar measurement with the same technique, we reduce the statistical error by 9% and the total error by 3% which is still dominated by the uncertainty on the HQE parameters. The $|V_{ub}|$ measurements are in good agreement with other inclusive $|V_{ub}|$ determinations obtained also with other kinematics cuts, obtained by BABAR CLEO and Belle experiments.

Our measurements are also compatible with the world average $|V_{ub}|$ value $(4.32 \pm 0.16_{exp} \pm 0.32_{theo})$ computed by HFAG using the BLNP theoretical approach. It is worth to mention here that the inclusive determinations, both the single measurements and the global average, are somewhat higher than the results based on exclusive $B \to \pi \ell \nu$ decays, which rely on the Lattice QCD to compute the needed form factor normalizations.

| Region       | $\Delta B(B \to X_u \ell \bar{\nu}) \times 10^{-3}$ | $|V_{ub}| \times 10^{-3}$ |
|--------------|--------------------------------------------------|--------------------------|
| $m_X$        | $1.17 \pm 0.08_{stat} \pm 0.06_{syst}$           | $4.11 \pm 0.14_{stat} \pm 0.11_{syst} \pm 0.31_{theo}$ |
| $P_+$        | $1.06 \pm 0.07_{stat} \pm 0.07_{syst}$           | $3.97 \pm 0.14_{stat} \pm 0.13_{syst} \pm 0.31_{theo}$ |
| $(m_X - q^2)$| $0.71 \pm 0.05_{stat} \pm 0.04_{syst}$           | $4.24 \pm 0.16_{stat} \pm 0.13_{syst} \pm 0.33_{theo}$ |

Table 8.3: $B \to X_u \ell \nu$ partial branching fraction for the $m_X$, $P_+$ and $(m_X - q^2)$ analyses and the $|V_{ub}|$ extracted with the BLNP framework.
### Table 8.1: Comparison of New Measurements with Previous Determinations

|                | Value ± Error
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average</strong></td>
<td>$4.32 ± 0.16_{\text{exp}} ± 0.32_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>NEW BABAR ($m_X$)</strong></td>
<td>$4.11 ± 0.18_{\text{exp}} ± 0.31_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>NEW BABAR ($P_+$)</strong></td>
<td>$3.97 ± 0.19_{\text{exp}} ± 0.31_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>NEW BABAR ($m_X$ - $q^2$)</strong></td>
<td>$4.24 ± 0.21_{\text{exp}} ± 0.33_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>NEW BABAR ($m_X$ - $q^2$) no cuts</strong></td>
<td>$4.43 ± 0.23_{\text{exp}} ± 0.25_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>CLEO ($E_e$)</strong></td>
<td>$3.94 ± 0.46_{\text{exp}} ± 0.37_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BELLE ($m_X$ - $q^2$)</strong></td>
<td>$4.33 ± 0.46_{\text{exp}} ± 0.35_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BELLE ($E_e$)</strong></td>
<td>$4.74 ± 0.44_{\text{exp}} ± 0.35_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BABAR ($E_e$)</strong></td>
<td>$4.29 ± 0.24_{\text{exp}} ± 0.35_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BELLE ($m_X$)</strong></td>
<td>$3.99 ± 0.26_{\text{exp}} ± 0.30_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BABAR ($m_X$)</strong></td>
<td>$4.13 ± 0.20_{\text{exp}} ± 0.32_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BABAR ($m_X$ - $q^2$)</strong></td>
<td>$4.41 ± 0.29_{\text{exp}} ± 0.36_{\text{theo}}$</td>
</tr>
<tr>
<td><strong>BABAR ($P_+$)</strong></td>
<td>$3.76 ± 0.24_{\text{exp}} ± 0.31_{\text{theo}}$</td>
</tr>
</tbody>
</table>

### Figure 8.1: Comparison between our new $|V_{ub}|$ measurements (red) and the previous determinations obtained by BABAR, CLEO and Belle experiments. Also the $|V_{ub}|$ world average value is shown.
8.2.2 $m_X - q^2$ analyses cutting away for high $q^2$

Following the same steps of the previous section we extract $|V_{ub}|$ measurements from partial branching ratios obtained exploiting phase space regions where no weak annihilation contributions are expected. In table 8.4 are reported the values of the partial branching fraction measured in these restricted regions, as described in Chapter 6, and the respective $|V_{ub}|$ extraction using the BLNP theoretical approach.

| Region ($m_X - q^2$) | $\Delta B(B \to X_u \ell \nu)$ ($10^{-3}$) | $|V_{ub}|$ ($10^{-3}$) |
|----------------------|------------------------------------------|----------------------|
| $m_X < 1.55$ GeV/c^2 |                                          |                      |
| $q^2 < 8$ GeV^2/c^4  | 0.51 ± 0.06 stat ± 0.03 syst             | 3.97 ± 0.25 stat ± 0.14 syst ± 0.72 theo |
| $q^2 < 10$ GeV^2/c^4 | 0.69 ± 0.07 stat ± 0.04 syst             | 4.04 ± 0.20 stat ± 0.13 syst ± 0.55 theo |
| $q^2 < 12$ GeV^2/c^4 | 0.78 ± 0.07 stat ± 0.05 syst             | 3.91 ± 0.19 stat ± 0.12 syst ± 0.42 theo |
| $q^2 < 14$ GeV^2/c^4 | 0.96 ± 0.08 stat ± 0.06 syst             | 4.02 ± 0.16 stat ± 0.13 syst ± 0.37 theo |

Table 8.4: Partial Branching Fraction of $B \to X_u \ell \nu$ decays measured from the $m_X - q^2$ analyses with $m_X < 1.55$ GeV/c^2 and $q^2 < 8, 10, 12, 14$ GeV^2/c^4 with respective $|V_{ub}|$ extractions obtained using BLNP theoretical approach.

The $|V_{ub}|$ measurements show a constant trend, as function of the $q^2$ cuts applied, and present an average value slightly smaller than $m_X$ analysis (see figure 8.2) and the ($m_X - q^2$) analysis. This disagreement should be interpreted as hint of possible Weak Annihilation contribution because this phenomena is concentrated in high $q^2$ region and should enhance the $|V_{ub}|$ measurement. Weak Annihilation measurement or possible its limit can be extrapolated knowing statistical, systematical and theoretical correlations. At the moment we did not study the correlation among the different analyses but we aim to compute these for the final publication.

8.3 $|V_{ub}|$ Extraction from Total Branching fractions $B(\bar{B} \to X_u \ell \bar{\nu})$

A method to reduce the model dependence is to measure the $B \to X_u \ell \nu$ rate over the phase space. Since no extrapolation is necessary to obtain the full rate, systematic uncertainties from $m_b$ and Fermi motion are much reduced. In table 8.5 is reported the value of the total branching ratio $B(\bar{B} \to X_u \ell \nu)$ measured in the full phase space, as described in Chapter 6, the respective theoretical decay rate $\Gamma$ and $|V_{ub}|$ obtained using the BLNP theoretical approach.

The above results are consistent with previous measurements but have substantially smaller uncertainties from $m_b$ and the modeling of Fermi motion.
Figure 8.2: \(|V_{ub}|(q^2)\) for the \(m_X < 1.55\ \text{GeV}/c^2\) analysis as function of the \(q^2\) cuts \((8.10,12,14\ \text{GeV}^2/c^4)\) in a phase space region with \(m_X < 1.55\ \text{GeV}/c^2\). These measurements are obtained using BLNP theoretical approach. Is also reported the \(|V_{ub}|\) value measured using the one dimensional fit on \(m_X\) described in section 6.2. In the figure the first error is experimental and the second error is theoretical.

| \(B(B \rightarrow X_{u}\ell\nu) (10^{-3})\) | \(\hat{\Gamma}\) | \(|V_{ub}| (10^{-3})\) |
|---------------------------------------------|----------------|----------------|
| \(2.23 \pm 0.16_{\text{stat}} \pm 0.16_{\text{syst}}\) | \(71.7 \pm 4.57_{SF} \pm 0.5_{ssf} \pm 5.0_{\text{scale}} \pm 1.3_{\text{WA}}\) | \(4.43 \pm 0.16_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.25_{\text{theo}}\) |

Table 8.5: \(B \rightarrow X_{u}\ell\nu\) branching ratio for the \((m_X - q^2)\) analyses without cuts on the phase space region, the theoretical \(\hat{\Gamma}\) and the respective \(|V_{ub}|\) value, extracted with the BLNP framework.
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