JOINT RESEARCH DOCTORATE IN FUSION SCIENCE AND ENGINEERING
CYCLE XXIV (2009/2011)

PhD THESIS

THE STUDY OF RESISTIVE WALL MODE IN REVERSED FIELD PINCH PLASMAS

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Start in Napoli
August 10, 2011
Summary

Fusion energy, with many attractive features in terms of safety, fuel reserves and minimal damage to the environment, is one of the best candidates to satisfy the rapidly increasing consumption of energy and to solve the problem of the energy resource limit in the world.

In order to realize controlled fusion, magnetic confinement fusion as an important approach to achieving the fusion conditions in the laboratory is proposed. Recently the burning plasma experiment ITER [1] based on the tokamak concept is designed and being constructed. However, in order to achieve the steady-state operation and the high performance in tokamak and the other most studied configurations e.g. reversed field pinch (RFP), many plasma instabilities must be mitigated and/or suppressed. Resistive Wall Modes (RWMs), as a kind of magnetohydrodynamic (MHD) instability, sets a severe beta limit on the advanced Tokamak (e.g. ITER) which points towards the steady-state operation. In RFP plasmas, RWM always appears as a potentially disruptive instability, whenever the duration of the discharge is longer than the penetration time of the passive conducting structure (resistive wall). Therefore, understanding the physics of RWM stabilization and control are important issues in both tokamak and RFP. The work presented in this thesis is dedicated to several subjects on the RWMs in RFP plasmas: the physical understanding of RWM behavior and on the active stabilization of RWM in the presence of a control system. The rotation stabilization of RWMs in the fluid theory is firstly studied. Moreover, the kinetic wave-particle resonance effect is included in the investigation of RFP plasmas. It is found that kinetic effects can significantly modify the results obtained by fluid theory. Finally, we make a detailed comparison of kinetic effects on RWMs between tokamak and RFP plasmas. Correspondingly, the difference of the physical mechanisms of kinetic effect in both configurations is clarified. To carry out these studies, two codes are employed and further developed:

**CMR** (Cylindrical Magnetohydrodynamic with Resistive wall) based on the cylindrical MHD model was developed, and takes into account the compressibility, longitudinal flow, viscosity tensor and resistive wall with a finite
thickness. This code is further integrated with a feedback system, which becomes CMR-F.

MARS-K [2] as a toroidal MHD-kinetic hybrid stability code, where the drift kinetic effects are self-consistently incorporated into the MHD formulation, was implemented into RFX-mod server. Later, we parallelized the kinetic calculation in MARS-K, which improves the code performance significantly. In order to gain an indepth physical understanding of kinetic effects on RWMs in tokamaks and RFPs, the analyses based on the quadratic potential energy components have been performed. A corresponding module was developed and integrated into MARS-K.

The chapters of this thesis are organized as follows:

- Chapter 1 briefly describes the concept of nuclear fusion and of two magnetic confinement devices: tokamak and reversed field pinch. Subsequently the basic physics of ideal MHD and resistive wall mode are introduced.

- Chapter 2 studies RWM instabilities using a (periodic) cylindrical model of MHD theory. In order to validate the model, a careful comparison with the experimental measurements in RFX-mod on the mode growth rates has been made by well matching the equilibrium parameters. The sensitivity of the growth rate to the equilibrium parameters is also discussed in detail. It is concluded that the model can provide consistent accuracy in studies of RWM in RFP plasmas.

- Chapter 3 is dedicated to the physical understanding on the nature of the instability spectrum of the RWM observed in RFP plasmas; specifically, the growth rates of the two groups of RWMs (internally non-resonant and externally non-resonant) have opposite dependence on the variation of the field reversal. Although these behaviors have been observed experimentally for years [3,4], the physical mechanism behind the observation has not been well understood yet. Our present study provides the answer to these questions by an analysis based on the balance of the potential energy components.

- Chapter 4 concentrates on the question of how the plasma responds to the feedback control action. The discussion based on the eigenmode equation and dispersion relation is carried out in order to further understand the
physics. It is found that actually the mode nature (eigenfunction) inside plasma is uniquely determined by the plasma equilibrium and independent of the feedback actions. The role of the feedback control is to modify the eddy current induced by the plasma perturbation on the resistive wall. Based on this result, the linear time evolutions of RWM under the feedback control can be easily studied by the calculation only in the vacuum without calculating the plasma responses for each time step, if the growth rate without feedback is known.

- Chapter 5 studies the kinetic effects of the thermal particles on RWM in RFP plasmas by adopting the toroidal hybrid stability code MARS-K where the drift kinetic effects are included self-consistently. It is found that the transit ion resonance can provide the ion acoustic Landau damping to stabilize the RWMs in high beta plasmas. The trapped particles do not play a significant role in the kinetic stabilization. The required critical flow rotation frequency is much smaller than what previously predicted by the fluid theory. The mostly unstable mode, having its rational surface closest to the plasma, can be stabilized for the wall near the plasma (e.g. n=6 mode in RFX-mod) with the rotation frequency in the ion acoustic range. For other RWMs with different toroidal wave numbers n the stabilization conditions depend on the wall position and plasma beta value. The analysis based on the potential energy components are carried out for the physical understanding. A preliminary study on the effects of the collisionality is also presented.

- Chapter 6: makes a comparison of kinetic physics on the RWM stability between RFP and Tokamak configurations in toroidal geometry. In tokamak, the kinetic effect can stabilize the mode with very slow, or vanishing plasma rotation, due to the mode resonance with the toroidal precession drift of thermal trapped particles. In RFP, instead, stabilization of the RWM comes mainly from the ion acoustic Landau damping (i.e. the transit resonance of passing particles) in the high beta region. The critical velocity required for the mode stabilization is predicted to be at least in the ion acoustic velocity range. Detailed physical analyses, based on the perturbed potential energy components, have been performed to answer the question of why the kinetic effects work differently in the two different
systems. The new physics insight into the stabilization condition of RWM is provided by an extensive analysis.
Prefazione

L'energia di fusione, con le sue molteplici caratteristiche interessanti in termini di sicurezza, di riserve di carburante e danno minimo per l'ambiente, è uno dei migliori candidati per soddisfare il rapido aumento del consumo di energia e per risolvere il problema del limite di risorse energetiche nel mondo.

Al fine di realizzare la fusione controllata, la fusione a confinamento magnetico è proposta come un approccio importante per ricreare in laboratorio le condizioni necessarie alla fusione. Recentemente l'esperimento ITER [1], dedicato allo studio di plasmi di interesse termonucleare, basato sul concetto tokamak, è stato progettato ed è in fase di costruzione. Tuttavia, al fine di ottenere un funzionamento stazionario ed alte prestazioni nel tokamak e nelle altre configurazioni più studiate, quale il reversed field pinch (RFP), molteplici instabilità del plasma devono essere mitigate e/o soppressse. Il resistive Wall Modes (RWMs), un tipo di instabilità magnetoidrodinamica (MHD), pone un severo limite nel parametro beta nel tokamak performante (i.e. ITER) che mira al funzionamento stazionario. Nei plasmi RFP, i RWM appaiono sempre come instabilità che possono causare potenzialmente disruzioni, ogni volta che la durata della scarica è più lunga del tempo di penetrazione della struttura conduttrice passiva (parete resistiva). Pertanto, la comprensione della fisica della stabilizzazione dei RWM e il loro controllo sono questioni importanti in entrambe le configurazioni Tokamak e RFP. Il lavoro presentato in questa tesi è dedicato a diversi aspetti dei RWMs in plasmi di tipo RFP: la comprensione del comportamento fisico dei RWMs e della loro stabilizzazione attiva in presenza del sistema di controllo. Inizialmente, la stabilizzazione tramite rotazione dei RWMs nella teoria dei fluidi è stata studiata. Inoltre, l’effetto cinetico della risonanza onda-particella è stato incluso nell’indagine dei plasmi RFP. Si riscontra che gli effetti cinetici possono modificare in modo significativo i risultati ottenuti dalla teoria dei fluidi. Infine, presentiamo un confronto dettagliato degli effetti cinetici dei RWMs tra i plasmi tokamak e RFP. In questo modo, viene chiarita la differenza dei meccanismi fisici alla base dell’effetto cinetico nelle due configurazioni. Per effettuare questi studi, due codici vengono utilizzati e sviluppati:
CMR (magnetoidrodinamica in geometria cilindrica e con parete resistiva), basato su un modello MHD cilindrico, è stato sviluppato, tenendo conto della comprimibilità, del flusso longitudinale, del tensore viscosità e della parete resistiva con uno spessore finito. Questo codice, ulteriormente integrato con il sistema di feedback, è chiamato CMR-F.

MARS-K [2], un codice MHD di stabilità ibrida, in geometria toroidale e basato sulla teoria cinetica, in cui gli effetti di deriva cinetici sono incorporati in maniera auto-consistente nella formulazione MHD, è implementato nel server di RFX-mod. In un secondo momento, abbiamo parallelizzato il calcolo cinetico in MARS-K, che migliora le prestazioni del codice in modo significativo. Per acquisire una comprensione approfondita della fisica dell’effetto cinetico dei RWMs nei tokamak e RFP, sono stati effettuati analisi basate sulle componenti quadratiche dell'energia potenziale. Il modulo corrispondente è sviluppato e integrato in MARS-K.

I capitoli di questa tesi sono organizzati come segue:

- Il Capitolo 1 descrive brevemente il concetto di fusione nucleare e di due dispositivi per il confinamento magnetico: tokamak e RFP. Successivamente vengono introdotti la fisica di base della MHD ideale e RWM.
- Il Capitolo 2 studia le instabilità RWM usando un modello cilindrico (e periodico) della teoria MHD. Al fine di validare il modello, un attento confronto con le misure sperimentali di RFX-mod dei tassi di crescita dei modi è stata condotta, soddisfacendo i parametri di equilibrio. La sensibilità del tasso di crescita con i parametri di equilibrio è anche discussa in dettaglio. Si può affermare che il modello può fornire una soddisfacente accuratezza per gli studi di RWM in plasmi RFP.
- Il Capitolo 3 è dedicato alla comprensione fisica della natura dello spettro delle instabilità RWM analizzati nei plasmi RFP; in particolare, i tassi di crescita dei due gruppi di RWMs (non-risonanti interni ed esterni) hanno una dipendenza opposta con la variazione dell’ inversione di campo. Anche se questi comportamenti sono stati osservati sperimentalmente già in passato [3,4], il meccanismo fisico che regola questo fenomeno non è stato ancora ben compreso. Il nostro studio fornisce la risposta a queste domande attraverso un'analisi basata sul bilanciamento delle componenti dell’energia potenziale.
Il Capitolo 4 è focalizzato su come il plasma risponda all'azione del controllo in feedback. La discussione, basata sull'equazione dell'autofunzione e della relazione di dispersione è stata realizzata al fine di comprendere meglio il meccanismo fisico. Si è constatato che effettivamente la natura modale (autofunzione) all'interno di plasma è univocamente determinata dall'equilibrio del plasma ed è indipendente dalle azioni di feedback. Il ruolo del controllo in feedback è di modificare le correnti parassite indotte dalla perturbazione di plasma sul muro resistivo. Sulla base di questo risultato, l'evoluzione lineare nel tempo dei RWM in presenza di controllo in feedback può essere facilmente studiata calcolando solo nel vuoto senza considerare la risposta di plasma per ogni passo temporale, se il tasso di crescita senza feedback è noto.

Il Capitolo 5 studia gli effetti cinetici delle particelle termiche sui RWM in plasmi RFP usando il codice di stabilità ibrida in geometria toroidale MARS-K in cui sono inclusi gli effetti cinetici in maniera auto-consistente. Si è constatato che la risonanza degli ioni passanti è in grado di fornire lo smorzamento delle onde acustiche ioniche secondo il modello di Laundau atto a stabilizzare i RWMs in plasmi ad alto beta. Le particelle intrappolate non svolgono un ruolo significativo nel processo di stabilizzazione cinetica. La frequenza di rotazione del flusso critica necessaria è molto più piccola di quanto è stato precedentemente previsto dalla teoria dei fluidi. Il modo più instabile, avendo la sua superficie razionale più vicino al plasma, può essere stabilizzato da una parete vicino al plasma (ad esempio il modo n = 6 in RFX-mod) con una frequenza di rotazione nel range delle onde acustiche ioniche. Per gli altri RWMs, con differenti numeri d'onda toroidale n, le condizioni di stabilizzazione dipendono dalla posizione del muro e dal valore del beta di plasma. L'analisi delle componenti di energia potenziale sono svolte per la comprensione del meccanismo fisico. Lo studio preliminare sugli effetti collisionali è presentato.

Il Capitolo 6 presenta un confronto tra la fisica cinetica sulla stabilità RWM tra le configurazioni in geometria toroidale Tokamak e RFP. Nel tokamak, l'effetto cinetico può stabilizzare il modo in presenza di rotazione di plasma bassa o quasi assente, grazie alla risonanza tra il modo con la deriva della precessione toroidale delle particelle termiche intrappolate. Invece, negli RPF, la stabilizzazione dei RWM deriva principalmente dal modello di
Laudau dello smorzamento delle onde acustiche ioniche (cioè la risonanza in transito delle particelle passanti) in regimi ad alto beta. La velocità critica necessaria per la stabilizzazione del modo è stimata essere almeno nel range delle onde acustiche ioniche. Analisi fisiche, basate sulle componenti perturbati dell’energia potenziale, sono state effettuate in dettaglio per rispondere alla domanda perché gli effetti cinetici funzionano in modo diverso nei due differenti sistemi. La nuova prospettiva fisica per ottenere la condizione di stabilizzazione dei RWM è fornita da un'analisi approfondita.
# Contents

1. **Nuclear fusion and magnetic confined plasma** .................................................. 1
   1.1 Fusion energy .................................................................................................. 1
   1.2 Magnetically confined plasmas ..................................................................... 6
      1.2.1 Tokamak and ITER ............................................................................... 7
      1.2.2 Reversed Field Pinch (RFP) and RFX-mod experiments ....................... 9
   1.3 Ideal magnetohydrodynamics and instabilities ........................................ 11
      1.3.1 Model of ideal magnetohydrodynamics ............................................. 11
      1.3.2 Ideal MHD instabilities ...................................................................... 14
   1.4 Resistive Wall Mode in RFP and Tokamak ............................................. 16

2. **Comparison between cylindrical model and experimental observation**. 19
   2.1 Cylindrical model of Ideal MHD ............................................................. 20
      2.1.1 Ideal MHD equations ......................................................................... 20
      2.1.2 Eigenmode equation .......................................................................... 21
   2.2 RFP equilibrium ......................................................................................... 23
   2.3 RWM instability spectrum in RFP plasma ............................................... 24
   2.4 Comparison with experimental results of RFX-mod ............................... 27
   2.5 The sensitivity of the RWM instability to the equilibrium parameters.... 32
   2.6 Summary ..................................................................................................... 37

3. **Physical understanding of RWM instability spectrum in RFP** .............. 41
   3.1 RWM dispersion relation and energy components ................................. 41
   3.2 Physical understanding of the instability spectrum of RWMs .................. 43
   3.3 Summary ..................................................................................................... 49

4. **Physical understanding of the feedback control of RWM in RFP** .......... 51
4.1 Feedback system and boundary condition ................................................ 53
4.2 Dispersion relation of RWM with feedback system ............................... 55
4.3 How does the plasma respond to the feedback control? ...................... 57
4.4 Effects of the location of the resistive wall and sensor in feedback control 62
4.5 Time dependent solution of RWM feedback control ............................ 64
4.6 Summary and discussion ....................................................................... 75

5 Kinetic damping on RWM in RFP plasmas .............................................. 79
5.1 Stabilization by plasma rotation and dissipation in fluid theory .......... 81
5.2 Model and formulations in toroidal geometry ......................................... 83
5.2.1 Kinetic model in MARS-K ................................................................. 83
5.2.2 Quadratic energy terms .................................................................... 86
5.3 General description of RWMs in RFP plasmas ....................................... 88
5.4 Kinetic Damping effects on RWMs in RFP ........................................... 91
5.4.1 Effects of various kinetic resonances on the RWMs ......................... 91
5.4.2 Effect of the wall position ................................................................. 102
5.5 Preliminary results of the investigation of collisionality ....................... 105
5.6 Summary and discussion ....................................................................... 106

6 Kinetic effects on RWM stability–comparison between RFPs and tokamaks ................................................................. 109
6.1 Characteristics of fluid RWM in RFPs and tokamaks ............................ 110
6.2 Drift kinetic effects on RWM in RFPs and tokamaks ............................ 113
6.2.1 Numerical results ............................................................................... 114
6.2.2 Physical understanding ...................................................................... 118
6.3 Non-perturbative versus perturbative results ....................................... 126
6.4 Summary and discussions .................................................................................. 128

Conclusions and further work ............................................................................. 133

Appendix .................................................................................................................. 139

Bibliography .......................................................................................................... 149

Publications .......................................................................................................... 155

Acknowledgements .............................................................................................. 157
Chapter 1

Nuclear fusion and magnetic confined plasma

In this chapter, we give an overview of the concept of nuclear fusion. Two configurations of magnetic confinement device with toroidal topology, tokamak and Reversed Field Pinch (RFP) are described. In these two important toroidal confinement systems, the ideal MHD instabilities which often lead to catastrophic loss of plasma are classified. The physics of macroscopic MHD instability, in particular Resistive Wall Mode (RWM), being the subject of this thesis, is introduced.

1.1 Fusion energy

Since the start of the industrial revolution, energy consumption has increased significantly, due to the fast developing industries, greatly increased industrial productions and consumption of human in modern society. The greenhouse effect is starting have an observable negative impact on the environment as a result of the large consumption of fossil fuels. However, if the production of greenhouse gases is to be reduced in the future, there are limits to how much energy can be generated from the primary fossil fuels: coal, natural gas, and oil. A further complication is that the known reserves of natural gas and oil will be exhausted within decades. The most significant alternative used so far to fulfill world energy needs is the nuclear power. The primary use of nuclear power is of the nuclear fission of plutonium and the uranium isotope U$^{235}$. In the process of the nuclear fission, usually plutonium and U$^{235}$ splits into two or more smaller nuclei, and emits energy. Even though the nuclear fission reactors provide about 20% of the world’s electricity, the use of fission energy is still controversial because of the problem of the disposal and the storage of radioactive nuclear waste. The actual
Nuclear fusion and magnetic confined plasma

safety records of nuclear power, Chernobyl accident, Three Mile Island accident, and Fukushima accident remind us the other issue of the safety of fission energy. Consequently, a sort of environmental friendly and largely reserved energy source is desired. Fusion, with many attractive features in terms of safety, fuel reserves and minimal damage to the environment, enters the picture. Its potential role in energy production is put into context by comparisons with the other existing energy options.

Fusion involves the merging of light elements, mainly hydrogen (H) and its isotopes deuterium (D) and tritium (T). The fusion of hydrogen is the main reaction that powers the sun. There are three main advantages of fusion power: largely fuel reserves, friendly environmental impact, and intrinsic safety. The Einstein relationship

\[ E = \Delta mc^2 \]  \hspace{1cm} (1.1)

where \( E \) is the energy produced by the mass defect \( \Delta m \), indicates how nuclear fusion produces energy. When two light nuclei fuse together and become one heavier nucleus, the process leads to the mass defect and releases energy.

In order to fuse two light nuclei, both nuclei must be sufficiently close for the short-range attractive nuclear force to overcome the barrier owing to the Coulomb repulsion among them. This can happen when two nuclei collide with sufficiently high energy, and this is indeed the strategy behind all current fusion research. There are three major nuclear reactions with lower temperature threshold: Deuterium-Deuterium (D-D), Deuterium-Helium-3 and Deuterium-Tritium (D-T). These reactions can produce a significant amount of nuclear energy, it can be written as

\[ D + T \rightarrow \alpha + n + 17.6MeV \]  \hspace{1cm} (1.2)
\[ D + D \rightarrow He^3 + n + 3.27MeV \]  \hspace{1cm} (1.3)
\[ D + D \rightarrow T + p + 4.03MeV \]  \hspace{1cm} (1.4)
\[ D + He^3 \rightarrow \alpha + p + 18.3MeV \]  \hspace{1cm} (1.5)
Nuclear Fusion and Magnetic Confined Plasma

The corresponding reaction rate as the function of temperature is shown in figure 1.1. The most favorable reaction is D-T reaction, which is the easiest of all the fusion reactions to initiate. It is also the most favorable reaction in the attainable condition in laboratory at present. Since the nuclear fusion occurs with such high temperature (10^8 to 10^9 K), all the atoms (D, T) are ionized and become the fourth state so called plasma, a quasi-neutral ensemble of ions and electrons[5]. The plasma must be confined for a sufficiently long time so that the nuclear energy due to the fusion reactions of ions exceeds the energy required to heat the plasma. Therefore, the energy confinement time is defined as \( \tau_E = \frac{W}{P_{\text{loss}}} \), which denotes the ratio between the plasma internal energy and the losses per time unit.

Two methods seem up to now to be rather promising to realize fusion conditions in the laboratory, i.e. the so called magnetic confinement and inertial confinement fusion.

Magnetic confinement fusion (MCF) [6], which will be focused on in this thesis, uses strong magnetic fields to confine the plasma.

On the other hand, Inertial Confinement Fusion (ICF) [7] experiments use small volumes of solid matter compressed to sufficiently high densities and heated to sufficiently high temperatures by firing high power lasers from many directions to achieve the critical conditions.

![Figure 1.1: Reactions rates versus the temperature for the three fusion reaction](image-url)
In order to obtain the positive energy balance of a fusion reactor, the energy produced by fusion reactions has to exceed that required to create and sustain the plasma itself. One form of energy loss for a D-T plasma with electron density $n(n_{D,T} = n/2)$ and temperature $T$ is bremsstrahlung radiation [5]. The power lost per unit volume due to the bremsstrahlung emission is defined as $P_b = b n^2 T^{3/2}$, where $b$ is a function of the effective charge $Z_{\text{eff}} = n^{-1} \sum_i n_i Z_i^2$ in a multi-species plasma.

In addition, power losses due to confinement degradation, e.g. through collisional and turbulence transport, have to be considered in the power balance. A simple estimate of the energy losses due to mechanisms different from bremsstrahlung can be made as $P_i = 3nT / \tau_E$.

The power generated by fusion reactions can be written as $P_n = W_{DT} <\sigma v>_r / 4$, where $W_{DT} = 17.59\text{MeV}$ is the energy released after a single D-T fusion reaction, and $<\sigma v>_r$ is the product of the reaction cross-section and the relative velocity of the reactants averaged over a Maxwellian velocity distribution, where the last term is a function of temperature. One assumes that the balance between the reaction power $P_n$ and the energy losses with a coefficient $\eta$.

The self-sustainment condition is:

$$P_b + P_i \leq \eta(P_b + P_i + P_n),$$

which can be rewritten as

$$n \tau_E \geq 3T \left( \frac{\eta W_{DT}}{1-\eta} \frac{<\sigma v>_r - bT^2}{4} \right)^{-1}$$

Equation 1.7 is known as Lawson’s criterion [8], containing the three most important factors for fusion reaction to occur: confinement time, plasma density and temperature. The most probable reactor scenario is that the $\alpha$ particles ($^4\text{He}$ nuclei) produced by fusion reactions are confined by the magnetic field and replace all the energy losses by transferring their energy to the plasma, whereas neutrons escape the plasma volume and their energy is converted to electric energy. In this case, the Lawson's criterion must be modified; it is called ignition criterion and is written as
Nuclear Fusion and Magnetic Confined Plasma

\[ P_b + P_i \leq P_a. \] (1.8)

It can also be further written as:

\[ n\tau_E \geq 3T \left( \frac{W_a}{4} < \sigma v > T - bT^2 \right)^{-1} \] (1.9)

where \( W_a = W_{DT}/5 \) is the energy of a single \( \alpha \) particle after a fusion reaction. This ignition curve has a minimum at \( T_i \approx 20\text{keV} \). Moreover, one obtains the classic form, triple product:

\[ n\tau ET \geq 3 \cdot 10^{21} \left[ m^{-3}\text{keV}s \right] \] (1.10)

The values of the triple product reached in magnetic confinement fusion devices since the beginning of experiments in plasma physics is reported in figure 1.2. It shows more and more encouraging results in fusion research in the last decades, but a major step demonstrating the feasibility of a nuclear fusion reactor is still missing.

Figure 1.2: Values of the fusion triple product of D-T reactions obtained in existing tokamak experiments versus the central ion temperature \( T_i \).
1.2 Magnetically confined plasmas

Plasma magnetic confinement is based on the nature of charged particle (ions and electrons) motion in the presence of magnetic fields due to the Lorentz force. The purpose of the magnetic field is to confine the hot plasma away from the wall. Several magnetic field geometries have been investigated to seek the best conditions for plasma confinement.

The toroidal coordinates \((r, \theta, \phi)\) is usually used in the toroidal configuration study, where \(r\) is radial coordinate, \(\theta\) and \(\phi\) are the poloidal and toroidal angles respectively. \(R_0\) and \(a\) denote the major and minor radius of the torus respectively. Magnetic field lines in toroidal experiments usually have both a poloidal component \(B_\theta\), and a toroidal component \(B_\phi\). Therefore, an important parameter crucially determining several features of plasma instabilities in toroidal configurations needs to be introduced. It is so called safety factor \(q\), which in the large aspect ratio approach \((a/R<<1)\) can be defined as:

\[
q(r) = \frac{rB_\phi(r)}{R_0B_\theta(r)}
\]  

(1.11)

This quantity represents the number of toroidal turns done by a helical field line per poloidal turn. In this thesis, we mainly focus on two toroidal configurations: Tokamak [9] and Reversed Field Pinch [10], which are characterized by different magnetic field, safety factor profiles, and instability behavior [11]. Moreover, in order to evaluate the merit of magnetic confinement, an important dimensionless number \(\beta\) is introduced to estimate how much plasma pressure is balanced by the magnetic field pressure:

\[
\beta = \frac{\bar{p}}{\bar{B}^2/2\mu_0},
\]  

(1.12)

where \(\bar{p}\) and \(\bar{B}\) are the averaged plasma pressure and the total magnetic field over the plasma volume, \(\mu_0\) is the vacuum permeability. The poloidal magnetic field can be used in the above definition, and in this case the parameter is called poloidal beta \(\beta_p\).
1.2.1 Tokamak and ITER

The configuration of tokamak is characterized by a relatively strong toroidal magnetic field $B_\phi$ and a much weaker poloidal magnetic field $B_\theta \ll B_\phi$ as shown in figure 1.4. The tokamak safety factor usually increases monotonically along the radial direction $r$, with the typical values greater or close to unity in the core, as represented in figure 1.9. For stability requirements, tokamak operation needs to have $q>1$, which is called Kruskal-Shafranov limit [11]. When a rational surface $q=1$ presents in the plasma, a strong internal kink instability can be induced. It usually appears as a strong sawtoothing dynamics, which causes severe confinement losses, and in some cases it can abruptly terminate the discharge. The Kruskal-Shafranov limit in fact sets a limitation on the maximum achievable plasma current and related Ohmic heating in tokamaks, so that auxiliary heating methods, e.g. neutral beam or radiofrequency heating, are needed to reach thermonuclear conditions.
Nuclear fusion and magnetic confined plasma

Figure 1.4: The sketch of tokamak configuration

The large experimental database obtained in the last decade in tokamaks and other toroidal configurations, and the improving capability of numerical simulations have provided the international community the physics basis for the design of a burning plasma experiment based on the tokamak concept, which is called International Thermonuclear Experimental Reactor (ITER) [12,13]. A schematic of ITER is reported in figure 1.5. On November 21, 2006, the seven participants (European Union, Japan, Russian Federation, People's Republic of China, South Korea, India and United States of America) formally agreed to fund the project, whose program is anticipated to last for 30 years - 10 years for construction and 20 years of operation. The designed parameters of the ITER machine are a major radius of the torus $R_\theta = 6.2\text{m}$, minor radius $a = 2\text{m}$, maximum magnetic field $B = 5.3\text{T}$ and plasma current $I_p = 15\text{MA}$. ITER is predicted to produce D-T burning plasma. The discharge duration is designed to be several hundred seconds, which can be regarded as a stationary condition on the time scales characteristic of the plasma processes. ITER is expected to achieve the ratio of fusion power to heating power $Q = Q_{\text{fus}} / Q_{\text{aux}} \geq 10$ and a nominal fusion power output of about 500MW. ITER is also designed to work in a steady-state operation with less power gain $Q \geq 5$, by using a large fraction of non-inductive current drive. The plasma of ITER will be with density $n = 10^{20}$ and core electron and ion temperatures of $T_e = 8.8keV$ and $T_i = 8keV$.

New physical regimes and a variety of technological issues will be explored with ITER. For example, conditions in which the $\alpha$ particles contribute
Nuclear Fusion and Magnetic Confined Plasma

significantly to the plasma pressure, with a class of plasma instabilities which can be studied in depth only with this new device. Again, a variety of technological issues could also be studied in ITER, e.g. the test of advanced materials facing very large heat and particle fluxes, the test of concepts for a tritium breeding module, and the superconducting technology under high neutron flux and many others. The auxiliary systems needed to achieve the conditions expected in ITER are an external heating and current drive capability of 73 MW and several advanced diagnostics for both analysis and plasma control. All of these requirements are expected to solve many of the scientific and engineering issues concerning the burning plasma and could allow to making a straightforward step towards the demonstration of a tokamak power plant.

Figure 1.5: Schematic of the International Thermonuclear Experimental Reactor (ITER)

1.2.2 Reversed Field Pinch (RFP) and RFX-mod experiments

Reversed field pinch (RFP) is different from tokamak. The magnitude of $B_\theta$ is comparable with $B_r$, as shown in figure 1.6. This implies that a simpler technology is required to generate the magnetic fields. The name RFP comes from the fact that the toroidal magnetic field reverses its sign near the plasma edge. Consequently, the safety factor at plasma edge also changes its sign at the same radius, as shown in figure 1.10. In RFP plasmas, two macroscopic parameters can be directly related to the equilibrium: the reversal parameter $F$ and the pinch parameter $\Theta$, which can be directly measured by experiments and defined as follows:
Nuclear fusion and magnetic confined plasma

\[ F = \frac{B_\phi(a)}{\langle B_\phi \rangle}, \quad \text{and} \quad \Theta = \frac{B_\phi(a)}{\langle B_\phi \rangle}, \]

where \( a \) is the minor radius of plasma, \( \langle \cdots \rangle \) denotes an average over the entire cross section.

The RFP has a low \( q \) profile device compared with tokamak. Low safety factor operation in the RFP does not lead to dangerous disruptive behavior. This is due to the magnetic relaxation coming with a magnetic reverse (which characterizes this configuration and has a strong stabilizing effect on the magnetic modes), of a conducting shell (which passively reacts against growing magnetic perturbations) and particularly in RFX-mod of a sophisticated feedback system for controlling plasma instabilities. For all the above reasons, the plasma current in the RFP is not limited by the Kruskal-Shafranov limit, as it happens in tokamaks. RFP can operate at plasma currents about ten times larger than in tokamaks with the same toroidal field, and it can reach \( \beta_p \approx 20\% \) through Ohmic heating only.

![Figure 1.6: The sketch of RFP configuration](image)

The RFX-mod experiment shown in figure 1.7 is the largest RFP machine in the world: it is based on a previously existent machine; after an accident, this machine has been modified so called RFX-mod \( (R_0=2\text{m}, a=0.459\text{m}) \) by replacing a thick passive aluminum shell with a thinner copper shell and by installing a set of 192 (4 poloidally \( \times \) 48 toroidally) feedback saddle coils with independent power supplies [14,15]. These active coils are used to control the plasma instabilities.
Nuclear Fusion and Magnetic Confined Plasma

They allow to mimic an ideally conducting wall by locally opposing the radial field (Virtual Shell, VS), or to actively control individual magnetohydrodynamic modes. The maximum toroidal bias magnetic field is \( B_\phi = 0.7T \) at the beginning of the discharge. The plasma current is 2MA.

![Figure 1.7: Picture of the RFX-mod Experiment](image)

1.3 Ideal magnetohydrodynamics and instabilities

1.3.1 Model of ideal magnetohydrodynamics

Ideal magnetohydrodynamics (MHD) is the most basic single-fluid model for determining the macroscopic equilibrium and stability properties of plasmas. The model describes how magnetic, inertial and pressure forces interact with an ideal perfectly conducting plasma in an arbitrary magnetic geometry. There is general consensus that a fusion reactor must satisfy the equilibrium and stability limits set by ideal MHD. If not, plasma disruption (catastrophic termination of plasma) is the usual consequence. The ideal MHD model is given by

Mass: \[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0
\]  \hspace{1cm} (1.13)

Momentum: \[
\rho \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p
\]  \hspace{1cm} (1.14)

Ideal Ohm’s law: \[
\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0
\]  \hspace{1cm} (1.15)
Maxwell:
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  
(1.16)
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]  
(1.17)
\[ \nabla \cdot \mathbf{B} = 0 \]  
(1.18)
Energy:
\[ \frac{d}{dt} \left( \frac{p}{\rho} \right) = 0 \]  
(1.19)

In these equations, the electromagnetic variables are the electric field \( \mathbf{E} \), the magnetic field \( \mathbf{B} \), and the current density \( \mathbf{J} \). The fluid variables are the mass density \( \rho \), the fluid velocity \( \mathbf{v} \), and the pressure \( p \), also, \( \Gamma=5/3 \) is the ratio of specific heats and \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \) is the convective derivative.

Since the main goal of ideal MHD is the investigation of macroscopic phenomena, the length scale of interest correspond to the macroscopic dimensions of the plasma denoted by \( L \sim a \) (\( a \) is MHD characteristic length). The typical time scale of MHD interest corresponds to \( \tau \sim a / V_{ir} \), \( V_{ir} = (2T_i / m_i)^{1/2} \), the ion thermal transit time across a macroscopic plasma dimension. This time scale is characteristic of many MHD plasma instabilities and represents the fastest time scale in which macroscopic plasma motion can occur. It should be noted that certain MHD waves and other phenomena can have time scales somewhat faster or slower than \( a / V_{ir} \). In determining the scaling relations, it is helpful to introduce the characteristic MHD frequency \( \omega \) and wave number \( k \) as follows:
\[ \omega \sim \frac{\partial}{\partial t} \sim \frac{V_{ir}}{a}, \]
\[ k \sim \nabla \sim \frac{1}{a}, \]
and, similarly, the resulting velocity
\[ \frac{\omega}{k} \sim v \sim V_{ir}. \]

In order to provide insight into which specific phenomena are not accurately described by ideal MHD, and perhaps more importantly, to indicate those phenomena that will be reliably treated even when the overall validity conditions are violated, ones define
\[ y = \frac{r_i}{a} \]
Nuclear Fusion and Magnetic Confined Plasma

\[
x = \left( \frac{m_i}{m_e} \right)^{1/2} \frac{V_T \tau_i}{a}
\]

where \( r_{gi} = \frac{V_T}{\omega_i} \) is the ion gyro radius, \( \tau_i \) is ion-ion collision time, \( m_i \) and \( m_e \) are ion and electron masses. There are three independent conditions which must be satisfied for ideal MHD to be valid. They are [11]

1. High collisionality \( x << 1 \)
2. Small gyro radius \( y << 1 \)
3. Small resistivity \( y^2 / x << 1 \)

It is known that the conditions of small gyro radius and small resistivity are well satisfied for plasmas of fusion interest. Note however, that the high collisionality assumption is never satisfied. This disconcerting conclusion is, however, inconsistent with the overwhelming empirical evidence demonstrating that ideal MHD provides a very accurate description of most macroscopic plasma behavior.

A more subtle examination of ideal MHD, one equation at a time, shows that the model is still reliable even though the collision-dominated assumption is not satisfied. Crucial to this argument is the claim that for problems involving MHD equilibrium and stability, the plasma motions of interest are incompressible. Specifically, the conservation of mass and the perpendicular momentum equation are shown to be valid in the collisionless limit [11,16]. However, both the parallel momentum equation and the energy equation are incorrect in this regime. Nevertheless, for incompressible motions neither of these equations plays an important role.

An analysis of Ohm’s law demonstrates that ideal MHD accurately calculates the inductive part of the electric field, but often incorrectly predicts the electrostatic part. The ideal MHD Ohm’s law produces a strong constraint on the allowable motions of the plasma because of the need to preserve field-line topology. This limits the types of instabilities (the tearing and reconnecting of magnetic field lines are forbidden [9]) that can develop. However, if an ideal MHD instability is excited, it is relatively robust since it does not depend on small subtle plasma physics effects. It is this robustness that makes ideal MHD modes so dangerous and has led to the consensus that these instabilities must be avoided in a fusion reactor.
1.3.2 Ideal MHD instabilities

Ideal MHD instabilities are briefly introduced and distinguished by the following three classification schemes [16].

**Internal and external modes:** If a well-confined plasma equilibrium separated from the first wall by a vacuum region exists, the distinction between internal and external instabilities is based on whether or not the surface of the plasma moves as the instability grows. For an internal mode the plasma surface remains fixed in place. These instabilities occur purely within the plasma and place constraints on the shape of the pressure and current profiles e.g. the saw tooth. Often they do not lead to catastrophic loss of plasma but can result in important experimental operational limits or enhanced transport. External modes, on the other hand, involve motion of the plasma surface, and hence the entire plasma. Since it is this motion that leads to a plasma striking the first wall as shown in figure 1.8, the external modes are particularly dangerous in a fusion plasma and must, in general, be avoided.

**Pressure-driven and current-driven modes:** The other way to classify plasma instabilities is by the driving source. In general, a plasma has both perpendicular and parallel currents and each can drive instabilities. The classification system for these instabilities is as follows. Since $\nabla p = J \times B$ in equilibrium, instabilities driven by perpendicular currents are often called “pressure-driven” modes. Actually, it is a combination of the pressure gradient and the field-line curvature that drives the instabilities. The curvature of the field lines can be favorable, unfavorable, or oscillate with respect to stability. The choice depends upon which way the radius of curvature vector points as compared to the direction of the pressure gradient. Instabilities driven primarily by the pressure gradient are usually further subclassified into one of two forms: the “interchange mode” or the “ballooning mode.” [11]

Instabilities driven by the parallel current are often called “current-driven” modes. These instabilities can exist even in the limit of low $\beta$, a regime where all pressure-driven modes are stable. In this regime, current-driven instabilities are
Nuclear Fusion and Magnetic Confined Plasma

often called “kink modes” because the plasma deforms into a kink like shape. Kink modes can be either internal or external. The external kink mode shown in figure 1.8 sets an important limit on the maximum toroidal current that can flow in a plasma.

In certain situations, the parallel current and pressure gradient (perpendicular current) combine to drive an instability. This is usually the most dangerous mode in a fusion plasma. It sets the strictest limits on the achievable pressure and current. Furthermore, it is an external mode, implying that violation of the stability boundary can lead to a rapid loss of plasma energy and plasma current to the first wall.

\[
\text{Ideal external kink}
\]

Figure 1.8: The sketch of external kink

Conducting wall vs. no wall configurations: The last classification scheme is based on whether a perfectly conducting wall is required or not. A close fitting perfectly conducting wall can greatly improve the stability of plasma against external kink modes. Since these modes set the strictest stability limits it would be highly desirable to avoid such modes by means of a perfectly conducting wall. The resulting gains in the $\beta$ and current limits due to wall stabilization are substantial, and may be mandatory for reactor viability in certain magnetic configurations.

However, a real experiment or reactor cannot maintain a superconducting wall (ideal wall) close to the plasma. The wall must be resistive, and this subjects the plasma to the resistive wall mode (RWM). Based on the simple mechanical analog, the presence of a resistive wall has no effect on the stability boundary of a plasma without a wall. In other words, while a perfectly conducting wall can raise the
Nuclear fusion and magnetic confined plasma

stability limit above that of the no-wall case, a resistive wall leaves the stability boundary unchanged and only reduces the growth rate.

The possibility of stabilizing the resistive wall mode by feedback, kinetic effect or other means is an area of active investigation. This can be a critical issue because certain configurations require a conducting wall even at $\beta = 0$ since they carry a large current e.g. RFP. Furthermore, the advanced tokamak scenarios such as ITER and future devices aiming at simultaneously maximizing the plasma pressure and operating in steady state require that all slowly evolving macroscopic MHD instabilities, particularly RWM, be stable.

1.4 Resistive Wall Mode in RFP and Tokamak

The resistive wall mode is normally understood as an ideal external kink mode, whose stability is significantly affected by the presence of conducting structures (resistive walls) surrounding the plasma. A highly conducting wall significantly reduces the growth rate of the ideal external kink mode, typically down to the inverse wall time $\tau_w$,

$$\gamma \sim 1/\tau_w.$$  

As well known, RWM sets a severe MHD limit for tokamak plasmas on the achievement of high $\beta$ values, and is therefore a critical issue for the advanced tokamak operational regime, aiming at maintaining the plasma in a high beta and high-bootstrap current fraction state. In Reversed Field Pinches (RFPs), the RWM instability appears as a potentially disruptive instability whenever the duration of the discharge is longer than the penetration time of the passive conducting structure (resistive wall). Thus, it is crucial to understand and control such a type of MHD instability for advanced fusion experimental devices and for all next step devices (ITER[12], DEMO[17] ...) which point toward to the steady state operation.

In order to introduce the basic physics of RMW, we adopt the cylindrical model for the following description. Therefore, one can assume that all the perturbations have the form

$$A(r,\theta,z,t) = A(r) \exp[-i\omega t + i(m\theta + kz)],$$
Nuclear Fusion and Magnetic Confined Plasma

\[ \omega = \omega_i + i \gamma \] where \( \omega_i \) is the mode frequency and \( \gamma \) denotes the mode growth rate, \( m \) is the poloidal wave number, \( k = n / R \) (\( n \) is the toroidal wave number, \( R \) is the major radius). For a given \((m,n)\) mode, its resonant surface corresponding to the safety factor \( q \) is located at the position where,

\[ q(r) = \frac{m}{n}. \]

In tokamaks: RWM is usually pressure driven with toroidal wave number \( n = 1 \); its \( \beta \) value should be in the range \( \beta_{\text{no-wall}} < \beta < \beta_{\text{ideal-wall}} \), which implies that, when an ideal wall is at infinity, the ideal external kink is stable when \( \beta \) is less than a critical value \( \beta_{\text{no-wall}} \), so that RWM is stable; if \( \beta \) is greater than a critical value \( \beta_{\text{ideal-wall}} \), the ideal external kink is unstable when an ideal wall is located at the resistive wall position. Due to the strong toroidal coupling effect in tokamak, many poloidal Fourier harmonics (different \( m \) wave number) with the same toroidal wave number \( n = 1 \) grow together; the resonant surface of these poloidal harmonics can have the resonant surfaces inside/outside plasma as shown figure 1.9.

![Figure 1.9: q profile of tokamak the poloidal Fourier harmonics of n=1 mode with resonant surfaces inside and outside plasma are presented.](image)

In RFP configuration: It is interesting to note that apart from sharing similar behaviours in both RFP and Tokamak configurations, RWMs in RFP have also
some specific characteristics which suggest independent studies. For instance, the RWMs in RFP plasmas are current driven instabilities; while in Tokamak the RWMs are normally driven by the plasma pressure. Furthermore, in RFPs the external kink instabilities, having their rational surfaces outside the plasma, are separated in two types, one is so called externally non-resonant modes (ENRM), which have their rational surfaces located at $q < q(a) < 0$ ($q(a)$ is the safety factor at the plasma edge), another is internally non-resonant modes (INRM) with rational surfaces corresponding to $q > q(0) > 0$ [10]. Furthermore, the RFP configuration has the characteristic of a strong poloidal magnetic field $B_\theta$, which is of the same order as the toroidal field $B_\phi$. Therefore, the poloidal asymmetry of the equilibrium magnetic field is much weaker than in a tokamak, which leads to weaker toroidal coupling effects.

Figure 1.10: Typical $q$ profile of RFP while two groups of RWMs (INRMs and ENRMs) are pointed out, and the modes with resonant surfaces inside plasma are also presented, which usually appears as the tearing modes.
In Reversed Field Pinches (RFPs), the RWM instability appears as potentially disruptive instability whenever the duration of the discharge is longer than the penetration time of the passive conducting structure (resistive wall). Apart from the first observation of RWM reported in HTBX1C in 1988 [18], numbers of studies have been reported in recent years, both experimentally and theoretically [19-25]; in particular, the successful feedback control of multiple unstable RWMs has shown to be achieved by the equipments implemented in the experiments [23-25]. For instance, in RFX-mod [26] experiments, the observations give an evidence that the system of the active control has a sufficient efficiency and a large flexibility [24], therefore can provide an excellent environment to apply and/or to test RWM physical modeling and various control scenarios [27], which can further provide a better understanding for RWM physics and the control strategy.

In this chapter, we study RWM instabilities using a (periodic) cylindrical model of MHD theory, in which the effects of the plasma pressure, compressibility, plasma inertia, longitudinal rotation and parallel viscosity (tensor) have been taken into account. The resistive wall is modeled with a finite thickness, which allows to treat a large equilibrium flow in the plasma and $\omega \tau_b >> 1$ ($\omega$ is the mode frequency and $\tau_b$ is the wall penetration time scale length). The model has been proposed in the work of 1999 [22], and used to the study focusing on the ENRMs. However there was no experimental result presented in existed RFP devices at that time, thus direct comparison with experiments was impossible. Recent experiments on RFPs provide very precise measurements on the growth rates of RWMs, which
Comparison between cylindrical model and experimental observation
give excellent bases to validate the theoretical models. Furthermore, the
observation shows that INRMs, having larger growth rates, are more important
than ENRMs in the current operations. In section 2.4, we make careful
comparisons with experimental results in RFX-mod on the mode instability
spectrum and the mode growth rates by well matching the equilibrium parameters
F, Θ, and β_p. In section 2.5, the sensitivity of the growth rate to the equilibrium
parameters is also investigated in detail; the corresponding physical understanding
is also provided by analysis based on the balance of the potential energy
components in chapter 3. Very good agreements are obtained between the theory
and experiments on both instability spectrum and growth rates, which give
confidence that cylindrical model is good enough for further studies on RWMs in
RFP plasmas.

2.1 Cylindrical model of Ideal MHD

2.1.1 Ideal MHD equations

We start with the MHD equations for the density ρ, fluid velocity v, magnetic field
B and pressure p.

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \]  \hspace{1cm} \text{(2.1)}

\[ \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla p + j \times B - \nabla \cdot \Pi \]  \hspace{1cm} \text{(2.2)}

\[ \frac{d}{dt} \left( \rho - \Gamma p \right) = 0 \]  \hspace{1cm} \text{(2.3)}

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B) \]  \hspace{1cm} \text{(2.4)}

where \( \frac{d}{dt} = \partial / \partial t + v \cdot \nabla \), \( \Gamma \) is the ratio of specific heats; and \( \Pi \) is the viscous stress
tensor, in which only the part related to the parallel viscosity (with coefficient \( \eta_o \))
is taken:

\[ \Pi = -3\eta_o (e||e|| - \frac{1}{3} I)s \]  \hspace{1cm} \text{(2.5)}
Comparison between cylindrical model and experimental observation

\[ s = e_\parallel \cdot \nabla (v \cdot e_\parallel) - v \cdot (e_\parallel \cdot \nabla e_\parallel) - \frac{1}{3} \nabla \cdot v , \text{ where } \tilde{e}_\parallel = \frac{B}{B} \]

The perturbed displacement has the form \( \xi = \xi_1(\sigma) \exp[-i\omega + i(m\theta + k_z)] \), \( \omega = \omega_r + i \gamma \) where \( \omega_r \) is the mode frequency and \( \gamma \) denotes the mode growth rate, \( m \) is the poloidal wave number, \( k = n/R \) (\( n \) is the toroidal wave number, \( R \) is the major radius). The velocity is written as \( v = v_0 + v_1 \), \( v_0 = v_{0z}(r)e_z \), and \( v_1 = \frac{d\xi}{dt} = -i(\omega - k \cdot v_0)\xi = -i\omega \xi \).

### 2.1.2 Eigenmode equation

From the linearized MHD equations 2.1-2.4, the following eigenmode equation can be obtained [22], written as

\[
\frac{d}{dr} \left( A(\sigma) \frac{d\sigma}{dr} \right) - C(\sigma)\sigma = 0
\]  \tag{2.6}

where \( \sigma = r\xi_r \),

\[
A = \frac{\rho}{r} \frac{1}{D + i\mu} \left( \sigma^2 - \omega_a^2 \right) \left[ (v_s^2 + v_A^2)(\omega^2 - \omega_h^2) - i \frac{\omega \eta_c E}{\rho} \right]
\]

\[
C = -\frac{\rho}{r} (\sigma^2 - \omega_a^2) + \frac{1}{D + i\mu} \left[ \frac{4k^2v_A^2B_0^2}{\mu_0 r^3} (\omega^2 - \omega_h^2) \right.
\]

\[
+ \left\{ \frac{4kB_0^3}{\mu_0 r^3 B^2} (\omega^2 - 3\omega_h^2) + \frac{4k^2B_0^2}{\mu_0 r^3} \frac{4}{3} \omega_a^2 - \frac{3B_0^4}{\mu_0 r^3 B^2} \frac{D}{v_A^2} \right\}
\]

\[
+ \frac{d}{dr} \left( \frac{B_0^2}{\mu_0 r^2} - \frac{1}{D + i\mu} \left( \frac{2kB_0^2}{\mu_0 r^2} \left( (v_A^2 + v_s^2)(\omega^2 - \omega_h^2) - i \frac{\omega \eta_c E}{\rho} \right) \right. \right.
\]

\[
+ \frac{i}{\rho} \frac{\omega \eta_c}{\mu_0 r^2 v_A^2} \left( \omega^2 - \omega_a^2 \right) \left( \omega^2 - 3\omega_h^2 \right) \right\}
\]

\[
D = \omega^4 - k_o (v_s^2 + v_A^2)\omega^2 + v_s^2 \omega_h^2 k_o^2
\]
Comparison between cylindrical model and experimental observation

\[
\mu = \frac{\bar{\omega} \eta_0}{\rho} (k_0^2 E + \frac{\bar{\omega}_a^2}{\nu_A^2} + 3 \frac{\bar{\omega}_a^4}{\nu_A^4} v_s^2),
\]

\[
E = \frac{\bar{\omega}^2}{3} - \frac{4}{3} \bar{\omega}_a^2 - \frac{3v_s^2}{\nu_A^2} \bar{\omega}_a^2, \quad \bar{\omega}_a^2 = \frac{F_B}{\mu_0 \rho},
\]

\[
\omega_n^2 = \frac{v_s^2}{\nu_A^2} \omega_s^2, \quad \omega_n^2 = \frac{v_s^2}{\nu_A^2} \omega_a^2, \quad F_B = kB_z + \frac{m}{r} B_0,
\]

\[
G = \frac{m}{r} B_z - kB_0, \quad k_0^2 = k^2 + \frac{m^2}{r^2},
\]

\[
v_A^2 = \frac{B^2}{\mu_0 \rho}, \quad v_s^2 = \frac{\gamma \rho}{\rho}.
\]

The corresponding boundary condition is obtained by integrating equation 2.6 over a thin layer across the plasma boundary and then taking the limit for the layer width to zero [22]. It is written as

\[
a B_0 \frac{a^2 E_0^2}{|k|a} = \frac{B^2}{D + i \mu} (\bar{\omega}^2 - \omega_s^2)(\bar{\omega}^2 - i \frac{\bar{\omega} \eta_0}{\rho v_s^2} E) \frac{a d \phi}{\mathcal{V} dr} - 2B_0^2
\]

\[
+ \frac{1}{D + i \mu} [2kB_0 G(\bar{\omega}^2 v_A^2 - i \frac{\bar{\omega} \eta_0}{\rho} E) + i \frac{\bar{\omega} \eta_0}{\rho} \frac{B_0^2}{\nu_A^2} (\bar{\omega}^2 - \omega_s^2)]
\]

where

\[
\Xi = \frac{K_a - (\frac{K_b'}{I_b'}) H_w I_a}{K_a' - (\frac{K_b'}{I_b'}) H_w I_a'}, \quad H_w = \frac{1}{1 - \frac{i k_0^2}{\omega \tau_b} \frac{\varsigma}{k^2 K_b' I_b' \tanh \varsigma}}, \quad \tau_b = \mu_0 \sigma bh,
\]

\[
\varsigma = [-i \omega \tau_b (h/b)]^{1/2}, \quad k_0^2 = k^2 + \frac{m^2}{b^2}
\]

where \( \sigma \) is the conductivity of the resistive shell and \( b, h \) are the shell minor radius and thickness respectively. \( K_i = K_m(|k|r) \) and \( I_i = I_m(|k|r) \) are the modified Bessel functions; the prime denotes the derivative of the argument. Equation 2.6 with the boundary condition of equation 2.7, is the eigenmode equation for the RWM used
Comparison between cylindrical model and experimental observation

in our study. Without the viscosity and the plasma flow, this equation is readily reduced to the commonly used MHD eigenmode equation including plasma compressibility [11,16].

2.2 RFP equilibrium

The commonly used modeling for RFP equilibrium [10] in cylindrical geometry is expressed by the following equations:

$$\nabla \times \vec{B}_0 = \mu_0 (r) \vec{B}_0 + \frac{\mu_0 \vec{B}_0 \times \nabla p}{\vec{B}_0^2}$$  \hspace{1cm} (2.8)

which can be decomposed into poloidal and longitudinal components as follows:

$$\frac{dB_z}{dr} = -\mu B_0 - \frac{\mu_0 B_z \frac{dp}{dr}}{B_0^2}$$  \hspace{1cm} (2.9)

$$\frac{1}{r} \frac{d}{dr} (rB_0) = \mu B_z - \frac{\mu_0 B_0 \frac{dp}{dr}}{B_0^2}$$  \hspace{1cm} (2.10)

where $B_0^2 = B_z^2 + B_0^2$. The $\mu(r)$ profile is based on the so-called “$\mu$–p” model which is commonly used for the data analysis in the experiments of RFX-mod. According to the model,

$$\mu(r) = \frac{2}{a} \Theta_0 [1 - \left(\frac{r}{a}\right)^\alpha]$$  \hspace{1cm} (2.11)

Note that $\Theta_0$ is related to the on-axis safety factor $q(0)=a/(R\Theta_o)$. The pressure profile $p(r)$ is adopted as the same as [22], where

$$\frac{dp}{dr} = -\chi \frac{r}{2\mu_0} \left(\frac{\mu B_0^2}{2B_0} - \frac{B_z^2}{r}\right)$$  \hspace{1cm} (2.12)

which gives Suydam’s necessary condition for stability when $\chi<1$; and the poloidal beta $\beta_p$ is defined as

$$\beta_p = \frac{2\mu_0}{B_0(a)^2} \frac{1}{\pi a^2} \int_0^a p(r) 2\pi r dr$$  \hspace{1cm} (2.13)
Comparison between cylindrical model and experimental observation

We would like to note that in the case without pressure, for a given $\mu(r)$ profile (by fixing $\alpha$ and $\Theta_o$) the reversal parameter $F$ and pinch parameter $\Theta$ introduced in chapter 1 are uniquely determined, which implies that the two parameter groups ($\alpha$, $\Theta_o$) and ($F$, $\Theta$) have one to one correspondence. Furthermore, giving any two among the four parameters (e.g. giving $\Theta_o$ and $F$), other two ($\alpha$ and $\Theta$) will be fixed too. When taking into account the plasma pressure, the parameters ($\alpha$, $\Theta_o$, $\chi$) will uniquely determine the parameters ($F$, $\Theta$, $\beta_p$). In other words, when the values of equilibrium parameters $\alpha$, $\Theta_o$ and $\chi$ are fixed, these equilibrium profiles can be determined. Since $\beta_p$ is a global parameter, different shapes of the pressure profiles may lead to the same value of $\beta_p$. The dependence on the shape of the pressure profiles will be discussed in next section.

In figure 2.1, the typical profiles of the magnetic fields $B_z$ and $B_\theta$, safety factor $q(r)$, and $\mu(r)$, as well as parallel and perpendicular currents densities are plotted for $F=-0.3$, $\Theta=1.57$ and $\beta_p=0.03$.

![Figure 2.1: Equilibrium profiles of Bz, B_θ, and q, in (a) and μ, J_∥ and J_⊥ in (b) for F=-0.3, \Theta=1.57 and \beta_p=0.03 are plotted against r/a. The value of J_⊥ in (b) are amplified 20 times.](image)

2.3 RWM instability spectrum in RFP plasma

The normalized eigenmode equation 2.6 [22] has been solved numerically under the boundary condition equation 2.7. The corresponding numerical code named CMR (Cylindrical Magnetohydrodynamic with Resistive wall code) is developed and applied to our study. The growth rates for both INRM and ENRM are computed for the parameters of RFX-mod, where $\sigma=5.88\times10^7$ $\Omega/$m, $b=0.514$m,
Comparison between cylindrical model and experimental observation

h=3×10^{-3}m, thus the wall penetration time \( \tau_b = 114 \text{ ms} \); the plasma minor radius is \( a=0.459\text{m} \), major radius \( R=2.0\text{m} \), and \( q(0) \) is in the range of \( 1/6>q(0)>1/7 \). In present RFX-mod, the rotation velocity \( v_0 \) measured at core plasma is around 10 km/s , which, for 1MA(or 600kA) discharge with plasma density of \( n\sim 4\times 10^{19}\text{m}^{-3} \) and ion temperature of \( T_i \sim 600-700\text{ev} \), corresponds to \( v_0/v_{\text{th}} \sim 1-1.5\% \). By considering the classical parallel viscosity [28] calculated for RFX-mod plasmas, normalized \( \eta_o \) is in the range around 0.01 or less. We found that the plasma rotation with viscosity in this range actually has no influence on the mode growth rate neither on the mode frequencies. The obtained mode frequencies for RFX-mod parameters always remain to zero. Therefore, in this work, we concentrate the study on the effects of equilibrium parameters \( F, \Theta, \) and \( \beta_p \), making comparison between the theory and the observation in RFX-mod experiments. As for the effect of the plasma rotation, we will discuss in detail elsewhere.

In figure 2.2, the growth rate of \((m=1)\) RWM instabilities in RFP for different parameter \( F \) are plotted by using the parameters of RFX-mod. All unstable RWMs are included. In RFP plasmas, for poloidal mode number \( m=1 \), there are many \( n \) modes unstable for a given equilibrium; as often used in RFP studies [19,21], we define INRMs with \( n<0 \) while ENRMs with \( n>0 \) in CMR , unless otherwise stated. Since the unstable mode spectrum is limited in a certain interval of \( k \) value (\( k \) is the normalized toroidal wave number, \( k=n\epsilon, \epsilon=a/R \)), a RFP machine with smaller aspect ratio \( \epsilon \) has more \( n \) modes being unstable. Figure 2.2 shows that deeper reversal (more negative \( F \)) causes more unstable ENRMs and less unstable INRMs; while shallow reversal (less negative \( F \)) leads to larger growth rates of INRMs and smaller growth rates for ENRMs. Among the INRMs the \( m=1 n=-6 \) mode is most unstable mode; as for ENRMs, \( n=3 \) has largest growth rate. All the characteristics mentioned above fully agree with the experimental observation and previous numerical calculations [29,30,40].

The plasma pressure enhances the instability of Current driven RWM in RFPs. Figure 2.3 shows the mode growth rates increase with \( \beta_p \) value for both INRMs and ENRMs.
Comparison between cylindrical model and experimental observation

Figure 2.2: For fixed $\Theta_0=1.5$ and $\beta_p=0.0$, the growth rates of RWM for various $F$ values (-0.05,-0.1,-0.2,-0.5,-1.0) with RFX-mod parameters ($a=0.459\text{m}$, $b=0.514\text{m}$, $h=3 \times 10^{-3} \text{m}$, $\sigma=5.88 \times 10^7 \Omega/\text{m}$, $\tau_b=114 \text{ms}$, $\varepsilon=a/R=0.23$) are calculated numerically.

Figure 2.3: The growth rates of INRMs($F=-0.05$) and ENRMs($F=-0.2$) plotted as function of wave number $n$ for different $\beta_p$ value (0, 0.05, 0.1, 0.2), when fixing $\Theta_0=1.5$. The other parameters of RFX-mod are the same as figure 2.2. For INRMs and ENRMs, the values of growth rates correspond to left and right vertical axes respectively.

In figure 2.4, the eigenfunctions of both the perturbed radial magnetic field $b_r$ and the plasma displacement $\xi_r$ are plotted. In the plasma $b_r = iF_b \xi_r$. It shows that for the external modes (ENRMs) the displacements are almost constant along the whole minor radius; while for internal modes (INRMs) $\xi_r$ decreases toward the plasma edge. The plot of corresponding $b_r$ is extended to the resistive wall and the vacuum regions outside the plasma and the wall. It shows that the value of $b_r$ decreases about 3% when penetrating the wall. This small change of $b_r$ when crossing the wall implies the small discrepancy of the results between thick shell
Comparison between cylindrical model and experimental observation

and the thin shell approximations (in “thin shell”, $b_r$ is constant inside the shell). Therefore, for the low level plasma rotation as observed in RFX-mod, the thin shell approximation for RWM is acceptable within the above tolerance.

![Figure 2.4: (a) The eigenfunctions of the plasma displacement $\xi_r$ of ENRM ($n=3$) and INRM ($n=-5$) vs. $r/a$ are plotted. (b) the perturbed radial magnetic field $b_r$ is shown as the function of $r/a$, where $b_r = iF\xi_r$ in the plasma, and the small picture shows the change of $b_r$ inside the resistive wall.](image)

2.4 Comparison with experimental results of RFX-mod

Since the growth rates of the RWMs are comparable to the resistive diffusion time of the metal structures (resistive wall) surrounding the plasma, its exponential growing can be easily observed in experiments. Furthermore, RFX-mod has a flexible feedback system, which can suppress the mode or leave it growing in any desired time interval of the operation. Therefore the growth rate of RWMs can be
Comparison between cylindrical model and experimental observation

precisely measured in a certain time interval. Figure 2.5 shows a typical experimental observation on the RWM in RFX-mod, where the mode growth rates are precisely measured. In figure 2.5(a), the radial magnetic field amplitude related to the most unstable RWM as measured by an array of saddle sensors located between the vacuum vessel and the resistive shell, is presented. For the same mode, in figure 2.5(b) an example of experimental determination of the growth rate is shown: once calculated the logarithm of the mode amplitude, an automatic procedure selects the time interval where a linear interpolation with constant parameters best fits; the same time interval is then recorded and used to average some reference plasma parameters. That time interval is used for example to give the experimental measurement of F and Θ in the comparison. This kind of operation provides an excellent base for the comparison between the experiments and the theoretical modeling.

Figure 2.5: Amplitude (a) and logarithmic amplitude (b) for the mode m=1,n=-6 in shot 18542. The discharge ends at t=0.15s, a long pure exponential growth (linear in (b)) is evident approximately from 0.05s to 0.10s, of which the growth rate is 24.0s⁻¹.
Comparison between cylindrical model and experimental observation

For current operation in RFX-mod, the magnetic reversal is often shallow, which means $|F|$ is small. Therefore, as mentioned before, INRMs appear as the most important unstable and well measured modes. We will concentrate the comparison with experiments on the INRMs.

The equilibrium profiles of the magnetic fields in RFP plasmas currently cannot be directly measured yet. However, the global parameters $F$ and $\Theta$ can be obtained by the direct measurement (using very reliable magnetic flux and field sensor), the parameter $\beta_p$ can also be provided as well. In addition, one has to consider that the experimental determination of $F$ and $\Theta$ is done by averaging many consecutive time samples (the minimum being of the order of a hundred) and that the automatic procedure discards all the cases where the time behavior of $F$ or $\Theta$ is not constant enough during the selected RWM growth phase. In this way experimentally, for a given time interval, a constant RWM growth rate is associated to average values of $F$ and $\Theta$ with a very small (below 1%) statistical deviation. Since the mode growth rates actually are rather sensitive to the global parameters $F$, $\Theta$ and $\beta_p$ (the detail of discussion on this point will be made in the next subsection), for the purpose of good comparison, the values of $F$, $\Theta$, $\beta_p$ calculated by the model should well match that given by experiments for every shot. Consequently, the mapping from $F$, $\Theta$, and $\beta_p$ to $\alpha$, $\Theta_o$, and $\chi$ should be created; the properties of mapping are nonlinear and one to one correspondence. If the values of $F$, $\Theta$, $\beta_p$ are given, the values of $\alpha$, $\Theta_o$, $\chi$ are uniquely determined. Owing to the nonlinear property, instead of finding continuous mapping, we have to enumerate the values of $\alpha$, $\Theta_o$, $\chi$ exhaustively and compute the corresponding values of $F$, $\Theta$, $\beta_p$ then create the matrix including $F$, $\Theta$, $\beta_p$ and $\alpha$, $\Theta_o$, $\chi$ as a discrete mapping. By searching the experimentally measured values of $F$, $\Theta$, $\beta_p$ in the matrix, the corresponding values of $\alpha$, $\Theta_o$, $\chi$ for the equilibrium model can be obtained. In figure 2.6, the parameter region covered by this equilibrium model calculated by the precreated matrix is plotted, and the experimental parameter range is also marked on the region.
Comparison between cylindrical model and experimental observation

Figure 2.6: The range of F and $\Theta$ is plotted by the calculation from the precalculated matrix according to the model equation 2.9-2.12 (dots area) the experimental measurements (circle), which are from the database for m=1,n=-5 and m=1,n=-6 modes, are also marked on the figure. The corresponding value of $\beta_p$ is in the range from 0.01 to 0.06.

By employing the matrix, a careful comparison are carried out with the parameters of RFX-mod machine ($a/R=0.2295$ and $b/a=1.12$). Seven typical shots with various wave numbers $n$ are taken from the experimental database for the comparison. Since the magnetic and pressure profiles vary in time, the growth rates of seven shots are measured in time intervals which lie in the RWMs’ linear phase. Table 2.1 reports the comparison of the results produced by well matching the values of F, $\Theta$ and $\beta_p$. The maximum discrepancies of F and $\Theta$ between experimental measurement and theoretical calculation are less than 1.4% and 0.8% respectively, and for $\beta_p$ is 2.8%.

Figure 2.7 reports the comparison of the growth rates for the seven shots corresponding to the table 2.1. Obviously growth rates predicted by theory are in good agreements with the experimental measurements. This implies that the cylindrical assumption is reasonable in RFP.
Comparison between cylindrical model and experimental observation

Figure 2.7: Plot of careful comparison of the growth rates between experiments and theoretical model with RFX-mod parameters for seven shots. The letters correspond to the shots in table 2.1. The growth rates measured in the experiments are marked as the letters with outline. The characters without outline represent the growth rates resulting from theoretical calculation.

Table 2.1: The comparison of the RWMs’ growth rates between the experiments and the theoretical calculation for very well matched values of $F$, $\Theta$ and $\beta_p$. For each shot, the columns of “Exp.” are the experimental measurements. The columns marked by “Theory” are calculated from the model. The last row is the comparison of the growth rates.

<table>
<thead>
<tr>
<th>Shot No.</th>
<th>18544(A)</th>
<th>18621(B)</th>
<th>18556(C)</th>
<th>20227(D)</th>
<th>25467(E)</th>
<th>17275(F)</th>
<th>22259(G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>-0.046</td>
<td>-0.0464</td>
<td>-0.017</td>
<td>-0.073</td>
<td>-0.0509</td>
<td>-0.047</td>
<td>-0.23</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.016</td>
<td>0.0161</td>
<td>0.015</td>
<td>0.0153</td>
<td>0.018</td>
<td>0.0332</td>
<td>0.021</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>1.4</td>
<td>1.396</td>
<td>1.36</td>
<td>1.44</td>
<td>1.452</td>
<td>1.43</td>
<td>1.41</td>
</tr>
<tr>
<td>$n$</td>
<td>-4</td>
<td>-4</td>
<td>-4</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>5.6</td>
<td>6.7</td>
<td>4.2</td>
<td>5.2</td>
<td>4.9</td>
<td>4.6</td>
<td>5.2</td>
</tr>
<tr>
<td>$\Theta_0$</td>
<td>1.45</td>
<td>1.4</td>
<td>1.54</td>
<td>1.47</td>
<td>1.485</td>
<td>1.56</td>
<td>1.48</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.09</td>
<td>0.085</td>
<td>0.11</td>
<td>0.19</td>
<td>0.12</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Comparison between cylindrical model and experimental observation

2.5 The sensitivity of the RWM instability to the equilibrium parameters

In the previous section, it has been shown that the growth rates of RWMs are sensitive to the changing of the reversal parameter \( F \), especially for the most unstable mode. In this section the dependence of the mode growth rates on the parameter \( \Theta \) and \( \beta_p \) as well as profiles of the pressure and \( \mu \) will be investigated.

In the study of the sensitivity to the pinch parameter \( \Theta \), the assumption of zero pressure is taken. Three values of \( F \) are considered; for each reversal depth, two different values of \( \Theta \) are chosen for comparing. Figure 2.8 shows the growth rates calculated as a function of wave number \( n \) for two different \( \Theta \) values. For INRMs, particularly \( n=-6 \) mode, the growth rates are very sensitive to the changing of \( \Theta \). When the magnetic reversal is shallow, \( F=-0.05 \), even the increment of \( \Theta \) is as small as 2.8\%, the discrepancy of growth rates almost reaches 50\%. Instead, ENRMs are less sensitive than INRMs for change in \( \Theta \). Among the unstable ENRMs, the \( m=1, n=3 \) mode is most unstable. When \( F=-0.3 \), the maximum growth rate is about 6s\(^{-1}\), which is still much smaller than that of INRMs. For (1,3) mode, when increasing the value of \( \Theta \) from 1.52 to 1.59 (about 4.5\%) and fixing \( F=-0.3 \), its growth rate raises about 7\%. Additionally, figure 2.8 also indicates that the discrepancy of growth rates decreased while the wave number \( n \) goes toward to zero.

![Figure 2.8: The growth rates vs. mode number \( n \) is plotted for different \( F \) and \( \Theta \). The sensitivity of growth rates to pinch parameter \( \Theta \) can be compared for fixed \( F \). The solid, hollow and the plus ‘+’ symbols denote the growth rate of different magnetic reversal](image)

Figure 2.8: The growth rates vs. mode number \( n \) is plotted for different \( F \) and \( \Theta \). The sensitivity of growth rates to pinch parameter \( \Theta \) can be compared for fixed \( F \). The solid, hollow and the plus ‘+’ symbols denote the growth rate of different magnetic reversal
depth $F=-0.05$, -0.15 and -0.3 for comparison. The diamond and circle symbols show two different values of $\Theta$, but corresponds to the same $F$.

As for the study on the impact of $\beta_p$ value, we notice that the change of $\beta_p$ value may influence the shape of the current profile. Therefore, the effects of the two driving mechanisms, pressure driven and current driven, usually cannot independently vary with $\beta_p$. The above study on figure 2.3 is the selected case of fixing the values of $\Theta_o$ and $F$ ($\Theta$ varies with $\beta_p$), in which the current profile has only a slight change when $\beta_p$ value increases from 0 to 0.1; thus increasing $\beta_p$ value mainly enhances the pressure driven mechanism. Figure 2.3 has shown that the plasma pressure, as an additional driving mechanism, can significantly enhance the instabilities. However, when fixing the values of $F$ and $\Theta$ and changing $\beta_p$, the investigation shows that the mode growth rates of RWMs are less sensitive to $\beta_p$ than the case in figure 2.3. The reason is when increasing $\beta_p$ value, the diamagnetic current density ($j_\perp$) increases on the edge of the plasma, which makes the parallel current density decreasing in the plasma center and thus the profile less peaked (the total toroidal plasma current is unchanged). This leads to a reduction in the current driven mechanism of the RWMs. In other words, in this case, increasing $\beta_p$ value enhances the pressure driven mechanism but reduces the current driven force due to modifying the parallel current density profile. The mode growth rate comes from the balance of these two effects. An example is for (1,-6) mode and fixed $F=-0.05$ and $\Theta=1.45$; while raising $\beta_p$ from 0 to 0.05, the mode growth rate is even decreased from 31.28 S$^{-1}$ to 29.01 S$^{-1}$ (about 7%) due to the reduction of the current driven effect. When $\beta_p$ continue to increase up to 0.1, due to the further increasing of the pressure driven effect, the growth rate increases again, becomes 31.11 S$^{-1}$. In summary of all cases we have studied, it is found that for given RFP parameters $F$ and $\Theta$, the RWM growth rates are less sensitive to $\beta_p$ value than to $F$ and $\Theta$, although the pressure is an additional driven mechanism.

The impact of the shape of pressure profiles on the RWM growth rate is also less sensitive, although the same ($\beta_p$, $F$, $\Theta$) value may correspond to different pressure profiles. For comparison, two types of pressure profiles are considered. One is the Suydam’s necessary condition of equation 2.12, the other is defined as $p=n(r)T(r)$, where $n(r)$ is the plasma density and $T(r)$ is the plasma temperature. The $n(r)$ and $T(r)$ are presented as $n(r)=n(0)(1-A_0 r^{B_0} + C_0 r^{D_0})$ and
Comparison between cylindrical model and experimental observation

\[ T(r) = T(0) \left( 1 - A_n r^{-n} + C_n r^0 \right) \]

The parameters of \( B_n, B_t \) and \( D_n, D_t \) usually are obtained by fitting the experimental data. Here we take one group as an example of \( A_n=7.0, B_n=10.0, C_n=6.0, D_n=8.0, A_t=0.9, B_t=4.0 \) and \( D_t=1.0 \).

Note that in the present RFX-mod, the plasmas have \( \beta_p \) value in the range of 0.02-0.04. In such a \( \beta_p \) range, analysis indicates that the pressure profiles modeled by equation 2.12 always satisfy the Suydam stability condition \( \chi < 1 \). Searching in wide range of the reversal parameter \( F \) (from 0 to -3.0), we found that the violation of the Suydam stability criterion may occur only when \( \beta_p \) exceeds a certain value \( \beta_{p_{\text{min}}} \). For \( F=0.008 \), \( \beta_{p_{\text{min}}} \sim 0.127 \); deeper reversal requires even higher \( \beta_{p_{\text{min}}} \). As for the experimental fitted profile, the calculation by using equation 2.12 shows that this type of profile satisfies the Suydam stability criterion for most plasma region (\( \chi(r) \) is much smaller than one), only in very edge of the plasma (outside the field reversal point) \( \chi(r) \) may larger than one, which implies the possible appearance of the resistive interchange instabilities[31]. This subject, however, is beyond the scope of present study on RWMs.

Figure 2.9 presents two different shapes of the pressure profiles, both lead to the same equilibrium parameters \( F=0.05, \Theta=1.41 \) and \( \beta_p=0.02 \). In the calculation, the most unstable \((m,n)=(1,-6)\) mode is considered for above two type of the pressure profiles and compared with the experimental data. As for the selection of parameters \( F \) and \( \Theta \), since there is a strong \( F-\Theta \) correlation observed experimentally as expressed by the hollow points in figure 2.10, the parameters \( F, \Theta \) are picked up from the fitted \( F-\Theta \) curve; and \( \beta_p \) is chosen as \( \beta_p=0.02 \). The result is presented in figure 2.11, which shows two pressure profiles with the same global values of \( F, \Theta \) and \( \beta_p \) lead to about 10% discrepancy. For other modes less unstable than \((1, -6)\) mode, the difference is even less. This comparison also reconfirms the tendency that the INRMs’ growth rates become smaller when the magnetic reversal goes towards deeper configurations.
Comparison between cylindrical model and experimental observation

Figure 2.9: Plot of two pressure profiles for the same equilibrium parameters $F=-0.05$, $\Theta=1.41$ and $\beta_p=0.02$, where the solid line is calculated by the model of equation 2.12, the dot line is the result from $p=n(r)T(r)$.

Figure 2.10: Plot of $\Theta$ as function of $F$ by fitting experimental data from the database ($m=1$, $n=-6$) which are marked by hollow circles. The solid squares and dots fitting to the curve are calculated by the model with Suydam’s necessary condition equation 2.12 and $p(r)=n(r)T(r)$ respectively.

In both figure 2.7 and figure 2.11, a slight underestimation of the growth rate from the theory is noticed when $n=-6$ where $|F|$ is small. The cause might be the toroidal effect or the external error field produced by more realistic wall condition, which are not presently included in the model, and/or the real plasma equilibrium profiles are not exactly described by our equilibrium model. However, the above investigation firmly shows that the present model is good enough for describing the RWMs in RFP plasmas.
Comparison between cylindrical model and experimental observation

At the end, apart from the above analysis of sensitivity of $F$, $\Theta$ and pressure, we would like to point out the influence of the shape of $\mu$ profile on the growth rate of RWMs. For this purpose we extend the equation 2.11 of “$\mu$-p” model as follows:

$$
\mu(r) = \frac{2}{a} \Theta_o \left\{ 1 - \left( \frac{r}{a} \right)^{\alpha_1} + \alpha_2 \left[ \left( \frac{r}{a} \right)^{\alpha_1} - \left( \frac{r}{a} \right)^{\alpha_2} \right] \right\}
$$

When $\alpha_2=0$, equation 2.14 is reduced to equation 2.11. The zero pressure assumption is taken as well in this study. By fixing $F=-0.1$ and $\Theta=1.44$, $\beta_p=0.0$, we find two $\mu$ profiles shown in figure 2.12, one is monotonically decreasing and the other is hollow with peaked value located around $r/a=0.4$. In this instance, when the $\mu$ profile becomes hollow, the growth rate of INRM $m=1$, $n=-6$ decreases about 20% from $14.8s^{-1}$ to $11.8s^{-1}$, as for ENRM $m=1$, $n=3$, its growth rate reduces from $1.67s^{-1}$ to $1.47s^{-1}$ varying about 11.5%. This indicates the noticeable influence of $\mu$-profile on the instability. In the current operated RFPs, the measurement of $\mu$ profile is still unavailable; we leave this study in the future works.

![Figure 2.11](image-url)

Figure 2.11: For $m=1$, $n=-6$ mode, the growth rates are calculated as the function of $F$, for two pressure profiles expressed in figure 2.9. The solid line with cubic corresponds to the pressure model equation 2.12. The dash line with dot are calculated by $p=n(r)T(r)$. The hollow circle and cubic with plus represent the growth rates of experiments for $\beta_p=0.02$ and $\beta_p \neq 0.02$ respectively.
Comparison between cylindrical model and experimental observation

Figure 2.12: Plot of profiles of \(\mu\) (circle), \(J_\parallel\) (diamond) by fixing \(F=-0.1\) and \(\Theta=1.44\), \(\beta_p=0.0\). The solid line is obtained when \(\Theta_0=1.5\), \(\alpha=5\) from equation 2.11. The dot line expresses the profile of \(\mu, J_\parallel\), when \(\Theta_0=1.43\), \(\alpha_1=1.8\), \(\alpha_2=1.5\) and \(\alpha_3=3.8\) from equation 2.14.

From the above analysis, we conclude that both the parameters \(F\) and \(\Theta\) in RFP can significantly influence the mode growth rate; while the sensitivities of RWMs to \(\beta_p\) and to the shape of pressure profile are less important. However, it is still noticeable for high \(\beta\) plasmas and for more precise calculation. When the plasma pressure is taken into account, a change of \(\beta_p\) value may cause the change of the parallel current density profile, thus affect both pressure driven and current driven mechanisms. This should be studied carefully. Therefore, in order to validate the model and draw a meaningful conclusion, a serious comparison between theory and experiments by well matching the three RFP equilibrium parameters \((F, \Theta, \beta_p)\) is recommended.

2.6 Summary

In this chapter, we numerically studied the RWMs stability properties in RFP plasmas by a cylindrical MHD model. The model takes into account the effects of plasma pressure, compressibility, inertial and plasma toroidal rotation, and parallel viscosity. An equilibrium model, which is commonly used in both RFP experiments and theory, is adopted. In order to validate the model, we concentrated the study on the careful comparison with the experimental observations in RFX-mod. Since the plasma flow in the current RFX-mod cannot
Comparison between cylindrical model and experimental observation

influence the growth rates of the RWM instabilities, the comparison concentrates on the variation in the equilibrium parameters $F$, $\Theta$ and $\beta_p$. The effects of the plasma rotation with various dissipation mechanisms on the stability of RWMs in RFP will be presented in chapter 5.

In the numerical calculation, the equilibrium parameters are selected carefully in the pre-computed matrix to well fit the experimental measured values of $F$, $\Theta$ and $\beta_p$. The obtained growth rates show a good agreement with the experimental results. The sensitivity of the mode growth rates to the equilibrium parameters $F$, $\Theta$ and $\beta_p$, as well as the pressure profiles have been investigated in detail. It is shown that a careful matching of the equilibrium parameters with experiments is necessary in the comparison. In particular, the parameter $F$ and $\Theta$ can significantly influence the mode growth rates. This fact should be noticed while applying a theoretical model to experiments and/or comparing the theoretical results with the experiments in order to draw a meaningful conclusion.

In this chapter, it shows that the cylindrical model is also consistent with experimentally observed RWM instability spectra. It is found that when the field reversal becomes shallower (less negative $F$), INRMs become more unstable while ENRMs are more stable; as the field reversal becomes deeper (more negative $F$), on the contrary, ENRMs becomes more unstable while INRMs are more stable. As regard with the parameter $\Theta$, larger $\Theta$ value implies more peaked plasma current profile (larger current gradient), which makes both types of the RWMs more unstable.

RWM physics is a complex field since several factors can contribute to the final RWM growth rates. The model in this work assumes a homogeneous resistive shell, which does not involve the more realistic boundary condition (such as shell gaps, diagnostic ports, mechanical support structures, coils, and vacuum vessels). [20] has shown that the shell gaps in RFX-mod may essentially increase the mode growth rate; however, it also pointed out that the introduction of additional conducting structures (vessel and mechanical structures) could slow down again the estimated growth rate. A final careful comparison involving all the terms, where their relative relevance is shown still has to appear and will be certainly a necessary subject in the future studies. Furthermore, at present probably the most critical aspect is the lack of knowledge of experimental internal plasma current profiles. The influence of the current ($\mu$) profile on the mode is
Comparison between cylindrical model and experimental observation
invented preliminarily in this work; we left further studies on this issue for future works.
Physical understanding of RWM instability spectrum in RFP

In chapter 2, a (periodic) cylindrical model of MHD theory, taking into account the effects of the plasma pressure, compressibility, plasma inertia, longitudinal rotation and parallel viscosity (tensor) has been employed for the study of RWM in RFP plasmas. The careful comparison with experimental results in RFX mod on the mode growth rates is made. A very good agreement is obtained between the theory and experiments. The sensitivity of the mode growth rate to the RFP equilibrium parameters are investigated in details.

This chapter concentrates on finding out what mechanism leads to the sensitive dependence of the RWM instability spectrum (mode growth rates as function of the toroidal mode number n) on the reversal parameter F; and why the variations of the growth rates with F are in the opposite directions between the ENRMs and INRMs. Although these behaviours have been observed experimentally since years [3,4]; the physical mechanism behind the observation is not yet well understood. Our present study provides the answer to these questions by an analysis based on the balance of the potential energy components. The physics of the sensitivity of the RWM instabilities to the pinch parameter Θ in RFP plasmas is also clarified.

3.1 RWM dispersion relation and energy components

Current RFP plasmas have slow rotation, for instance, in RFX-mod plasma, the velocity of the plasma flow \( v_0 \) is in the range of \( v_0 \approx 0.01-0.015v_{A0} \), therefore it is
Physical understanding of RWM instability spectrum in RFP

found $\omega \approx k v_o << \omega_a$ is always satisfied. By ignoring $\omega$ and the viscosity ($\eta_0=0$), the eigenmode equation 2.6 is reduced to well-known Newcomb equation [32]

$$\frac{d}{dr} \left[ f(r) \frac{d \xi_r}{dr} \right] - g(r) \xi_r = 0$$

where $f = \frac{rF^2}{k_o^2}$; and $g(r) = \frac{2k^2}{k_o^2} (\mu_e p)^{1/2} + \left( \frac{k^2 r^2}{k_o^2 r^2} - 1 \right) rF^2 + \frac{2k^2}{r k_o} (kB - \frac{mB^2}{r}) F_R$

Correspondingly, the boundary condition equation 2.7 actually gives a dispersion relation for the RWM in a cylindrical RFPs. By taking $\omega << \omega_a$ and ignoring the viscosity $\eta_0$, equation 2.7 becomes

$$\left[ \frac{F_B \hat{F}_B}{k_o^2} + \frac{F^2}{k_o^2} \left( \frac{r \xi'_r}{\xi_r} \right) \right] = \frac{a^2 F^2_B \Xi}{ka}$$

where $\hat{F}_B = kB - \frac{m}{r} B_0$. Equation 3.2 can be further expressed as

$$-i \omega \tau_b \tanh \xi = \frac{k^2}{k^2 + k^2 b a b' \left( 1 - \frac{K_b' I_b}{K_a' I_a} \right)} \frac{(\delta W_p + \delta W_v)}{(\delta W_p + \delta W_v)}$$

where

$$\delta W_p = \frac{2 \pi^2 R}{\mu_o} \left[ \frac{F_B \hat{F}_B + F^2_B \left( \frac{r \xi'_r}{\xi_r} \right)}{k_o^2} \right]$$

$$\delta W_v = \frac{2 \pi^2 R}{\mu_o} \left[ r^2 \Lambda \xi \xi_a \right]$$

$$\delta W_v = \frac{2 \pi^2 R}{\mu_o} \left[ r^2 \Lambda \xi \xi_a \right]$$

$$\delta W_v = \frac{2 \pi^2 R}{\mu_o} \left[ r^2 \Lambda \xi \xi_a \right]$$

$$\Lambda = -\frac{K_a}{K_a'} | k | a, \quad \Lambda_b = A \Lambda, \quad A = \left[ \frac{1 - (K_b' I_b) / (I_b' K_a)}{1 - (K_b' I_b') / (I_b' K_a')} \right], \quad \xi_a = \xi_b(a)$$

$\delta W_p$ is the potential energy in the plasma, $\delta W_v$ is the potential energy in vacuum region when the perfect conducting wall is located at $r=b$, and $\delta W_v$ is also the
Physical understanding of RWM instability spectrum in RFP

vacuum contribution when the wall is at infinity [33]. Here \( \delta W_p \) is negative, represents the potential source for driving external kink instability; both \( \delta W_{v\infty} \) and \( \delta W_{vb} \) are positive and playing stabilizing role, which are induced by the perturbed magnetic field (bending and compression) in the vacuum region due to the instability. In RFX-mod the resistive shall is rather close to the plasma, resulting to \( A \gg 1 \) and \( \delta W_{vb} \gg \delta W_{v\infty} > 0 \). The present RWMs study is for \( \delta W_p + \delta W_{v\infty} < 0 \) and \( \delta W_p + \delta W_{vb} > 0 \).

The above potential energy components \( \delta W_p, \delta W_{v\infty}, \) and \( \delta W_{vb} \) are calculated by substituting the eigenfunction \( \xi \). The growth rates obtained by using equation 3.3 coincide with those obtained from the eigenmode equation 2.6. For \( \xi \ll 1 \) (\( \omega \tau_b \sim 1 \) and \( h/b \ll 1 \)), thus \( \tanh \xi / \xi \approx 1 \); Equation 3.3 coincides with equation (9.105) in [11]. It can also recover the dispersion relation used for cylindrical Tokamaks when taking the approximation of \( ne \ll 1 \) [34].

3.2 Physical understanding of the instability spectrum of RWMs

In the previous chapter, we investigated the effect of parameter \( \Theta \) on the RWM instabilities in RFP plasmas. In this section, the physical understanding of RWM behaviour in figure 2.8 based on the introduced potential energy components is provided. When the parameter F is fixed, the pinch parameter \( \Theta \) influences the mode growth rate sensitively. This is due to the changes of the plasma current profiles. When \( \Theta \) increases, the current gradient increases, even though \( B_z (a) \) remains unchanged. It is found that when \( \Theta \) increases, the vacuum contributions to the potential energy \( \delta W_{vb} \) and \( \delta W_{v\infty} \) remain almost unchanged, while the plasma contribution \( \delta W_p \) significantly increases, which leads to larger growth rate. In figure 3.1 we plotted the variation of plasma potential energy components \( \Delta W_p \) as a function of \( \Theta \), and compared it with the variation of vacuum contribution \( \Delta W_{vb} \) (normalized by the value at \( \Theta = 1.41 \)) for \( \Theta \) varying from 1.41 to 1.45.
Physical understanding of RWM instability spectrum in RFP

Figure 3.1: By denoting the variations of the components of potential energy as $\Delta W_j = \delta W_j(\Theta)/\delta W_j(\Theta=1.41)$, $j= P, Vb$. $\Delta W_p$, $\Delta W_{Vb}$ and $-\delta W_\infty/\delta W_b$ are plotted as functions of $\Theta$ by keeping $F=-0.05$, $\beta_p=0.0$. Evidently, the change in growth rate (which is proportional to $-\delta W_\infty/\delta W_b$) depends on $\delta W_p$. The solid and dash lines are $\Delta W_p$ and $\Delta W_{Vb}$, the dot line with triangle represents the $-\delta W_\infty/\delta W_b$ ($\delta W_\infty=\delta W_p+\delta W_{V_\infty}$ and $\delta W_b=\delta W_p+\delta W_{Vb}$).

In fact, as shown in figure 2.10 of section 2.5, for each RFP machine, the reversal parameter $F$ and the pinch parameter $\Theta$ cannot change independently for all discharges. It has been found that all of the discharges are following a fixed $F-\Theta$ curve ($F$ is a monotonic function of $\Theta$) [35,36]. An example of the fitted curve for RFX-mod is shown in figure 3.2. In order to study how the instability spectrum of RWM varies with $F-\Theta$ curve, we pick up 4 points with different reversal depths ($F$ values) and corresponding values of $\Theta$ from the curve as marked in figure 3.2. The growth rates of each $n$ (toroidal mode number) mode for the four $F-\Theta$ values are computed by the CMR code and plotted in figure 3.3. The calculation is for zero $\beta$, since the RWMs in RFP are current driven modes, and $\beta$ values span in the current RFX-mod plasmas gives little influence on the mode growth rate as discussed in section 2.5.

The result in figure 3.3 coincides with the experimental observation and the earlier numerical calculations [4,30,37]. Figure 3.2 indicates, as our previous study, when magnetic reversal goes shallower, the growth rates increase for INRMs, decrease for ENRMs and vice versa.
Physical understanding of RWM instability spectrum in RFP

Figure 3.2: The plot of reversal parameter $F$ vs. pinch parameter $\Theta$. The hollow circles are the data from the experimental database for $m=1, n=-6$ mode. The curve is obtained by fitting the experimental data. Four solid points are taken for the instability spectrum calculation.

Figure 3.3: The mode growth rates as a function of the toroidal mode number $n$ are plotted for different $F$ and $\Theta$ value taken from the $F$-$\Theta$ curve. In the calculation, RFX-mod parameters ($a=0.459m$, $b=0.514m$, $h=3\times10^{-3}$ m, $\sigma=5.88\times10^7 \Omega/m$, $\tau_b=114$ ms, $\varepsilon=a/R=0.23$) are adopted.

The purpose of the following analysis is to understand the above behaviour of RFP plasma, where we chose the most unstable mode ENRM $m=1,n=3$ and INRM $m=1,n=-6$ respectively. Three potential energy components, $\delta W_p$, $\delta W_c\infty$ and $\delta W_{vb}$, which vary with the reversal parameter $F$ are plotted in figure 3.4. It can be seen that the plasma potential energy $\delta W_p$ is almost a constant value for each mode along the entire $F$-$\Theta$ curve. This phenomena seems to imply that, whatever type of
the field reversal of the discharge, RFP plasmas automatically adjusted the current profiles and the $\Theta$ value, intending to keep its potential energy unchanged and remaining the perturbation as small as possible.

![Figure 3.4](image)

Figure 3.4: (a) plots the energy potential components of INRM $m=1$, $n=-6$ (solid circle) and ENRM $m=1$, $n=3$ (hollow circle) as the function of reversal parameter $F$ along $F-\Theta$ curve of figure 3.2, where $\delta W_j = \delta W_p$, $\delta W_{V_e}$, $\delta W_{vb}$ are defined by equation 3.3 and presented by solid line, dash line and dot line respectively. (b) The growth rates of two modes are plotted as a function of $F$.

The plasma potential energy can be further written in the well-know three potential energy components form as $\delta W_p = \delta W_{mag} + \delta W_{cur} + \delta W_{pre}$, where $\delta W_{mag}$ is the stabilizing term from the magnetic bending and compressibility, $\delta W_{cur}$ and $\delta W_{pre}$ are the destabilizing terms from the current driven and pressure driven mechanisms; the definition of these three components can be found in [11]. Since $\beta_p = 0$, the pressure driven term $\delta W_{pre}$ vanishes in this study. These energy components as functions of the parameter $F$ for two RWM modes are plotted in figure 3.5. It shows, for both INRM and ENRM, the current driven term becomes stronger while $F$ becomes deeper ($\Theta$ becomes to larger value too), meanwhile the stabilizing term $\delta W_{mag}$ also increases. The balance of these two results in the fact that the total plasma energy, $\delta W_p$, remains a constant. Therefore, $\delta W_p$ does not change along the $F-\Theta$ curve.
Figure 3.5: Plasma potential energy $\delta W_p$ and its components, $\delta W_{\text{mag}}$ and $\delta W_{\text{cur}}$ for INRM $m=1, n=-6$ (solid line) and ENRM $m=1, n=3$ (dash line) in figure 3.2 are plotted as a function of $F$.

The variation of the mode growth rates with $F$ is due to the changes of the vacuum energies (mainly due to $\delta W_{\text{vb}}$, since $\delta W_{\text{v}}\ll\delta W_{\text{vb}}$). Figure 3.4 indicates that the vacuum energy $\delta W_{\text{vb}}$ changes significantly with parameter $F$ and $|\delta W_{\text{vb}}|>|\delta W_p|$. From equations 3.3b and 3.3c, it is found that the change in $\delta W_{\text{vb}}$ with reversal parameter is due to the change in the quantity $F_B(a)$, where $F_B(a) = k \cdot B(a) = (k\parallel B)_a$, which, after normalization [22], is expressed as $F_B(a) = [1 + kB_z(a)]$. We note here that $B_z(a) < 0$. For INRMs, $k<0$, the helical winding of the plasma displacement induced by the kink instability is in the opposite direction to the equilibrium magnetic field at the plasma edge; deeper reversal (Larger $|B_z(a)|$) leads to larger $F_B(a)$, as shown in figure 3.6(a), which implies stronger bending of the magnetic field line in the vacuum by the RWM perturbation, and larger $\delta W_{\text{vb}}$. This leads to stronger stabilizing role. Therefore, deeper reversal results in smaller growth rates of the INRMs. As for ENRMs, in the contrary, $k > 0$, the helical winding of the displacement is in the same direction with the equilibrium magnetic field at the edge. Figure 3.6(b) shows the schematic of ENRM case. Deeper reversal implies smaller $F_B(a)$, therefore smaller $\delta W_{\text{vb}}$, which leads to a larger growth rates for the ENRMs. Vice versa for the situation of the shallower reversal (smaller $|B_z(a)|$).
As for the dependence of the growth rates on the mode number n, it is found that the dominant stabilizing effect of $\delta W_{vb}$ is a decreasing function versus wave number k, due to the fact that $\Lambda_\infty$ (in equation (3.3c)) decreases when the wave number k increases. Therefore, the mode should be more unstable when k (thus mode number n) increases for both INRMs and ENRMs. In fact, this is true for all numbers of unstable INRMs and n=1-3 ENRMs. However, there is an exception for ENRM with large k where the growth rate decreases until zero. This is due to the fact that the driving source $\delta W_p$ for the externally non-resonant kink instability tends to change the sign when k is large enough. So the externally (non-resonant) kink mode with large k is stable in RFP even without wall. This is clearly shown in figure 3.7(a), where the three component of $\delta W$ are plotted as functions of wave number n. For n=4, $\delta W_p$ becomes almost zero and for n=5 it becomes positive. The variation in $\delta W_p$ can be also seen from equation (3.3a) where, for ENRMs,
Physical understanding of RWM instability spectrum in RFP

\[
\left(\frac{r_{s}^2}{\xi}\right)_{a} \approx 0, \text{ hence } \delta W_{p} = \frac{2\pi^{2}R}{\mu_{0}} \left[ \frac{k^{2}B_{z}^{2} - 1}{k^{2}_{0}} \right] \xi_{a}^{2}, \text{ which can clearly show } \delta W_{p}
\]
changes sign when k goes large. The values of \(- (\delta W_{p} + \delta W_{v_{\infty}})/(\delta W_{p} + \delta W_{v})\), which are proportional to the growth rate (when b, and k=nc are fixed), is shown in figure 3.7(b) which indicate the conclusion of this analysis.

Figure 3.7: Energy potential components of INRM and ENRM for different mode number \(n\), and keeping \(\Theta_{0}=1.5\) and \(\beta_{p}=0.0\). The solid circle and hollow circle represent the cases \(F=-0.05\) and \(F=-1.0\) respectively. (a) plots the components of potential energy against \(n\), where \(\delta W_{j} = \delta W_{v_{b}}\) (solid line), \(\delta W_{v_{\infty}}\) (dash line), \(\delta W_{p}\) (dot line) are from equation 3.3. (b) \(-\delta W_{v_{\infty}}/\delta W_{b}\) is plotted as the function of wave number \(n\), where \(\delta W_{\infty} = \delta W_{p} + \delta W_{v_{\infty}}\) and \(\delta W_{b} = \delta W_{p} + \delta W_{v_{b}}\).

3.3 Summary

Based on the above analysis, our findings are summarized as follows. As for the pinch parameter \(\Theta\), which is related to the gradient of the current profile, it mainly influences the potential energy in the plasma. Therefore, raising \(\Theta\) value (fixing \(F\) value) implies enhancing the driving source, thus enforcing the instability for both INRMs and ENRMs. Then we explain why the variations of the growth rates with \(F\) are in the opposite directions between the ENRMs and INRMs in RFP plasmas. It is found that when the RFP plasma changes its equilibrium by following the fixed \(F-\Theta\) curve, the normalized plasma potential energy \(\delta W_{p}/\xi_{a}^{2}\) of a given mode is unvaried, which implies that RFP plasmas intends to keep the potential energy
Physical understanding of RWM instability spectrum in RFP

\( \delta W_p \) being a constant value while following a fixed F-\( \Theta \) relation. This phenomena seems to imply that for whatever type of the field reversal of the discharge, RFP plasmas automatically adjusted its current profiles and the \( \Theta \) value, intending to keep its potential energy unchanged and remaining the perturbation as small as possible. Therefore the variation of the RWM growth rates versus F is determined only by the change of the vacuum energy, mainly by \( \delta W_{vb} \) which plays the stabilizing role to RWMs. Due to the opposite helical winding between INRM and ENRM in RFPs, the vacuum potential energies (\( \delta W_{v∞}, \delta W_{vb} \)) for these two modes reflect the change of F oppositely; thus the changes of the growth rates versus F for the two types of the modes go in different directions. One should also notice that the vacuum potential energy is a decreasing function of the wave number k. The growth rate of RWMs is determined by the balance of above components of the potential energy according to the equation 3.3.
Chapter 4

Physical understanding of the feedback control of RWM in RFP

The study of RWM and feedback control is an important issue for fusion researches. In tokamak plasmas, RWM is normally driven by plasma pressure; the instability occurs when the plasma $\beta$ value exceeds the no-wall limit [38]. In RFPs, the RWM always appears as a current driven potentially disruptive instability whenever the duration of the discharge is longer than the wall penetration time.

A numbers of studies by experiments and theories have indicated that in Tokamak plasmas, RWMs can be suppressed by a plasma rotation [34,39-41]. Recent studies point out even little or no plasma rotation due to the kinetic damping mechanisms may be effective at changing the instability conditions of the RWM [42-45]. Nevertheless, the feedback stabilization of the RWM is still important. This is not only for the current operating machines in high $\beta$ performance, where the RWMs are easily trigged to be unstable by interacting with other instabilities [46-48], but also for the next step devices, e.g. ITER [1] and DEMO [17], which point toward to the steady state operation. In RFP plasmas, the rotational stabilization of RWM predicted by the existing fluid and kinetic theories in [22] and chapter 5 requires quite higher plasma rotation and higher beta value which still can not yet be provided by the present RFP experiments. Therefore, an active feedback control of RWMs is essential to RFP operation.

In recent years, many studies have reported the successful feedback controls for RWMs in both RFP and Tokamak plasmas [3,24,23,27,49,50]. For instance, the feedback system equipped on RFX-mod [26], having sufficient efficiency and flexibility, has been giving a successful active control on multi-unstable RWMs in
Physical understanding of the feedback control of RWM in RFP

RFP plasmas. These activities provide valuable knowledge not only for how to control RWMs in RFP plasmas but also for future Tokamak devices.

In order to well study the RWM behaviors and its feedback control in the RFP plasmas, a better physical understanding is an important issue, and many efforts have been already devoted to this subject [51-58]. In this chapter, we concentrated our study on the following two topics via both analytical and numerical analyses.

(1) To investigate how the plasma responds to the feedback control action. The investigation by the linear theory considers two cases, without and with plasma rotation. It is found that actually the eigenfunction inside plasma is uniquely determined by the plasma equilibrium (except a normalization factor) and independent of the feedback actions. The discussion based on the nature of the eigenmode equation and of the dispersion relation is carried out in order to further understand the physics.

(2) Furthermore, based on the above results, the time evolutions of RWM under various scenarios of the feedback control are studied, and compared with the steady state solution. The effects of wall proximity, the probe location and the system response time scale on the consequences of the controls are investigated; and various feedback scenarios are discussed in details.

In this chapter, the periodic cylindrical model and the corresponding RFP equilibrium described in chapter 2 are adopted for the study, which is extended and involves a simple model of an active feedback system. A generalized dispersion relation including proportional-integral-derivative (PID) controller is derived in terms of the potential energy components. The corresponding optimized numerical code CMR-F (Cylindrical Magnetohydrodynamic with Resistive wall code integrated with Feedback control) is applied, which has been carefully benchmarked with experimental data for the RWMs growth rates in chapter 2 and active control of the mode rotation [27,59]. To start with, the effect of toroidal coupling is not included in this study, therefore only single helical RWM is considered. This is reasonable since the existing studies in RFPs have shown a weak influence of the toroidal coupling effects on the growth rate of the RWM modes [20,21]. Actually, only $m=1$ poloidal modes are originally unstable in RFPs. Hence we leave the mode coupling effects to chapter 5. The plasma rotation and the viscous dissipation considered in this chapter are for the study of the feedback
Physical understanding of the feedback control of RWM in RFP

effect on the rotating plasmas. The subject of the stabilization of RWM by plasma rotation in RFP will be discussed in chapter 5.

4.1 Feedback system and boundary condition

Figure 4.1 illustrates the cylindrical RFP integrated with a feedback system. The plasma has the minor radius \( r=a \), which is surrounded by a circular resistive wall at \( r=b \) with a thickness \( h \) and conductivity \( \sigma \). The feedback coil and the radial magnetic sensors are located at \( r_f (r_f>b) \) and \( r_s (a<r_s\leq b) \) respectively, where the magnetic perturbation of the mode is assumed to be measured by radial magnetic sensors accurately; For studying the effect of RWM feedback control on RFP plasmas, we concentrate on controlling one Fourier mode. The magnetic coil is continuous and produces the magnetic field for controlling the mode without error field and sidebands.

![Image of cylindrical model with feedback system](image_url)

Figure 4.1: The configuration of cylindrical model of RFP with a feedback system. The locations of radial magnetic sensor, resistive wall and feedback coil are presented. \( b_r \) is the sketch of the magnetic perturbation of RWM.

For modeling an active feedback system, the well known PID controller, which has been successfully applying to RWM control in RFP experiments [27,57], is integrated in CMR-F. Therefore, the feedback circuit for a single mode \((m, n)\) can be expressed as

\[
i \omega L_r I_{pid} \psi_s = R_I I_r, \tag{4.1}
\]
Physical understanding of the feedback control of RWM in RFP

where the PID controller has the form

\[ G_{\text{pd}} = G_p + \frac{iG_i}{\omega} - i\omega G_d, \]  \hspace{1cm} (4.2)

and \( \psi_r = -i r b_r(\tau_r) \) is the perturbed radial magnetic flux and \( b_r(\tau_r) \) is the magnetic perturbation measured at sensor (Note that the radial magnetic perturbation \( b_r(\tau) \) in plasma is related with \( \varphi = r \xi_r \), where \( b_r = \frac{iF_B}{r} \varphi \)). \( L_f \) and \( R_f \) are the effective inductance and resistance of feedback coil. \( G_p \) is the proportional gain, \( G_i \) is the integral gain, \( G_d \) is the derivative gain. From the point of view of control theory [60], \( G_p \) provides the overall control action to the error (the difference between the sensor signal and reference), the integral gain plays a role to eliminate the steady state error, and the derivative gain is responsible for the transient response of the controller. Nevertheless, it’s still essential to further study the RWM feedback control associating with the response of plasma.

For this purpose, the periodic cylindrical model and the corresponding RFP equilibrium described in chapter 2 are adopted; the boundary condition including feedback system are obtained similarly to the manner in [22]. By deriving the perturbed magnetic fields in different vacuum regions, these solutions are then matched across the thick shell (which allows to treat the large equilibrium flow \( \omega \tau_b > k_{v_0} \gg 1 \), \( \tau_b = \mu_0 \sigma_b h \) is the wall penetration time) and feedback coil. Finally, the solution outside the plasma should match at the condition in plasma boundary. This matching condition is obtained by integrating equation 2.6 over a thin layer across the plasma boundary and then taking the limit for the layer width going to zero [22, 27]. The extended boundary condition including control system is then written as

\[
\begin{align*}
\frac{a^2 F_{\theta}^2}{k a} \Xi &= \frac{B^2}{D + i \mu} (\omega^2 - \omega_s^2) (\omega^2 - i \frac{\bar{\omega} \eta_e}{\rho v_A^2} E) \frac{a}{\rho} \frac{d \varphi}{dr} - 2B_\theta^2 \\
&+ \frac{1}{D + i \mu} [2kB_\theta G(\omega^2 v_A^2 - i \bar{\omega} \eta_e E) + i \frac{\bar{\omega} \eta_e}{\rho} B_\theta^2 v_A^2 (\omega^2 - \omega_s^2)] \hspace{1cm} (4.3)
\end{align*}
\]

where
Physical understanding of the feedback control of RWM in RFP

\[ \Xi = \frac{K_a - (K_b')}{K_a'} I_w I' \quad \frac{H_w}{1 + i k^2/\omega_0} \quad \frac{1 + K_{\infty} / K_b}{1 + \frac{i k^2}{\omega_0 I_b} k^2 K_b \tanh \zeta} \quad \text{and} \quad \Re = i \frac{K_b'}{\omega_0 I_b (1 - \omega_1)} G_{\text{pid}}; \]

\( \tau_f = L_f/R_f \) is the effective response time of the feedback system;
\( \zeta = [\text{i} \omega_0 (h/b)]^{1/2} \); \( k^2_{\omega_0} = \frac{m^2}{b^2} + k^2 \); \( K_r = K_m(|k|r) \) and \( I_r = I_m((|k|) \) are the modified Bessel functions; the prime denotes the derivative on the argument. The feedback system is introduced from the terms \( \Re \). While canceling the feedback control \( G_{\text{pid}} = 0 \), the boundary condition equation 4.3 will return to the result in section 2.1 and [22,61]. Thus, equation 2.6 accompanied by B.C. equation 4.3 is the eigenmode equation integrated with feedback system for our RWM study.

4.2 Dispersion relation of RWM with feedback system

Current RFP plasmas have slow rotation, for instance, in RFX-mod plasma, the velocity of the plasma flow \( v_0 \) is in the range of \( v_0 \sim 0.01-0.015v_{A_0} \), therefore it is found \( \omega_0 \approx kv_0 \ll \omega_a \) is always satisfied. By ignoring \( \omega_0 \) and the viscosity (\( \eta_0 = 0 \)), the eigenmode equation 2.6 is reduced to well-known Newcomb equation [32]

\[ \frac{d}{dr} \left[ f(r) \frac{d\xi}{dr} \right] - g(r) \xi = 0 \quad (4.4) \]

where \( f = \frac{r F_b^2}{k_o^2} \); and \( g(r) = \frac{2 k^2}{k_o^2} (\mu_o p) + (\frac{k^2 r^2}{k_o^2} - 1) r F_b^2 + \frac{2 k^2}{r k_o^2} (k B_z - \frac{m B_o}{r}) F_b \)

Correspondingly, the B.C. equation 4.3 is easily simplified as

\[ \frac{a^2 F^2_{\beta}}{ka} \Xi = \left[ \frac{F_{\beta} - \frac{F_{\beta}^2 (r \xi)}{k_o^2}}{k_o^2} \right] \quad (4.5) \]

Equation 4.5 actually can provide the following dispersion relation, written as

\[ (\gamma - \text{i} \omega_r) \tau_d = - \frac{\delta W_p + \delta W_{v_{\infty}}}{\delta W_p + \delta W_{v_b}} - \frac{\delta W_p + \delta W_{v_{\infty}}}{\delta W_p + \delta W_{v_b}} \widetilde{G}_{\text{pid}} \quad (4.6) \]
Physical understanding of the feedback control of RWM in RFP

\[
\delta W_p = \frac{2\pi^2 R}{\mu_0} \left[ \frac{F_B \hat{F}_B}{k_0^2} + \frac{F_B^2}{k_0^2} \left( \frac{\xi'_r}{\xi_r} \right) \right] a \xi_a^2
\]  
(4.6a)

\[
\delta W_{\text{vac}} = \frac{2\pi^2 R}{\mu_0} \left[ r^2 \Lambda \Lambda F_B^2 \right] a \xi_a^2 ;
\]

(4.6b)

\[
\delta W_{\text{vi}} = \frac{2\pi^2 R}{\mu_0} \left[ r^2 \Lambda \Lambda F_B^2 \right] a \xi_a^2 = A_i \delta W_{\text{vac}}
\]

(4.6c)

where \( \Lambda = \frac{-K_a}{K_a'} | k | a \), \( \Lambda_i = A \Lambda \), \( A_i = \left[ \frac{1}{1-(K_a'I_a')/(I_a'K_a')} \right] \), \( i=b,s \)

\[
\xi_a = \xi_r(a) \text{, and } \hat{F}_B = kB_z - \frac{mB_0}{r},
\]

(4.6d)

\[
\tilde{G}_{\text{pid}}(r_s, r_f, \tau_f, G_f) = \frac{\tilde{G}_{\text{pid}}}{1 - i\omega \tau_f},
\]

(4.6e)

\[
\Gamma_\tilde{G}(r_s, r_f) = -\frac{\theta}{\xi} \xi_r r_s k^2 I'_f \left( 1 - \frac{K_a'I_a'}{I_a'K_a'} \right) > 0 ,
\]

\[
\tau_d = \frac{\tanh \xi}{\xi} \frac{K_b'I_b}{k_0^2} \left( 1 - \frac{K_b'I_b}{K_b'I_b} \right) > 0
\]

where the definitions of \( \delta W_p \), \( \delta W_{\text{vac}} \) and \( \delta W_{\text{vi}} \) were already given out in chapter 3. We repeat them here for the convenience of the following study, and further introduce the new terms relating to the control system. \( \delta W_p \) is the potential energy in the plasma, which is negative, describes the potential energy source for driving external kink instability; \( \delta W_{\text{vac}} \) and \( \delta W_{\text{vi}} \) denote the potential energy in vacuum region when the perfect conducting wall is located at sensor position \( r=r_s \) and wall position \( r=b \) respectively, and \( \delta W_{\text{vac}} \) is also the vacuum contribution when the wall is at infinity [11]. \( \delta W_{\text{vac}}, \delta W_{\text{vi}} \) and \( \delta W_{\text{vb}} \) are positive and playing stabilizing role, they are induced by magnetic field bending in the vacuum region due to the instability. \( \tau_d \) is the coefficient relating to \( \tau_b \) and determined mainly by the wall parameters as well. Therefore, we may call it the effective wall penetrating time. If taking the thin shell approximation, \( \omega \tau_b << h/b \) and \( \xi << 1 \), thus \( \tanh \xi / \xi \approx 1; \tau_d \)
Physical understanding of the feedback control of RWM in RFP

coincides with the coefficient of equation (9.105) in [11]. The first term of RHS of equation 4.6 is related to the RWM growth rate without feedback. The RWM instability is for \( \delta W_p + \delta W_{vo} < 0 \) and \( \delta W_p + \delta W_{vb} > 0 \). The second term is contributed by the feedback control. \( \tilde{G}_{pid} \) is the effective feedback coefficient, which depends on the sensor position \( r_s \), the active coil position \( r_f \), the response time \( \tau_f \), and the value of feedback gain \( G_{pid} \). If the sensor is near the wall, then \( r_s \approx b \), and \( \frac{\delta W_p + \delta W_{vb}}{\delta W_p + \delta W_{vb}} \approx 1 \). The dispersion relation is simplified to

\[
-i\omega \tau_d = -\frac{\delta W_p + \delta W_{vo}}{\delta W_p + \delta W_{vb}} - \tilde{G}_{pid}.
\] (4.7)

Without the feedback control \( \tilde{G}_{pid}=0 \), the dispersion relation equation 4.6 returns to the result in [61]. Further considering thin shell approximation, equation 4.6 returns to usual dispersion relation for RWM [11].

4.3 How does the plasma respond to the feedback control?

In order to understand how the feedback control impacts on the RWMs inside the unstable plasmas, we carry out the study on the most unstable \( m=1, n=-6 \) and the secondary unstable \( m=1, n=-5 \) INRMs by using CMR-F code and present the results in the following figures 4.2-4.4. Figure 4.2 reports, without plasma rotation and viscosity, the perturbed radial magnetic field \( b_r(r) \) in both plasma and vacuum regions for three cases \( \gamma>0 \) (without feedback), \( \gamma \approx 0 \) and \( \gamma < 0 \) due to the feedback control. The equilibrium parameters are taken as \( F=-0.05, \Theta=1.417 \) and \( \beta_p=0.02; \) the magnetic sensor and feedback coil are located near the wall \( b/a=1.12, \) and \( r_f/a=1.268 \) according to the RFX-mod feedback system configuration. While solving the eigenmode equation by the shooting method, the starting point of shooting at plasma center \( r=0 \) is set to be the same for the three cases as \( \xi_{p}(r=0)=i \xi_{p}(r=0) \) (implying \( \xi_{p}(r=0)=1 \)). Figure 4.2 shows that, for three values of \( G_{p} \), the control system affects the eigenfunction \( b_{i}(r) \) (perturbed radial magnetic field) only outside the resistive wall (region II, IV in figure 4.1); the shape of perturbed magnetic field in plasma and vacuum region I is not changed. Even changing the wall position, moving the sensor, using the other PID parameters \( G_{i} \).
and $G_d$ etc., the shapes of the eigenfunction in plasma are still unchanged for each case. This conclusion is also valid for the case when taking into account the plasma rotation and viscosity. The following discussion may give explanation on the above phenomena.

Figure 4.2: For $m=1 \ n=-6$ mode, the eigenfunction of magnetic perturbation $b_r(r)$ is plotted for $G_p=0$ (unstable mode), $G_p=0.943$ (marginal stability) and $G_p=1.45$ (damping mode), where $F=-0.05$, $\Theta=1.417$ and $\beta_p=0.02$, $r_s/a=b/a=1.12$, $r_f/a=1.268$, $\tau_b=114$ms, $\tau_f=2$ms, and $b_r(r=0)=iF_B(r=0)$ while solving the eigenmode equation.

1) When the plasma flow is much slower than the critical stabilizing velocity $v_{oc}$, which is the case of RFX-mod plasmas, where the modes do not rotate at all, thus, $\omega << \omega_\phi$ and $\omega$ can be neglected in equation 2.6, which actually reduces to the Newcomb equation 4.4. For given $m$ and $n$, the solution $\xi$, of equation 4.4 is uniquely determined by the equilibrium quantities of the plasma except for a normalization factor. Hence the value of $\left(\xi',\xi\right)$ (an important factor to determine $\delta W_p$) is also uniquely determined by the equilibrium. The boundary condition, which includes the effects of the resistive wall and feedback system, provides the constrain for the matching between the vacuum solution of $b_r(r)$ and the fixed value corresponding to $\left(\xi',\xi\right)$ at the plasma edge given by the plasma equilibrium. This matching finally determines the total eddy current induced in the resistive wall (both by the plasma perturbation and by the feedback coils) and the magnetic perturbation at the wall. The mode growth rate is determined by the eddy current at the wall which is proportional to the jump of the derivative of $b_r$ at the wall

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Physical understanding of the feedback control of RWM in RFP
Physical understanding of the feedback control of RWM in RFP

\[
\left[ \frac{db}{dr} \right]_{r=b^+} = \left[ \frac{db}{dr} \right]_{r=b^-}. \text{ Therefore adjusting the feedback system only changes the value of} \\
\left[ \frac{db}{dr} \right]_{r=b^+}, \text{ i.e. the eddy current at the wall, the eigenfunction } \xi_a(r) \text{ (and also } b_i(r)) \text{ inside plasma is certainly unchanged. From the point of view of the dispersion relation equation 4.6, since the quantity } \left( \frac{\xi'_a}{\xi_a} \right)_b \text{ is independent of the feedback action, the value of } \delta W_p/\xi_a^2 \text{ (relating to } \xi_a/\xi_a) \text{ is also independent. The linear combination of first and second terms of equation 4.6 indicates that the feedback control just affects the growth rate of RWM. The plasma is still potentially unstable to ideal kink mode.}

Furthermore, because the value of \( \xi'_a/\xi_a \) is unchanged while the feedback applied, the first term of RHS in equation 4.6 is unchanged either. Thus, let 
\[
-i\omega_0 \tau_d = -\frac{\delta W_q + \delta W_{ve}}{\delta W_q + \delta W_{vb}}, \text{ where } \omega_0 = \omega_0 + i\gamma_0. \text{ The dispersion relation of equation 4.6 can be written as}
\]
\[
-i\omega_0 \tau_d = -i\omega_0 \tau_d - \tilde{G}_{pid} \quad (4.8)
\]

Without mode rotation, \( \omega_0 = i\gamma_0, \gamma_0 \) denotes the original growth rates of RWMs without feedback control. Several observations can be obtained from the dispersion relation equation 4.8; (1) If the feedback gain \( G_{pid} \) has an imaginary part, the control system gives the mode a rotating frequency, although the original mode without feedback does not rotate. This kind of mode rotation is already observed in the experiments [27]. (2) Since \( \frac{\delta W_q + \delta W_{ve}}{\delta W_q + \delta W_{vb}} \) is only determined by the equilibrium and the wall position \( b \), if \( \tau_b \) is changed by taking different conductivity and thickness of the wall, the value of \( \tau_b \gamma_0 \) is unvarying. (3) As well known, \( \tau_d \gamma_0 \) is related to the eddy current at the wall induced by the magnetic perturbation of the plasma, the second term of RHS in equation 4.8 represents the eddy current induced by feedback system. It implies, for stabilizing the RWM, feedback control contributes a new eddy current to modify the original eddy current induced by RWM in the wall, and the total eddy current is zero when the unstable mode is stabilized by feedback, \( \gamma = 0 \). This subsequently changes the jump of derivative of perturbed magnetic field on the wall.
Physical understanding of the feedback control of RWM in RFP

2) When the plasma rotation and the viscosity are taken into account, the Newcomb equation is no longer valid. The typical velocity scans of m=1, n=-5 mode are investigated numerically by using eigenmode equation 2.6 for F= -0.05, Θ=1.36, β_p=0.0, and the RFX-mod parameters are adopted. Figure 4.3 plots the mode growth rate and frequency versus flow velocity for fixed wall position, when the parallel normalized viscosity η_0 is taken as 0 and 0.01 (the typical range of viscosity of RFX-mod from 0.003 to 0.01 is estimated by the method in [62]). It shows that in the region of v_0<v_{oc}, when the plasma flow velocity increases, the mode frequency almost remains to be zero, and the growth rate slightly increases. When the velocity reaches the critical value v_{oc}= 0.474v_{0A}, the mode begins to rotate (ω_r=0.001ω_{0A}) and the growth rate reaches to a maximum value, where the flow velocity approximately matches the resonant condition (0<k_{||}v)_k≈ω_s^2 (here ω_0=ω_0(0)=(k_{||}v_{0A})_{r=0}, k_{||} is the parallel RWM wave number). As for v_0>v_{oc}, the rotating frequency of the mode increases significantly and has a rather high value e.g. ω_r=0.145ω_{0A}, when v_0=0.6v_{0A}. Without the viscous dissipation, the growth rate decreases from the maximum value but still keeps a positive value, However, when the viscosity is included, the growth rate becomes negative and the mode become stable. Therefore, v_{oc} is the critical flow velocity to stabilize RWM if a necessary dissipation exists. Similar results are reported in earlier work of [22] for ENRMs, where ω_0 in the resonant condition should be replaced by ω_0=ω_0(a)=(k_{||}v_{0A})_{r=a} [59]. The detailed study of rotational stabilization of RWM in RFP plasmas is presented in chapter 5.
Figure 4.3: The plot of (a) the growth rate $\gamma$ (normalized by $\tau_b^{-1}$) and (b) the rotating frequency $\omega_r$ (normalized by the Alfven frequency at plasma edge $\omega_{\theta A}=a/v_{\theta A}$) of $m=1$, $n=-5$ mode as the function of plasma rotation $v_0$ for $\eta_0=0$ and $\eta_0=0.01$, when $F=-0.05$, $\Theta=1.36$, $\beta_p=0.0$. RFX-mod parameters are used in this calculation.

Figure 4.4: (a) and (b) plot the real and imaginary parts of the eigenfunction of magnetic perturbation $b_r(r)$ for $m=1, n=-5$ mode when $v_0=0.2v_{\theta A}$ and $v_0=0.6v_{\theta A}$ respectively, where
Physical understanding of the feedback control of RWM in RFP

viscosity $\eta_0=0.01$. In both (a) and (b), the line marked by square is the case without feedback, and the circle denotes the mode is controlled by proportional control, where $\tau_f=2\text{ms}$ is evaluated for RFX-mod, and the other parameters are the same as that in figure 4.3.

Figure 4.4 is the plot of the magnetic perturbation $b_r$ without/with feedback system for two cases, where (a) is for $v_0=0.2\nu_{0A} < v_{oc}$, and (b) is for $v_0=0.6\nu_{0A} > v_{oc}$ (the velocity is much higher than that the normal velocity of RFP plasmas.). While taking into account the plasma rotation, the eigenfunctions become a complex. For the case $v_0 =0.2\nu_{0A}$, $\eta_0= 0.01$, the mode does not rotate and it can be stabilized by the feedback system. The magnetic perturbations $b_r$ is given in figure 4.4(a), the marginal stability is achieved when $G_p=0.6$. It shows that, similarly to the case of ignoring plasma flow, the shape of magnetic perturbation $b_r$ is changed outside the wall by feedback system, but unchanged inside the wall. However, without plasma rotation, the required $G_p$ for getting the marginal stability of the mode is 0.436. This result implies that a larger $G_p$ should be used for stabilizing the mode if the plasma rotation exists. As for the case of $v_0 =0.6\nu_{0A}$, $\eta_0= 0.01$, where the RWM has fast rotation ($\omega_r=0.145\omega_{0A}$) and is stabilized by plasma rotation and the viscosity, the feedback system cannot affect either the value of the growth rate or the shape of eigenfunction $b_r(r)$ of the mode for whatever large value of gain $G_p$. Figure 4.4(b) is the plot of the eigenfunction $b_r(r)$ and its extension in vacuum region for $G_p=0$ and $G_p=2$ (both cases are coincided). It clearly shows that both the magnetic perturbation inside the plasma and the possible small magnetic signal from the response of the feedback coil (also having high frequency) are shielded by the resistive wall because $\omega \gg \tau_b^{-1}$.

4.4 Effects of the location of the resistive wall and sensor in feedback control

While fixing the positions of the magnetic sensor and feedback coil, for different wall position ($r_s<b<r_f$), the growth rate of the mode $m=1$, $n=-6$ versus proportional gain $G_p$ is plotted in figure 4.5. It shows that, without feedback the mode has larger growth rate if the resistive wall is farther from plasma; however, the decreasing slope of the mode growth rates due to $G_p$ is steeper and proportional
Physical understanding of the feedback control of RWM in RFP

to \(-\frac{1}{\tau_d} \frac{\delta W_p + \delta W_{vs}}{\delta W_p + \delta W_{sh}}\), which implies that, if the resistive wall is closer to the feedback coils, the feedback system could affect the eddy current in the wall more efficiently. However, the same value of \(G_p\) is required for getting the marginal stability for all of the wall positions. This conclusion can also be obtained from the dispersion relation equation 4.6 by setting \(\omega=0\),

\[
G_p = -\frac{1}{\Gamma_G} \frac{\delta W_p + \delta W_{vs}}{\delta W_p + \delta W_{vs}}. \quad (4.9)
\]

where \(\Gamma_G\) is the function of the sensor and feedback location \(r_s, r_f\).

Since RHS of equation 4.9 are constant for given equilibrium and independent of the wall parameters, it indicates that the value of \(G_p\) for achieving the marginal stability is determined by sensor position, and coil position.

![Figure 4.5: The effect of wall position on feedback system. The growth rates of m=1 n=-6 mode as the function of Gp are plotted for different wall position, where the calculation is carried out by using the parameters of figure 4.2, r_s/a=1.01 and r_f/a=1.268.](image)

For studying the effect of sensor position in RWM control, we put the magnetic sensor on different position, the calculation of mode growth rates varying with \(G_p\) are performed numerically. Figure 4.6 reports the feedback control for stabilizing the m=1, n=-6 mode, If ignoring \(\tau_f (\gamma \tau_f << 1)\), when the sensor is closer to the plasma, the less proportional gain \(G_p\) is required. If considering the finite feedback
Response time $\tau_f$, the variation of the mode growth rate with the value of $G_p$ is not linear. As for getting the marginal stability, required $G_p$ are the same for $\tau_f \neq 0$ and $\tau_f=0$. This result suggests that, for stabilizing the unstable mode, the feedback system can be more efficient, if moving sensor to the plasma as close as possible and shortening the feedback response time.

Figure 4.6: The effect of sensor position on feedback system. The growth rates of $m=1$ $n=-6$ mode as the function of $G_p$ are calculated for different sensor position by using the same parameters of figure 4.2. The lines with square, circle and triangle are calculated by assuming $\tau_f=0$, the line with diamond is for the case considering the effect of $\tau_f=0.45\gamma_0^{-1}$ ($\tau_f=20\text{ms}$).

4.5 Time dependent solution of RWM feedback control

The study of section 4.3 has shown that the important quantity $\zeta'/\zeta_a$, which describes the plasma response to the external kink perturbation, is not influenced by the feedback activity, but only by the plasma equilibrium. By assuming the variation of the plasma equilibrium is negligible during the feedback control, the time-dependent solution (linear theory) of the RWM during the application of the feedback control can be studied easily only by analyzing the vacuum region, if $\gamma_0$ (without feedback) is known. It is unnecessary to calculate the plasma responses for each time step.
Physical understanding of the feedback control of RWM in RFP

In the following study the ignorable plasma flow (thus $\omega < \omega_0$) and thin shell approximation are considered. In the vacuum regions I, II, IV (see figure 4.1), the perturbed magnetic field can be presented as,

$$b_1'(r,t) = ik\left[C_1(t)I_m'(|k|r) + C_2(t)K_m'(|k|r)\right]$$  \hspace{1cm} (4.10)

$$b_2''(r,t) = ik\left[C_3(t)I_m'(|k|r) + C_4(t)K_m'(|k|r)\right]$$  \hspace{1cm} (4.11)

$$b_3''(r,t) = ik|C_5(t)K_m'(|k|r)$$  \hspace{1cm} (4.12)

The magnetic perturbation on the wall (region III) satisfies

$$\tau_b \frac{\partial b_1(t)}{\partial t} = \left[\frac{\partial b_1'(r,t)}{\partial r}\right]_{r=b^+} - \left[\frac{\partial b_1'(r,t)}{\partial r}\right]_{r=b^-}$$  \hspace{1cm} (4.13)

$$b_3''(r,t) = b_1'(r,t)_{r=b^-} = b_2''(r,t)_{r=b^-}$$  \hspace{1cm} (4.14)

In the feedback coil, we have

$$-ik_0^2 I(t) = \left[\frac{\partial b_2''(r,t)}{\partial r}\right]_{r=t^+} - \left[\frac{\partial b_2''(r,t)}{\partial r}\right]_{r=t^-}$$  \hspace{1cm} (4.15)

$$b_3''(r,t)_{r=t^-} = b_1''(r,t)_{r=t^-}$$  \hspace{1cm} (4.16)

where $k_0^2 = \frac{m^2}{\tau_i^2} + k^2$, the effective coil current $I(t)$ in the active feedback coil satisfies the equation

$$\tau_f \frac{\partial I(t)}{\partial t} + \left(G_p\psi_s(t) + G_d\int_{\psi_s}^t \psi_s(t) dt + G_d \frac{d\psi_s(t)}{dt}\right) + I(t) = 0,$$  \hspace{1cm} (4.17)

where $\psi_s(t) = -ir_b r(t)$.

From equation 4.5 and dispersion relation equation 4.6, the relation $\frac{C_1(t)}{C_2(t)}$ can be obtained as

$$\frac{C_1(t)}{C_2(t)} = \frac{k_0^2 I_b K_m^2 \gamma_0}{k_0^2 k_0^2 K_b I_b \gamma_0}$$  \hspace{1cm} (4.18)
where $\gamma_0$ is the known mode growth rate without feedback. In the derivation the following relation have been used

$$\Xi = \frac{K_s \left( C_1(t) - C_2(t) \right)^I_s}{K_s' \left( C_1(t) - C_2(t) \right)^I_s}$$  \hspace{1cm} (4.19)

We set the initial values for the perturbed magnetic field on the sensor and coil as

$$b_r(r_s, 0) = b_{r0} \hspace{1cm} (4.20)$$

$$I(0) = 0 \hspace{1cm} (4.21)$$

where $t=0$ denotes the moment starting to observe the mode growth. Six unknown ($C_1(t)$-$C_5(t)$ and coil current $I(t)$) should be solved with above six equations (equations 4.13-4.18). The solutions for the perturbed magnetic field on sensor $b_{r}(r_s,t)$ and the coil current $I(t)$ can be obtained both analytically and numerically.

For the analytical solution, we consider the approximation $\gamma \tau_f \ll 1$, which means the response time of the feedback system is much faster than the time scale of the mode growth. Equation 4.17 can be reduced to

$$I = G_p \psi_s(t) - G_i \int_0^1 \psi_s(t) dt - G_d \frac{d\psi_s(t)}{dt}$$  \hspace{1cm} (4.22)

The solutions $b_r(b,t)$ and $I(t)$ can be solved and expressed as

$$b_r(b,t) = \frac{b_{r0}}{\gamma_+ - \gamma_-} \left( \gamma_+ e^{\gamma_+ t} - \gamma_- e^{\gamma_- t} \right)$$  \hspace{1cm} (4.23)

$$I(t) = \frac{tb_p \gamma_+}{\gamma_+ - \gamma_-} \left[ \left( G_p \gamma_+ + G_i + G_d \gamma_+^2 \right) e^{\gamma_+ t} - \left( G_p \gamma_- + G_i + G_d \gamma_-^2 \right) e^{\gamma_- t} \right]$$  \hspace{1cm} (4.24)

where,

$$\gamma_\pm = \frac{1}{2 \left( \tau_d + \alpha G_d \right)} \left[ \left( \gamma_0 \tau_d - \alpha G_p \right) \pm \sqrt{\left( \gamma_0 \tau_d - \alpha G_p \right)^2 - 4 \alpha \left( \tau_d + \alpha G_d \right) G_i} \right]$$  \hspace{1cm} (4.25)

where $G_j = \Gamma_j G_j$, $j = p,i,d$, $\alpha = \frac{\delta W_p + \delta W_{ai}}{\delta W_p + \delta W_{ab}}$. It shows that the solutions of equations 4.23 and 4.24 are both composed by two exponential terms. $\gamma_\pm$ can be identified as
Physical understanding of the feedback control of RWM in RFP

the roots (the growth rate) of the dispersion relation equation 4.6 as well. When \( G_p=G_i=G_d=0 \), the solution equations 4.23 is reduced to \( b_r(b,t) = b_r(b,0)e^{\gamma t} \) which describes the behavior when the mode is free to grow.

For seeking simplicity, we assume the sensor is near the wall, \( r_s \approx b \), therefore \( \alpha \approx 1 \). The numerical calculation by solving the equations 4.13-4.18 directly is performed also under these conditions.

1) When \( G_p \neq 0, G_i=0, G_d \neq 0 \),

the time dependent solutions become

\[
b_r(b,t) = b_0e^{\gamma t}, \quad (4.26)
\]

\[
I(t) = ib_0b(G_p+\gamma,G_d)e^{\gamma t}, \quad (4.27)
\]

where,

\[
\gamma = \frac{\gamma_0\tau_d - \overline{G}_p}{\tau_d + \overline{G}_d} \quad (4.28)
\]

Another index \( \gamma \) is zero in this case. It clearly shows that the RWMs can be linearly stabilized by the sufficient proportional gain \( \overline{G}_p \). Without \( \overline{G}_p \), increasing \( \overline{G}_d \) can decrease the growth rate. However for approaching the zero \( \gamma \), \( \overline{G}_d \) must go to infinity. Thus, \( \overline{G}_d \) can enhance the stabilizing role of \( \overline{G}_p \), but not completely stabilize the mode.
Physical understanding of the feedback control of RWM in RFP

Figure 4.7: The time evolution of the magnetic perturbation $b_r(b,t)$ on the sensor and coil current $I(t)$ for $m=1, n=-6$ mode are plotted in (a) and (b), where $F=-0.05$, $\Theta=1.417$ and $\beta_p=0.02$, $r_b/a=b/a=1.12$, $\tau_b=114\text{ms}$ $r_f/a=1.268$, and $\tau_f=2\text{ms}$. The solid, dash and dot lines present three cases of partially compressed, marginally stabilized and damped mode due to feedback control respectively.

The time evolution of the perturbed magnetic field $b_r(b,t)$ and the coil current $I(t)$ solved numerically for controlling $m=1, n=-6$ mode by proportional gain is illustrated in figure 4.7. The response time of feedback system $\tau_f$ is also considered and set to 2ms as evaluated for RFX-mod. The feedback control is launched at $t=50\text{ms}$ with a constant proportional gain $G_p$. Before $t=50\text{ms}$, the $m=1, n=-6$ mode is free to grow as the exponential form $b_r(b,0)e^{\gamma_0 t}$ where $\gamma_0 = 2.63 \tau_b^{-1} = 23s^{-1}$. The mode growth rate $\gamma_0$ without feedback is calculated by solving equation 2.6 and 4.3 with the parameters of RFX-mod. These parameters are also used in all following calculation unless otherwise specified. After 50ms, the feedback system is launched with constant gain. There are three cases plotted with different gain values. The solid line shows the instability of the mode is suppressed but not completely stabilized ($0<\gamma<\gamma_0$). The dash line shows the marginal stability of the mode. The dot line presents that the mode is damped by the feedback system ($\gamma<\gamma_0$). Figure 4.7(b) plots the corresponding coil current rises when $G_p$ applies, larger $G_p$ leads to higher current. The current goes to zero only when the mode amplitude is decayed to zero. These kinds of evolution are already observed in
Physical understanding of the feedback control of RWM in RFP

RWM feedback control experiments of RFX-mod [24,27]. We note that, there is \( \pi/2 \) phase difference between \( b_0(b,t) \) and \( I(t) \).

Figure 4.8 gives the time evolution of the eigenfuction of the magnetic perturbation \( b_r \) of \( m=1 \) \( n=-6 \) mode for \( \overline{G}_p = 1.7 \gamma_0 \tau_d \) before and after launching the feedback system. It shows the amplitude of magnetic perturbation is suppressed by feedback system in both plasma and vacuum regions.

2) For PI control, \( G_p \neq 0, G_i \neq 0, G_d = 0 \). The coefficients of equation 4.23 become

\[
\gamma_\pm = \frac{1}{2 \tau_d} \left[ \gamma_0 \tau_d - \overline{G}_p \pm \sqrt{ \left( \gamma_0 \tau_d - \overline{G}_p \right)^2 - 4 \tau_d \overline{G}_i } \right]
\]

(4.29)

It shows that the minimum value of the mode growth rate is \( \left( \tau_d \gamma_0 \overline{G}_p \right) / 2 \tau_d \), when the square root of equation 4.29 is zero. The time dependent solutions become

\[
b_r(b,t) = b_{d0} e^{\left( \gamma_0 \tau_d - \overline{G}_p \right) t/2 \tau_d} \left[ \frac{1}{2 \tau_d} \left( \gamma_0 \tau_d - \overline{G}_p \right) t + 1 \right]
\]

(4.30)
Physical understanding of the feedback control of RWM in RFP

\[ I(t) = \frac{b_0 e^{i(\gamma_0 \tau_d - \bar{G}_p)t/2 \tau_d}}{G_p} \left( \gamma_0 \tau_d - \frac{\bar{G}_p}{2 \tau_d} + G_p t + 1 \right) + G_p t \]  

(4.31)

It implies that the stability of the mode is determined by the sign of \( \gamma_0 \tau_d - \bar{G}_p \).

Further increasing \( G_i \), the square root of equation 4.29 becomes an imaginary number, the two indexes \( \gamma_\pm \) become complex conjugate. The solutions of magnetic perturbation at sensor and coil current can be obtained

\[ b_i(b,t) = \frac{b_0}{\cos \beta} e^{i(\gamma_0 \tau_d - \bar{G}_p)t/2 \tau_d} \cos \left( vt/2 - \beta \right) \]  

(4.32)

\[ I(t) = \frac{b_0 e^{i(\gamma_0 \tau_d - \bar{G}_p)t/2 \tau_d}}{G_p} \left( \frac{G_p}{\cos \beta} \cos \left( vt/2 - \beta \right) - \frac{2 G_i}{v} \sin \left( vt/2 \right) \right) \]  

(4.33)

where \( iv = \frac{1}{\tau_d} \sqrt{\left( \gamma_0 \tau_d - \bar{G}_p \right)^2 - 4 \tau_d G_i}, \beta = \arcsin \frac{\gamma_0 \tau_d - \bar{G}_p}{2 \sqrt{G_i \tau_d}} \).

Obviously, it shows that two waves having the opposite direction of propagation are obtained, the combination of two waves then becomes a standing oscillation; and the mode stability depends whether \( \gamma_0 \tau_d - \bar{G}_p \) is positive (unstable) or negative (stable). In figure 4.9(a) and (b) the effect of \( G_i \) is numerically studied by setting \( \bar{G}_p = \gamma_0 \tau_d / 2, \bar{G}_d = 0 \) and \( \tau = 2 \text{ms} \). The time evolution of \( b_i(b,t) \) and \( I(t) \) are plotted separately, when \( \bar{G}_i \) is less than, greater than, and equal to \( \bar{G}_i = \left( \gamma_0 \tau_d - \bar{G}_p \right)^2 / 4 \tau_d \).

Figure 4.9(a) shows three cases of the evolution of \( b_i(b,t) \). The standing oscillation is induced (dot line) when \( \bar{G}_i > \bar{G}_c \). In figure 4.9(b), the corresponding coil current for three cases is plotted. From equation 4.32, it’s known that the mode oscillating frequency is \( v / 4 \pi \); the envelop of the wave grows exponentially (the growth rate is \( \gamma_0 / 4 \)). Figure 4.9(c) plots the solution of equation 4.23 and its two exponential terms \( \frac{b_0 \gamma_+ e^{\gamma_+ t}}{\gamma_+ - \gamma_-} \) and \( \frac{b_0 \gamma_- e^{\gamma_- t}}{\gamma_+ - \gamma_-} \) respectively for the case of \( \bar{G}_i < \bar{G}_c \). Although both of \( \gamma_\pm \) are greater than zero (the amplitudes of two terms increase with time), the signs of two terms of equation 4.23 are opposite. In figure 4.9(c), it shows that the \( e^{\gamma_+ t} \) term is positive and dominant, and the \( e^{\gamma_- t} \) term contributes a smaller negative value and gives a minor modification of the \( e^{\gamma_+ t} \) term.
Figure 4.9: For different integral gain, the time evolution of the magnetic perturbation $b_t(b,t)$ on the sensor and coil current $I(t)$ for $m=1, n=-6$ mode are plotted in (a) and (b) respectively. (c) is the plot of two exponential terms of equation 4.23 (dash and dot lines) and their combination (solid line) corresponding to the solid line case in (a).

3) The influence of the wall penetration time $\tau_b$ on feedback control is investigated and presented in figure 4.10, where the time dependent solutions for different wall penetration time ($\tau_b$ is equal to 114ms, 57ms and 228ms) are plotted for the same value of the proportional gain $G_p=1.78 \tau_d$. Since $\gamma_0 \tau_d$ is constant for the given equilibrium and the fixed wall position, the solid, dash and dot lines during the initial state have growth rates $23s^{-1}$, $26s^{-1}$ and $12.5s^{-1}$ respectively. Figure 4.10 shows that, when $\tau_b$ is small (dash line), the mode has large growth rate intrinsically. However, for this case, the mode is also damped much faster by the feedback system with the same $G_p$. After 0.1s, the mode is more stable than the other two case (solid and dot line), and less coil current is required. From another
Physical understanding of the feedback control of RWM in RFP

point of view, if one set the same amplitude of magnetic perturbations for launching the feedback system, a conducting wall having proper small penetration time should be more efficient for the control system.

Figure 4.10: When fixing the proportional gain $\overline{G}_p = 1.7\gamma_0\tau_d$, the time evolution of the magnetic perturbation $b_r(t,b)$ of $m=1$, $n=-6$ mode on the sensor and coil current $I(t)$ are plotted in (a) and (b) for different $\tau_b$. The solid, dash and dot lines correspond to the growth rates of $23s^{-1}$, $26s^{-1}$ and $12.5s^{-1}$ respectively before feedback $t \leq 50ms$.

4) Now we discuss the effect of the response time $\tau_f$ of the feedback system by using the proportional control $G_p \neq 0, G_i = 0, G_d = 0$.

$$\tau_f \frac{\partial I(t)}{\partial t} + G_p \psi_s(t) + I = 0$$ (4.34)

Solving equations 4.13-4.16, 4.18, and 4.34 associating with the initial value and setting the sensor near the wall $r_s \approx b$, the time dependent solutions of perturbed magnetic field $b_r(t,b)$ and coil current $I(t)$ are derived.

$$b_r(t,b) = \frac{b_m}{\gamma_+ - \gamma_-} \left[ \left( \frac{1}{\tau_f + \gamma_+} \right) e^{\gamma_+ t} - \left( \frac{1}{\tau_f + \gamma_-} \right) e^{\gamma_- t} \right]$$ (4.35)
Physical understanding of the feedback control of RWM in RFP

\[ I(t) = \frac{ib_0 b G_p}{\tau_f (\gamma_+ - \gamma_-)} (e^{i\omega t} - e^{-i\omega t}) \]  \hspace{1cm} (4.36)

where

\[ \gamma_\pm = \frac{1}{2} \left[ (\gamma_0 - 1/\tau_f) \pm \sqrt{(\gamma_0 + 1/\tau_f)^2 - \frac{4 \bar{G}_p}{\tau_f \tau_d}} \right] \]  \hspace{1cm} (4.37)

\( \bar{G}_p = \Gamma_0 G_p \). Obviously, the indexes \( \gamma_\pm \) are also the roots of the following dispersion relation rewritten from equation 4.6 when \( G_i = G_d = 0 \).

\[ \tau_f \gamma^2 + (1-\gamma_0 \tau_f) \gamma - \left( \gamma_0 - \frac{\bar{G}_p}{\tau_d} \right) = 0 \]  \hspace{1cm} (4.38)

With respect to equation 4.28, the finite \( \tau_f \) actually introduces a new root. It is found, when \( 0 < \bar{G}_p / \tau_d < \gamma_0 \), one index is always greater than zero, \( \gamma_+ > 0 \); it means that the mode cannot be stabilized by the feedback system.

When \( \bar{G}_p / \tau_d \geq \gamma_0 \), the square root \( \sqrt{(\gamma_0 + 1/\tau_f)^2 - \frac{4 \bar{G}_p}{\tau_f \tau_d}} \) could be real or imaginary number. After a short calculation, it is found that, when \( \frac{C_-}{\gamma_0} < \tau_f < \frac{C_+}{\gamma_0} \), where

\[ C_\pm = \left( \frac{2 \bar{G}_p}{\gamma_0 \tau_d} - 1 \right) \pm \sqrt{\left( \frac{2 \bar{G}_p}{\gamma_0 \tau_d} - 1 \right)^2 - 1} \], the square root in equation 4.37 is imaginary; beyond this range, the square root is real. Hence, associating with the condition \( \bar{G}_p / \tau_d \geq \gamma_0 \), four different ranges of \( \tau_f \) are found,

1) \( 0 \leq \tau_f \leq \frac{C_-}{\gamma_0} \), \hspace{1cm} 2) \( \frac{C_-}{\gamma_0} < \tau_f < \frac{1}{\gamma_0} \)

3) \( \frac{1}{\gamma_0} < \tau_f < \frac{C_+}{\gamma_0} \), \hspace{1cm} 4) \( \frac{C_+}{\gamma_0} \leq \tau_f \).

In case 1, the indexes equation 4.37 are both less than zero \( \gamma_\pm < 0 \), thus the mode can be stabilized by the feedback system directly. For case 2 and 3, the square root
Physical understanding of the feedback control of RWM in RFP

of equation 4.37 becomes imaginary number. It implies the feedback system induces a standing oscillation of the mode. However, the mode decays in case 2, but diverges in case 3. For both cases, the growth rate of the mode is equal to $\left(\gamma_0 - 1/\tau_f\right)/2$. For the last case, the square root of equation 4.37 is real and positive, thus both roots are positive. The mode amplitude is no more oscillating but exponentially growing since $\gamma_+ > 0$. The reason is the response of the control system is too slow to control the mode.

Figure 4.11 (a) and (b) plots four time dependent solutions corresponding to the cases 1-4. The solid line expresses the region where $\tau_f$ satisfies the condition of case 1 and the mode is stabilized by the feedback system without oscillation. The dash line presents the mode is damped by feedback with oscillation. The dot line shows, when $\tau_f$ is in the case 3, the oscillation of the mode is divergent. For the case 4, the mode grows monotonically. Figure 4.11 (c) plots the analytical solution of equation 4.35 corresponding to the solid line case in (a) and its two exponential terms $b_0 \frac{1/\tau_f + \gamma_+}{\gamma_+ - \gamma_-} e^{\gamma_+ t}$ and $-b_0 \frac{1/\tau_f + \gamma_-}{\gamma_+ - \gamma_-} e^{\gamma_- t}$. Obviously, it shows that the combination of two damping terms with opposite sign leads to the evolution of the mode. When $G_p$ start to be launched, the time evolution of $b_r(b,t)$ does not coincide with the term $e^{\gamma_+ t}$ due to the finite $\tau_f$. With the time going, the $e^{\gamma_+ t}$ term is dominant, and the $e^{\gamma_- t}$ term decays faster than the $e^{\gamma_+ t}$ term.

In summary, for stabilizing the mode by feedback, the conditions $\bar{G}_p / \tau_d \geq \gamma_0$ and $\tau_f \leq \frac{1}{\gamma_0}$ must be satisfied. To get better mode control performance (non oscillating) for a given $\bar{G}_p$, $\tau_f$ should be less than $C_- / \gamma_0$. In fact, the feedback system on current RFX-mod also has enough capability and power to cope with this requirement for small $\tau_f$. 
Physical understanding of the feedback control of RWM in RFP

Figure 4.11: For a given proportional gain, the evolution of the magnetic perturbation $b_r(b,t)$ on the sensor and coil current $I(t)$ of $m=1,n=-6$ mode for different system response time $\tau_f$ are presented in (a) and (b) respectively. Four status of the mode due to different $\tau_f$ are given out, where $\tau_f = 0.045\gamma_0^{-1}$, $0.45\gamma_0^{-1}$, $1.12\gamma_0^{-1}$ and $20.2\gamma_0^{-1}$ corresponds to $0.002s$, $0.02s$, $0.05s$ and $0.9s$ in reality. (c) plots the solution of equation 4.35 and its two exponential terms corresponding to the solid line case in (a).

4.6 Summary and discussion

We introduced the feedback system into the cylindrical model described in chapter 2 in RFP plasmas, where the plasma pressure, compressibility, plasma rotation, parallel viscosity and a resistive wall with finite thickness are included in this model. The eigenmode equation with appropriate boundary condition which is generalized to include the PID feedback controller is derived for the study; and the dispersion relation for RWM with feedback control in terms of the potential energy components are derived under the assumption of $\omega \ll \omega_a$. 
Physical understanding of the feedback control of RWM in RFP

The RFP plasmas usually have slow rotation (without external momentum injection such as tangential N.B.) and the RWMs in RFPs do not rotate \((\omega_r = 0)\) except the occurrence of very high speed of plasma rotation \((v_0 \geq v_{oc})\). In addition, RWMs in RFPs are non-resonant modes, having large \(k_\parallel\) and \(\omega_a = k_\parallel B_0 \neq 0\) inside the plasma. Therefore, the mode frequency \(\omega\) can be ignored in the eigenmode equation, which leads to the solution inside the plasma (eigenfunction of RWMs, \(\xi_r(r)\) or \(b_r(r)\)) being uniquely determined by the equilibrium parameters (except a normalization factor). The quantity \((\xi'_r/\xi_r)_a\) is independent from the feedback application. This conclusion is valid also for the case having plasma rotation and the viscous dissipation if \(v_0 < v_{oc}\). The relative plasma potential energy \(\delta W_p/\xi_a^2\) is not influenced by the feedback, which shows to be potentially unstable to the external kink even when the feedback stabilization of RWM exists. The role of the feedback control is to modify the magnetic perturbation outside the wall, and thus to modify (reduce) the eddy current induced by the plasma perturbation on the resistive wall. As for \(v_0 > v_{oc}\), the RWM is stabilized by the plasma rotation and dissipation, and the mode rotates with high speed. Both the rotating magnetic perturbation and the possible signal from the feedback coils are totally shielded by the resistive wall because \(\omega \gg \tau_b^{-1}\); And the feedback system play no role in this situation.

Based on the above conclusion, the linear time evolution of the RWM under the feedback control (linear theory) is easily to be studied by the calculation only in the vacuum region if the growth rate without feedback, \(\gamma_0\) is known. There is no need to calculate the plasma responses for each time step. The effects of the wall proximity and of the location of the sensor have been discussed. The time dependent solution for different feedback scenario of PID controller has also investigated in details. The influence of the response time scale of the feedback system on the stabilizing the RWMs is also studied.

In RFP plasmas, only \(m=1\) RWMs are originally unstable. The toroidal coupling can induce some sideband mode (with \(m=1 \pm 1, m=1 \pm 2\ldots\) and the same \(n\)) growing with small amplitudes and the same growth rates. When the \((m=1,n)\) mode is stabilized, its sideband modes are also disappear. Therefore we don’t expect the toroidal coupling will essentially change the above conclusion on the feedback stabilization.
Physical understanding of the feedback control of RWM in RFP

Since in RFPs, the linearly growing RWMs are non-resonant ($k_\parallel \neq 0$ inside plasma) and non-rotating modes ($\omega_r$ is negligible), the feedback stabilization is particularly simple as described above. These analyses could be also valid for the large aspect ratio tokamak if having the RWM rational surface outside the plasma and with slow plasma rotation. As for the case where the RWM having the rational surface inside the plasma, the eigenvalue $\omega$ is not negligible in the eigenmode equation; or there are more than one poloidal mode being originally unstable and with strong coupling; this issue needs to be studied carefully.

Moreover, in a real RFP device, the gap on the shell and additional conducting structures around the machine may have an influence on the RWM. The discrete structure of feedback coils also produces a sideband during the control. These may slightly modify the character of the time evolution solution of RWMs under the feedback application, and should be further studied.
Two approaches to stabilize RWMs have been extensively investigated in recent years, namely, active control and rotational stabilization. Active control technique uses a set of sensors to measure the magnetic perturbation induced by RWM; then it applies the magnetic field generated by a set of active coils to control the mode instability. This technique was extensively studied theoretically [56, 63-69]. Many experiments have reported successful application of this technique for controlling RWMs in both RFP and tokamak plasmas [23,24,27,70-72]. In particular, we studied RWM feedback control in RFP plasmas in chapter 4.

The RWM suppression via plasma rotation is due to the presence of various dissipations, such as the plasma viscosity and/or resistivity, the continuum spectra damping (Alfvén continuum, sound wave continuum), and kinetic resonances between the mode and the plasma particles. Both theory and experiments have shown that, in tokamak plasmas, RWMs can be suppressed by the plasma rotation [34,40,73-75]. In particular, recent experiments from DIII-D [43,72] and JT-60U [44], with balanced neutral beam injection, produce RWM stable plasmas at very slow, or even vanishing, toroidal flow. Theoretical models, based on drift kinetic theory, have been proposed for physical understanding and accurate prediction of the RWM stabilization. It is found that the main dissipation is due to the mode resonance with the precession drift motion of trapped particles [42,45,76] in tokamak plasmas.

In contrast, rotational stabilization of the RWM on presently operating RFP devices has not been observed. Theoretical studies in section 5.1, based on the fluid theory involving the plasma viscosity, predicted the critical toroidal flow velocity, for stabilizing the mode, being in the Alfvén velocity range [22]. RFP
plasmas require much higher critical velocity than tokamak plasmas, due to the fact that the resonant surfaces of the mode in RFPs are much farther from the plasma edge than those in tokamaks [59]. Such a high velocity is not observed in the present day RFP experiments.

The effects of the RWM resonance with the particle motions were previously investigated for RFP plasmas [77], but without detailed physics clarification of which conditions that make the kinetic effects important and that contribute to the RWM stabilization in RFPs. In this chapter, the present work fills in this gap. The hybrid toroidal stability code MARS-K is adopted for this study, and a new module for computing various potential energy components has been integrated into the MARS-K code. This allows the in-depth analysis and a better understanding of the physical mechanisms behind the numerical data.

In section 5.4, it is found that the transit resonance of the passing ions may provide possible stabilizing effects at slow plasma rotation, particularly for high beta RFP plasmas. The critical rotation velocity, for suppression of the most unstable mode, is reduced to the ion acoustic range, which is much smaller than the previous prediction based on the fluid theory in section 5.1. High $\beta_p$ plasma regime is relevant to the RFP configuration, which is characterized by achieving high $\beta_p$ plasmas; even in the currently operating RFP devices (e.g. MST), it is possible to reach the $\beta_p$ value up to 20% [78], by performing the so called Pulsed Poloidal Current Drive (PPCD). Another critical ingredient in bringing the kinetic effect to play an important role in the RWM stabilization is the resistive wall position $b$, which has been discussed in early MHD studies [73,74]. This work provides the explanation from the point of view of potential energies. It is found that, by moving the resistive wall farther from the plasma edge and close to the critical wall position of (marginally stable) ideal kink, the kinetic effect can become much more important for the RWM stabilization. Numerical results indicate that kinetic stabilization always occurs when the wall locates near the critical position $b_c$ where $\delta W_b \approx 0$ ($\delta W_b = \delta W_F + \delta W_{vb}$, $\delta W_F$ is the plasma potential energy component and $\delta W_{vb}$ the vacuum energy component for ideal wall with minor radius $b$, see detailed definitions in the next section). In fact, both factors -- increasing $\beta_p$ and farther wall position $b$ -- can lead to the decrease of $\delta W_b$, and hence a more significant role from the kinetic resonant energy $\delta W_k$. In section 5.5, preliminary investigation of the plasma collisionality effect is also studied, which
Kinetic damping on RWM in RFP plasmas shows that the collisionality plays a minor role in the kinetic stabilization of the RWM in low beta RFP plasmas.

5.1 Stabilization by plasma rotation and dissipation in fluid theory

Firstly, we study the rotational stabilization of RWM based on the fluid theory involving the plasma viscosity (equation 2.5). In this study, the cylindrical MHD model introduced in section 2.1 and the corresponding CMR code are taken into account. It is found that in comparison with tokamak, the non resonant RWM in RFPs requires much higher plasma rotating frequency for the stabilization (with dissipation). The present model only takes into account the viscosity damping as equation 2.5. The kinetic resonances effects will be discussed in the next sections. For the case having only viscosity effect ($\beta=0$), it is found that when the velocity of the plasma rotation increases, due to the momentum input from the plasma via the dissipation, the mode slip frequency w. r. to the wall increases, then the stability window appears in an interval of the resistive wall position ($b/a$). This window starts to open in the vicinity of the critical wall distance for the ideal external kink mode with ideal wall at $r=b_c$. When the rotation velocity further increases, the window size extends toward the plasma boundary, where figure 5.1(a) and (b) shows these numerical results for $m=1, n=-5$ mode.

![Figure 5.1(a)](image-url)
Figure 5.1: The plot of (a) the growth rate $\gamma$ and the rotating frequency $\omega_r$ (normalized by the poloidal Alfvén frequency $\omega_{0A}=a/V_{A0}$, $V_{A0}$ is the poloidal Alfvén velocity at plasma edge) of $m=1$, $n=-5$ mode as a function of the wall position $b$ for various plasma rotation velocities $v_0$ with $\eta_0=0.01$, $F=-0.05$, $\Theta=1.41$, and $\beta_p=0.0$. The uniform toroidal flow is assumed.

For a given wall position, there is a critical value of the rotation velocity $V_{oc}$, above which the stability can be reached if the viscosity is sufficient. The value of $V_{oc}$ varies with the mode number and equilibrium parameters. As the wall falls closer to the plasma, the larger $V_{oc}$ value is needed. In the case of a resistive wall located near the plasma edge, the critical velocity required for the stabilization satisfies the condition [74] of $kV_{oc} \approx (k_{||}V_A)_{a}$ or $kV_{oc} \approx (k_{||}V_A)_{r=0}$, where $k_{||}$ is the parallel RWM wave number. Both INRM and ENRM, being the non-resonant modes, have their rational surfaces far from the plasma edge, so $k_{||}(a)$ and/or $k_{||}(0)$ are larger ($k_{||}(a) \approx 0.2-1$ for RFX-mod) than that of the RWM in Tokamaks (which has $k_{||}(a) \approx 0$). Therefore, the stabilization of RWMs in RFPs by plasma rotation requires higher $V_{oc}$ than in tokamak; in particular, $V_{oc}$ is much larger than the rotation velocity of current operated RFP plasmas where no external momentum source (e.g. neutral beam injection) is present. In figure 5.2(a) the values of $kV_{oc}$ for both INRM and ENRM are plotted and compared with the values of $(k_{||}V_A)_{r=0}$ and $(k_{||}V_A)_{r=a}$ for wall positions $b/a=1.12$ (RFX-mod) and $r=1.01$. The corresponding values of $V_{oc}$ for different modes are plotted in figure 5.2 (b). The critical toroidal plasma rotating velocity $V_{oc}$, required for the RWM stabilization, is in the range of Alfvén velocity (around $0.2V_{A0}$-1.0$V_{A0}$, where $V_{A0}$ is Alfvén velocity calculated by the edge poloidal magnetic field).
Kinetic damping on RWM in RFP plasmas

5.2 Model and formulations in toroidal geometry

5.2.1 Kinetic model in MARS-K

Since many works suggest that the kinetic effect may play an important role in RWM stabilization [42,45,76], it is necessary, therefore, to investigate the kinetic effect on RWM in RFP plasmas. For this study, we start from the single fluid MHD equations in the presence of the toroidal plasma flow. The core equations, where drift kinetic effects are involved, are solved numerically by the hybrid toroidal stability code MARS-K in the self-consistent approach. We assume that all the perturbations have the form of $A(s, \chi, \phi, t) = A(s, \chi) e^{-i \alpha s - i \omega t}$ for a given
Kinetic damping on RWM in RFP plasmas

curvilinear coordinate system \((s, \chi, \phi)\), where \(s\) is the normalized radial coordinate with respect to the equilibrium flux surface, \(\chi\) is the generalized poloidal angle, and \(\phi\) is the toroidal angle. The hybrid MHD equations involving the kinetic terms are briefly written in the Eulerian frame

\[
-i(\omega + n\Omega)\hat{\xi} = \mathbf{v} + (\hat{\xi} \cdot \nabla\Omega) R^2 \nabla \phi \tag{5.1}
\]

\[
-i\rho(\omega + n\Omega)\mathbf{v} = -\nabla \cdot \mathbf{p} + \mathbf{j} \times \mathbf{B} + \mathbf{J} \times \mathbf{Q} - \rho \left[ 2\Omega \hat{Z} \times \mathbf{v} + (\mathbf{v} \cdot \nabla\Omega) R^2 \nabla \phi \right] \tag{5.2}
\]

\[
-i(\omega + n\Omega)\mathbf{Q} = \nabla \times (\mathbf{v} \times \mathbf{B}) + (\mathbf{Q} \cdot \nabla\Omega) R^2 \nabla \phi \tag{5.3}
\]

\[
i(\omega + n\Omega) p = \mathbf{v} \cdot \nabla P \tag{5.4}
\]

\[
\mathbf{j} = \nabla \times \mathbf{Q} \tag{5.5}
\]

where \(\omega = i\gamma - \omega_r\) is the complex eigenvalue (\(\gamma\) being the mode growth, \(\omega_r\) the mode rotation frequency in the laboratory frame). The mode frequency is corrected by a Doppler shift \(in\Omega\), with \(n\) - the toroidal mode number, \(\Omega\) - the plasma rotation frequency in toroidal direction \(\phi\). \(\hat{\xi}, \mathbf{v}, \mathbf{Q}, \mathbf{j}, \mathbf{p}\) represent the perturbed quantities: plasma displacement, perturbed velocity, magnetic field, current, and pressure tensor, respectively. Moreover \(\rho\) is the unperturbed plasma density. \(R\) is the plasma major radius. \(\hat{Z}\) is the unit vector in the vertical direction. The \(\mathbf{B}, \mathbf{J}, \mathbf{P}\) represents the equilibrium quantities: magnetic field, current and pressure respectively. A conventional unit system is assumed with the vacuum permeability \(\mu_0 = 1\). For RWM study, a set of vacuum equations for the perturbed magnetic field \(\mathbf{Q}\) and the resistive wall equation based on the thin-shell approximation are solved together with equations 5.1-5.5 [66]. The plasma resistive term in the Ohm’s law is also dropped. Note that the ordinary \(5/3 PV \cdot v\) term is dropped from equation 5.4 for the perturbed fluid pressure. This term is replaced by the drift kinetic terms which are involved into the MHD equation via the perturbed kinetic pressure tensors \(\mathbf{p}\)

\[
\mathbf{p} = p\mathbf{I} + p_\parallel \hat{b}\hat{b} + p_\perp (\mathbf{I} - \hat{b}\hat{b}) \tag{5.6}
\]
Kinetic damping on RWM in RFP plasmas

where \( \hat{b} = B / B \), \( B = |B| \), \( \mathbf{I} \) is the unit tensor. \( p \) is the scalar fluid pressure perturbation, \( p_{\parallel} (\xi_{\parallel}), p_{\perp} (\xi_{\perp}) \) are the parallel and perpendicular perturbations of the kinetic pressure, computed by

\[
p_{\parallel} e^{-i\omega t} = \sum_{\varepsilon} \int d\Gamma M v_{\parallel}^2 f_{\parallel}^{i}
\]

\[
p_{\perp} e^{-i\omega t} = \sum_{\varepsilon} \int d\Gamma M v_{\perp}^2 f_{\perp}^{i}
\]

The summation in equation 5.7-5.8 is over the electron and ion components, the integral is carried out over the particle velocity space \( \Gamma \), \( M \) is the particle mass. \( v_{\parallel}, v_{\perp} \) are the parallel and perpendicular components of the particle velocities, \( f_{\parallel}^{i} \) is the perturbed particle distribution function, which is derived by solving the perturbed drift kinetic equations for each particle species. Following the approaches by Antonsen [79] and Porcelli [80], the expression for \( f_{\parallel}^{i} \) is

\[
f_{\parallel}^{i} = -f_{e}^{0} e^{-i\omega t} \sum X_{m} H_{m|n q l i} \lambda_{ml} e^{i(\Phi(t) + i\omega_{\alpha} t)}
\]

\( f_{e}^{0} \) is the energy derivative of particle equilibrium distribution function (which is assumed Maxwellian, since only the thermal particles are considered in this work), \( \varepsilon \) is the particle total energy. \( \varepsilon_{k} = \varepsilon - Z \varepsilon \Phi \) is the kinetic energy of the particle, being \( \Phi \) the equilibrium electrostatic potential with \( Z \varepsilon \) the charge number. The symbol \( \tilde{\phi}(t) = \phi(t) - \langle \phi \rangle t \) denotes the periodic part of the particle motion projected along the toroidal direction being \( \langle \cdot \rangle \) the average over the particle bounce period, \( m, n \) correspond to the toroidal and poloidal wave numbers respectively. \( l \) is the harmonic number in the bounce orbit expansion. Both \( X_{m} \) and \( H_{m|n q l i} \) are related to the perturbed particle Lagrangian [81]. \( X_{m} \) denotes the poloidal Fourier harmonics with respect to the perpendicular fluid displacement and the magnetic field perturbation; \( H_{m|n q l i} \) is the geometrical factor associated with the equilibrium quantities. \( \lambda^{\alpha}_{ml} \) represents the mode-particle resonance condition,

\[
\lambda^{\alpha}_{ml} = \frac{n \left[ \omega_{\alpha} + \left( \hat{\omega}_{k} - 3/2 \right) \omega_{\perp} + \Omega \right] + \omega}{n \omega_{d} - \left[ \alpha (m - n q) + l \right] \omega_{k} + n \Omega + \omega + i \nu_{eff}}
\]
where $\omega_\lambda$ and $\omega_r$ are the diamagnetic drift frequencies with respect to the density and temperature gradients respectively, $q$ is the safety factor, $\nu_{eff}$ is the effective collision frequency, $\hat{\varepsilon}_i = \varepsilon_i / T$ is the particle kinetic energy normalized by the temperature. In the integration over the particle bounce orbit, it is assumed that the effect of finite radial excursion width of particles across the magnetic surfaces is negligible. $\omega_d$ is the bounce-orbit-averaged precession drift frequency. For trapped particles, $\alpha = 0$, $\omega_b$ is the bounce frequency. For passing particles, $\alpha = \sigma$, $\sigma = \text{sign}(v_i)$, $\omega_p$ represents the transit frequency. In the following discussion, we denote $\omega_p$, the transit frequency, in order to be distinguished from the bounce frequency. Equation 5.10 includes the bounce and precession resonances of the trapped particles and the transit resonance of passing particles. The imaginary part of the resonant operator represents the energy transfer between the mode and the particles. A simple collisionality effect is also included in the resonant operator. The detail of kinetic pressure equations are presented in appendix.

We note that in the self-consistent approach, the unknown eigenvalue $\omega$ enters into the resonance operator equation 5.10. This is an important aspect for the self-consistency of the kinetic formulation. It requires an iterative loop to find the converged eigenvalue.

### 5.2.2 Quadratic energy terms

In order to gain an insight into the physics of the rotation stabilization of RWMs, the computation of the quadratic form consisting of various energy components have been integrated into MARS-K code. The well known quadratic energy terms are constructed, by multiplying equation 5.2 by $\xi_\perp$ and integrating over the plasma volume $V^p$. Then the following energy components of fluid potential energy $\delta W_F$ [11,33] and kinetic potential energy $\delta W_K$ are defined as,

$$\delta W_F = \delta W_{mb} + \delta W_{mc} + \delta W_{pre} + \delta W_{cur}$$

(5.11)

$$\delta W_{mb} = \frac{1}{2} \int_{r^p} |Q_\perp|^2 Jd\phi$$

(5.11a)
Kinetic damping on RWM in RFP plasmas

\[
\delta W_{mc} = \frac{1}{2} \int_{\gamma_p} B^2 \left| \nabla \cdot \xi_{\perp} + 2 \xi_{\perp} \cdot \kappa \right|^2 J ds d\phi \tag{5.11b}
\]

\[
\delta W_{pre} = -\frac{1}{2} \int_{\gamma_p} (\xi_{\perp} \cdot \nabla P) (\kappa \cdot \xi_{\perp}^* ) J ds d\phi \tag{5.11c}
\]

\[
\delta W_{cur} = -\frac{1}{2} \int_{\gamma_p} J_{||} (\xi_{\perp}^* \times b) \cdot Q_{||} J ds d\phi \tag{5.11d}
\]

In equation 5.11, \( J \) is the Jacobian of the flux coordinates, \( \kappa \) is the curvature of the magnetic field line. \( \delta W_{mb} \) is the magnetic bending term representing the energy required to bend magnetic field lines, \( \delta W_{mc} \) is the energy necessary to compress the magnetic field. Both \( \delta W_{mb} \) and \( \delta W_{mc} \) are stabilizing terms. \( \delta W_{pre} \) and \( \delta W_{cur} \) represent the potential sources of instability, referred as pressure driven term and current driven term respectively. Both of them can be negative and drive the instabilities. The surface term \( \delta W_S \) vanish due to zero surface current. In the energy calculation, we neglect the Coriolis force terms in RHS of equation 5.2 assuming subsonic equilibrium. One gets the drift kinetic energy \( \delta W_k \) from the kinetic pressure tensor term in equation 5.2,

\[
\delta W_k = \frac{1}{2} \int_{\gamma_p} J ds d\phi \left[ p_{||} \frac{1}{B} \left( Q_{||} + \nabla B \cdot \xi_{\perp}^* \right) + p_{\perp} \kappa \cdot \xi_{\perp}^* \right] \tag{5.12}
\]

In the kinetic energy, a surface term \( \frac{1}{2} \int_{S_p} p_{||} \xi_{\perp}^* \cdot n J ds d\phi \) is negligible if the equilibrium pressure vanishes at plasma edge \( P=0 \) (the perturbed kinetic pressure is roughly proportional to the equilibrium pressure), where \( S_p \) is the plasma surface, \( J_s = |\nabla \Phi| J \) is the surface Jacobian, \( n \) is an outward normal vector to the vacuum region. By taking into account equations 5.7, 5.8 and 5.9, the kinetic energy can be further written

\[
\delta W_k = \frac{\sqrt{\pi}}{2 B_0} \sum_{e,j} \int d\Psi P_{e,j} \left\{ \int d\vec{\ell} \int d\tau_{||} e^{-\tau_{||}} \sum_{\sigma} \left[ \int d\lambda \sum_{j} \lambda_{j}^{*} \tau_{\lambda} \left( e^{-d_{||} \lambda_{j}} \right) \frac{\partial H_{j}}{\partial \lambda_{j}} \right] \right\} \tag{5.13}
\]

where a derivation of equation 5.13 is presented in appendix, \( \Psi \) is the equilibrium poloidal flux, \( P_{e,j} \) denotes the ion and electron equilibrium pressure, \( \Lambda = B_0 \mu / e_k \) (B_0
Kinetic damping on RWM in RFP plasmas

is the on-axis field strength), $\sigma = \text{sign}(v_{||})$, $\hat{l} = l + am$. The integration is taken in both the real and the velocity space. The sum is over the poloidal Fourier harmonics $m$ and bounce harmonics $l$, the passing and trapped particles, as well as the particle species $(e, i)$. For trapped particles, $\alpha = 0$, $\nu = 1/2$ and $\tau_b$ is the bounce period normalized by a factor $\sqrt{M/2\varepsilon}$; for passing particles, $\alpha = \sigma$, $\nu = 1$ and $\tau_b$ represents the normalized transit period. $H_L$ is the perturbed particle Lagrangian [81].

$$H_L(s, \chi, \Lambda) = 2 \left(1 - \frac{\Lambda}{h}\right) \xi_\perp \cdot \kappa + \frac{\Lambda}{B_0} \left( Q_\perp + \xi_\perp \cdot \nabla B \right)$$  \hspace{1cm} (5.14)

where $h = B_0 / B$.

The vacuum energy without wall $\delta W_{ve}$ and with an ideal wall $\delta W_{vb}$ at the minor radius $b$ are computed as well.

$$\delta W_{ve} = \frac{1}{2} \int_{s_v} |Q_\perp|^2 Jd sd \chi d\phi = -\frac{1}{2} \int_{s_v} b^n_1 \tilde{\nabla}_i J d \chi d \phi$$  \hspace{1cm} (5.15)

$$\delta W_{vb} = \frac{1}{2} \int_{s_v} |Q_\perp|^2 Jd sd \chi = -\frac{1}{2} \int_{s_v} b^n_1 \tilde{\nabla}_i J d \chi d \phi$$  \hspace{1cm} (5.16)

where $b^n_i$ is the normal component of magnetic perturbation; $\tilde{\nabla}_i^{\text{con}}$ is the conjugation of perturbed magnetic scalar potential determined by the ideal wall position and $b^n_i$ [11]. These two terms are associated with the magnetic perturbation in the vacuum region, induced by the plasma instability. They are always positive and playing a stabilizing role in RWM instability. Moreover, $\delta W_{ve}$ and $\delta W_{vb}$ can be transformed into a surface integral as shown in equations 5.15 and 5.16.

Equations 5.11-5.13, 5.15 and 5.16 are integrated into MARS-K code, which will be applied to the analyses of RWM physics in the present work.

5.3 General description of RWMs in RFP plasmas

In RFPs, the RWM instabilities are current driven modes with a rich spectrum of toroidal mode number $n$. The dominant poloidal mode number is $m=1$. The RWMs
in RFPs always have their resonant surfaces outside the plasma, and thus are called ‘non-resonant modes’. They are separated into two types: one is the so-called externally non-resonant modes (ENRMs), having their rational surfaces located at q<q(a)<0 (q(a) is the safety factor at plasma edge r=a); the other is the internally non-resonant modes (INRMs) with rational surface corresponding to q>q(0)>0 as described in chapter 1. In toroidal geometry the toroidal coupling effect may cause many poloidal harmonics of RWM coupled and all together contribute to the mode growth rate. Therefore the mode growth rate is calculated for a given n number, with all poloidal harmonics (m) being involved. Specifically, in the following study, we choose n>0, it means for a given n number, each poloidal harmonic with m>0 corresponds to INRM; the poloidal harmonics with m<0 represents ENRMs. Figure 5.3 plots the poloidal harmonics of the eigenfunction, the normal component (in the normal direction to the equilibrium flux surface) of the plasma displacement for the most unstable INRM n=6 in shallow reversal case[69,82]. It is found that the amplitude of m=1 mode is much larger than that of the other poloidal harmonics, which is dominant and gives the major contribution to n=6 mode growth rate. In contrast to Tokamak plasmas, figure 5.3 clearly shows the weak mode coupling of poloidal harmonics in RFPs. This is due to the fact that the strong poloidal field makes the poloidal asymmetry weaker than that in Tokamak, which leads to less importance of toroidal effect. From another point of view, the stronger poloidal field makes low q (safety factor) configuration; and the distance between two neighboring rational surfaces of RWMs for a given n with different m (e.g. m and m±1), is much larger than that in tokamak, which leads a weaker poloidal mode coupling. This character of RWMs also suggests that the cylindrical model could be a good approach in RFP [61]. The peak of resonant m=0 mode is induced by Alfvén resonance, since the condition \( |\omega - n\Omega| \approx |k_iV_a| = 0 \) is satisfied at q=0, where \( k_i \approx m - nq \) is parallel wave vector, \( V_a \) is the Alfvén velocity. Our calculation finds that the m=0 harmonic, as secondary mode, in fact only slightly affects mode instability.
 Kinetic damping on RWM in RFP plasmas

In RFP the existing MHD study [22,59] and the previous section on the dissipation of classical viscosity in cylindrical geometry has shown that the critical toroidal plasma rotating velocity $V_{oc}$, required for the RWM stabilization, is in the range of Alfven velocity around $0.2V_{A\theta}$-$1.0V_{A\theta}$. The stabilization of RWMs in RFPs by plasma rotation requires higher $V_{oc}$ than that in tokamak. However, more complete description of RWM physics should include kinetic damping effects.

In the following study, we chose the density profile modeled as $n(s) = n_0 (1-s^2)$ for both electrons and ions. The pressure profile is given by the polynomial $P(s) = P_0 \left(1 + a_1 s^2 + a_2 s^4 + a_3 s^6\right)$. For sake of simplicity, a constant toroidal plasma flow velocity is considered. Different values of poloidal beta $\beta_p = \frac{8\pi <P>}{I_p^2}V_{tot}$ are chosen, where $<P>$ is the equilibrium plasma pressure averaged over the plasma volume $V_{tot}$, $I_p$ is the plasma current. The other parameters are mostly taken similarly from RFX-mod, such as the inverse aspect ratio $\varepsilon = a/R =0.2295$, ion and electron density at magnetic axis $n_{i0} = n_{e0} = 2.5 \times 10^{19} / m^3$ and the temperature ratio between the ion and electron is $T_i/T_e =0.7$. The reversal parameter $F = B_p(a)/<B_p>$ is chosen as $F\sim0.06$. When $\beta_p$ value changes the following parameters will be kept almost invariant:

Figure 5.3: The normal component of the plasma displacement for n=6 RWM in RFP. The amplitude of its poloidal Fourier harmonics (m=-5 to 5) are plotted along the minor radius, by the fluid model in MARS-F with $\beta_p =0.03$, $F=-0.063$, $\Theta=1.47$, $q(0)=0.146$, $q(a)=-0.01$ and without plasma rotation. A straight field line coordinate system is used. No plasma rotation is assumed.
Kinetic damping on RWM in RFP plasmas

F~0.06, and \(q(0)\sim0.144, q(a)\sim0.01\) unless otherwise stated. Obviously, the pinch parameter \(\Theta = B_\parallel(a)/<B_\parallel>\) has to be changed with \(\beta_p\) correspondingly. Moreover, several locations of the resistive wall will be taken from \(b/a=1.12\) (which is the case of RFX-mod) to \(b/a=2.0\), where \(b\) is the wall minor radius and \(a\) is the plasma radius.

5.4 Kinetic Damping effects on RWMs in RFP

In this study, we consider the kinetic damping effects on the RWM stabilization in RFP plasmas, where various kinetic mechanisms are taking into account. As described in the resonance operator equation 5.10, the following effects are included: the mode resonance with the precession motion of the trapped particles (both ions and electrons), the bounce motion of the trapped ions, and the circulating (transit resonance) of the passing ions. It will be shown that these are most important effects in the currently studied subject. In the following sections, the word “full kinetic” will be used and refer to the combined resonant effects mentioned above. The bounce and transit frequencies of electrons are much higher than that of ions, resulting in negligible contribution to the kinetic resonance effect, and will be neglected in the following study.

5.4.1 Effects of various kinetic resonances on the RWMs

(1) Stabilization for wall position at \(b/a=1.12\)

i) Results for the most unstable \(n=6\) mode

In this section, the investigation is carried out by using the parameters of RFX-mod, with the wall located rather close to the plasma edge, \(b/a=1.12\).

For the shallow reversal parameter \(F\sim0.06\), \(n=6\) INRM is the most important unstable RWM in RFX-mod. Actually the dominant \(m=1\ n=6\) mode is the mode with the closest resonant surface to the plasma. Therefore, \(n=6\) is the easiest to be stabilized mode in RFPs although it has the largest growth rate either. Since \(\delta W_\kappa\) is proportional to equilibrium pressure \(p\) (see equation 5.13), increasing \(\beta_p\) value could amplify the kinetic resonance with RWM. We perform the velocity scan in
Kinetic damping on RWM in RFP plasmas

the range of $\Omega/\omega_A = 0 - 0.1$ ($\omega_A = \frac{B_0}{R_0 \sqrt{\mu_0 \rho_0}}$ is the Alfvén frequency at the magnetic axis.) for five $\beta_p$ value ($\beta_p = 0.03, 0.06, 0.11, 0.15$ and $0.17$) and keeping almost the same $q(r)$ profiles, $q(0) = 0.144$, $q(a) = -0.01$. The growth rates of $n=6$ RWMs as function of normalized plasma rotation frequency for different $\beta_p$ values are plotted in figure 5.4. It is observed that for $\Omega < 0.02 \omega_A$ the mode instability is enhanced by high $\beta_p$ value; while further increasing the plasma rotation, the mode growth rates decrease due to the kinetic damping effect. In higher $\beta$ cases, the growth rates significantly decrease and finally vanish at a critical rotating frequency $\Omega_c$, where the modes become stable. It shows that $\Omega_c = 0.041 \omega_A$ for $\beta_p = 0.15$ and $\Omega_c = 0.028 \omega_A$ for $\beta_p = 0.17$. This results indicates that in the high $\beta_p$ plasma, the kinetic effects lead to a significant reduction of the critical rotating frequency $\Omega_c$, which is in the ion acoustic range, and much smaller than $\Omega_c \approx k V_A / n \approx 0.16 \omega_A$ given by the previous fluid study [59] for stabilizing $n=6$ mode.

![Figure 5.4](image)

Figure 5.4: The $n=6$ RWM growth rate $\gamma$ versus the plasma rotation frequency $\Omega$ for various $\beta_p$ values. The full kinetic effect is taken into account. The other parameters are kept almost unchanged, with $F \approx -0.06$, $q(0) \approx 0.145$, $q(a) \approx -0.01$. The $\gamma$ and $\Omega$ values are normalized by the Alfvén frequency $\omega_A$ in the plasma center.

In order to clarify which kinetic mechanism plays a principle role in the stabilization of $n=6$ mode, we choose two cases, $\beta_p = 0.11$ and $0.17$, for the analysis. In accordance with the resonant factor $\lambda_{nm}$ of equation 5.10, several types
Kinetic damping on RWM in RFP plasmas

of resonances may occur: (1) the precession resonances with both trapped ions and
electrons when \( \Omega \sim \omega_{de,i} (l = 0) \) is satisfied. (2) If \( l \neq 0 \), the mode resonance with
bounce frequency of trapped ion occurs when \( \Omega \sim l \omega_b / n \). (3) For the passing ions,
the contribution of transit resonance could become significant when the condition
\( \Omega \sim (m - nq) \omega_p / n (l = 0) \) is satisfied. Note that \( l = 0, m = 1 \) is the minimal resonant
transit frequency of \((m - nq + l) \omega_p / n \) for the passing ions. Figure 5.5 plots the
radial profile of each frequencies averaged over the velocity space of the
equilibrium distributions and over the flux surface for both cases of \( \beta_p =0.11 \) and
0.17. It is shown that the above frequencies are approximately in the following
regions: \( \omega_{de}, \omega_{di} \sim 0.01 \omega_A, \omega_b \sim 0.01-0.03 \omega_A, \) and \( (m - nq) \omega_p / n \sim 0.02-0.07 \omega_A \). These
frequency regions can be ranked as \( \omega_{di} > \omega_{de} < \omega_b / n \leq (m - nq) \omega_p / n \); Figure 5.5 also
demonstrates that these frequencies can be slightly increased by increasing the
plasma beta.

![Figure 5.5](image)

Figure 5.5: The radial profiles of various frequencies of trapped and passing thermal
particles averaged over the velocity space and over the poloidal angle. The diagmagnetic
frequency(\( \omega_\alpha \)), the precession frequencies of trapped ion (\( \omega_{di} \)) and electron (\( \omega_{de} \)), the
bounce frequency of trapped ions (\( \omega_b / n \)), as well as the resonant transit frequencies (m-
(nq))\( \omega_p / n \) of passing ions are plotted for \( \beta_p =0.11 \) and \( \beta_p =0.17 \) cases.

In figure 5.6, we plot the n=6 RWM growth rates versus the normalized plasma
rotation frequency for high \( \beta \) plasmas (\( \beta_p =0.17 \)) by considering different kinetic
resonance mechanisms: full kinetic effects of both trapped and passing particles,
Kinetic damping on RWM in RFP plasmas

ion transit, and fluid theory (without kinetic effects). It is clearly shown that taking into account only the ion transit resonance can lead to the stabilization of the RWM at the critical rotation frequency $\Omega_c/\omega_A = 0.02$. The full kinetic effect can stabilize the mode at $\Omega_c/\omega_A = 0.028$. This result indicates that the most unstable INRM $n=6$ can be stabilized by the transit ion motion, which provides the ion acoustic Landau damping in high $\beta$ RFP plasmas. The required flow velocity for the stabilization is in the acoustic speed region. The fluid theory can not predict the mode stabilization due to the incomplete description of ion acoustic damping, which is lacking of the physics of the wave-particle resonance. Moreover, figure 5.6 demonstrates that the critical rotation frequency $\Omega_c$ is slightly increased considering the full kinetic effects instead of the only transit resonance. This may imply the existence of a slight cancellation of the effects between the transit resonances and other kinetic effects (e.g. precession resonance). Figure 5.7 can further justify these conclusions.

Figure 5.6: The n=6 RWM growth rate $\gamma$ versus the plasma rotation frequency $\Omega$ excluding kinetic effect (fluid theory, △) and including full kinetic effect,(□), transit resonance of passing particles alone (○) are plotted respectively, at $\beta_p=0.17$.

Figure 5.7 shows the kinetic energy components $\delta W_k$ computed by taking into account each different kinetic mechanism: the precession motion of trapped particles, the ion bounce motion, and the transit motion of the passing ions. Each $\delta W_k$ is plotted as function of the flow rotation frequency $\Omega$ with $\beta_p = 0.11$. It is observed that the $\delta W_k$ contributed from ion transit resonance becomes a dominant one above the other contributions when $\Omega/\omega_A$ larger than 0.02, and plays a
Kinetic damping on RWM in RFP plasmas

stabilizing role on RWM. The results shown in figure 5.6 coincide with this conclusion. Moreover, we notice that in the region around \( \Omega \sim 0.02-0.03 \, \omega_A \), the real parts contributed by transit motion of passing ions has the opposite sign to that contributed by the precession motion of trapped particles; and this may result in the slight cancellation of the transit stabilization as shown in figure 5.6. Concerning the kinetic energy contributed by the precession of trapped ions and trapped electrons, it shows that the real part of \( \delta W_k \) is relatively large for small rotation \( \Omega \sim 0-0.01 \). In this \( \Omega \) region, the precession is the most important kinetic mechanism. The imaginary part, instead, is almost zero in the region, since the contribution from the precession resonances of trapped ions and of trapped electrons cancels each other [45]. As the results, the stabilization does not occur in the precession frequency region. This maybe partially due to the small fraction of the trapped particles, and their contribution to \( \delta W_k \) being not large enough to stabilizing the mode. The \( \delta W_k \) contributed from the bounce frequency is shown even smaller than that from the precessions, though the resonant frequency range (0.1- 0.2 \( \omega_A \)) can be very close to the transit resonance. This is because, besides the small fraction of the trapped particles w. r. to the passing ones, the dominant contribution from the transit resonance comes from \( l = 0 \) component of Lagrangian \( H_L \), whereas the largest contribution from bounce resonance is \( l = 1 \) harmonic, while in RFP configuration \( l = 0 \) component of Lagrangian \( H_L \) is much more important than other bounce harmonics (\( l \neq 0 \)) (see equation 5.13). Figure 5.8 is the plot of the comparison between the bounce ion contributed \( \delta W_k \) and the transit ions contributed \( \delta W_k \), where the various Fourier harmonic \( l \) of the \( \delta W_k \) expansion in the bounce period are shown. It is found that the major contribution of bounce resonance comes from \( l = 1 \) harmonic; although many other harmonics of \( l = -1, \pm 2, \pm 3 \ldots \) also contribute to \( \delta W_k \), the total kinetic energy from the bounce resonance is quite small comparing to \( \delta W_k \) from the transit resonance. For the passing particles, the kinetic energy \( \delta W_k \) corresponding to the \( l = 0 \) harmonic is almost equal to the total \( \delta W_k \) (summing up all of \( l \), for both real and imaginary parts). The other \( l \) harmonics give negligible contribution to \( \delta W_k \). This analysis also convinces us that the major role of stabilization is played by the resonance with \( l = 0 \) transit frequency of passing ions, when \( \Omega \sim (m-nq)\omega_p / n \).
**Kinetic damping on RWM in RFP plasmas**

Figure 5.7: The computed kinetic energy of \( n=6 \) mode contributed by the precession resonance, bounce resonance, transit resonance are plotted as the function of rotation frequency \( \Omega \) respectively, in RFP plasmas with \( \beta_p = 0.11 \).

Figure 5.8: The kinetic energy contributed by the bounce resonance and the transit resonance in the expansion of the Fourier harmonic \( l \) of the bounce/transit period respectively for \( n=6 \) mode when \( \Omega = 0.02 \omega_i \). The equilibrium is chosen with \( \beta_p = 0.17 \). The real and imaginary parts of total \( \delta W_k \) of bounce resonance (dotted line) and the transit resonance (solid line) are also plotted, respectively.

In summary, the numerical results from MARS-K code show that the transit resonance of passing ions can provide the acoustic Landau damping on the RWM in high \( \beta \) RFP plasmas, which can stabilize the most important RWM – INRM \( n=6 \) with the critical plasma rotation in the region of the ion sound speed. It is much lower than the previous predicted region of the Alfvén speed by the fluid theory.
In contrast with the tokamak plasmas, neither the precession resonance nor the bounce resonance can stabilize the RWMs in RFPs.

ii) Results for other n modes

Besides the most unstable \( n=6 \) mode, it is also necessary to clarify if the other unstable non-resonant \( n \) modes can be stabilized by the kinetic effect when the wall is close to plasma edge. We investigate two other representative modes \( n=3 \) and \( n=5 \), which usually have smaller growth rates than \( n=6 \) with RFX-mod parameters. The \( m=1 \) poloidal harmonic is still dominant in \( n=3 \) and \( n=5 \) modes in the shallow reversal case \( F_\perp \sim 0.06 \). These two modes have their rational surfaces farther from the plasma than that of \( n=6 \) \( m=1 \) mode. The rotation scan of the mode complex eigenvalues (growth rates and frequencies) till \( \Omega = 0.3 \omega_i \) are plotted in figure 5.9, involving the full kinetic effect when \( \beta_p = 0.17 \). The result shows that \( n=3 \) and \( n=5 \) modes cannot be stabilized by kinetic damping effect when the resistive wall is at \( b/a=1.12 \). The growth rate of \( n=5 \) is slightly decreased in the region of \( 0.15 \omega_i < \Omega < 0.22 \omega_i \) due to the transit resonance damping but cannot vanish, the value of the mode frequency also increases in the same region. For \( n=3 \) mode, the mode growth rate and frequency increase monotonically with the plasma rotation frequency.

These results indicate that the kinetic effects cannot give important influence on the other \( n \) mode when the wall is located near the plasma edge at \( b/a=1.12 \).
iii) Physical understanding of the results

In order to understand why the kinetic effect can stabilize $n=6$ mode but not $n=3$ and $n=5$ modes in the same high beta RFP plasmas, we make a physical analysis based on the computation of the potential energy components, equations 5.11-5.13, 5.15 and 5.16. For the purpose of physical understanding, we consider the formation of the generalized dispersion relation [40,83]

$$\gamma \tau_w = -\frac{\delta W_r + \delta W_k}{\delta b + \delta W_k}$$

(5.17)

where $\delta W_r = \delta W_F + \delta W_{\infty}$; $\delta W_b = \delta W_F + \delta W_{vb}$; $\delta W_{vb}$ and $\delta W_{\infty}$ are the vacuum energy with an ideal wall at minor radius $b$ and without the wall, respectively. $\tau_w$ characterizes the magnetic penetration time of a resistive wall, $\delta W_F$ is the fluid energy component as given in equation 5.11, $\delta W_k$ is the potential energy from the kinetic resonance presented in equation 5.13. The inertial effects are found to contribute with a very small term $\delta W_{iner}$, which has been neglected in equation 5.17. All energy components are calculated by using the RWM eigenfunctions which are obtained self-consistently as described in section 5.2. We note that, even though not fully corresponding to the self-consistent computation, equation 5.17 does approximately describe the RWM physics in the region we investigate here.

The kinetic energy $\delta W_k$, composed by the resonant (imaginary) part and non-resonant (real) part, is further written as $\delta W_k = \delta W_k^{re} + i \delta W_k^{im}$. It follows that the stabilization of RWM requires the condition derived from equation 5.17,

$$\delta W_r \delta W_k + \delta W_k^{re} (\delta W_k + \delta W_r) + (\delta W_k^{re})^2 + (\delta W_k^{im})^2 > 0$$

(5.18)

Generally, the imaginary part of $\delta W_k$ always gives a stabilizing effect; the real part can be either stabilizing or destabilizing.

The four groups of potential energy components computed from equations 5.11-5.13, 5.15 and 5.16 are presented in figure 5.10 for $\Omega = 0.0283 \omega_f$, where the
Kinetic damping on RWM in RFP plasmas

full kinetic effects are taking into account. Group (a) is chosen for n=6 mode when \( \beta_p = 0.06 \) (unstable mode). Group (b) is near marginal stable n=6 mode with \( \beta_p = 0.17 \). Group (c) and (d) are for n=5 and n=3 modes respectively when \( \beta_p = 0.17 \). The energy components in each group are normalized by the total driven energy \( \delta W_{\text{driven}} = -(\delta W_{\text{pre}} + \delta W_{\text{cap}}) \) and presented separately in several columns. The first column is the total driving energy components (current driven and pressure driven). The second column represents the stabilizing energy components including the magnetic field line bending \( \delta W_{\text{mb}} \), magnetic compressibility \( \delta W_{\text{mc}} \) and vacuum magnetic energy \( \delta W_{\text{v∞}} \). The third one is the vacuum energy for an ideal wall being located at minor radius \( b \), \( \delta W_{\text{vb}} \), which is also the stabilizing component and, together with \( \delta W_F \), determines the critical wall position for the ideal kink instability. The fourth column in the group expresses \( \delta W_\infty \) and \( \delta W_b \) as given in equation 5.17. The last two columns represent the real and imaginary parts of \( \delta W_k \) respectively.

First of all, comparing group (a) with (b), we observe that the fraction of the pressure driven energy in Group (b) (\( \beta_p = 0.17 \)) is significantly greater than that of group (a) (\( \beta_p = 0.06 \)). It demonstrates that if \( \beta_p \) increases, the magnetic compression term becomes larger; \( \delta W_{\text{vb}} \) and \( \delta W_{\text{v∞}} \) significantly decrease. This leads to smaller \( \delta W_b \) and \( \delta W_\infty \). Therefore, kinetic potential energy \( \delta W_k \) becomes more significant in the dispersion relation equations 5.17 and 5.18, which causes significant reduction of the growth rate. The mode is therefore near marginal stability. The completely stable mode can be obtained if the \( \beta_p \) value further increases. By further analysing the above results, we found that when plasma \( \beta \) increases, the parallel magnetic perturbation near the plasma edge increases, inducing a larger magnetic compression in RFP, thus a larger \( \delta W_{\text{mc}} \). In the meantime, increasing the parallel magnetic perturbation leads to decrease the normal component of the magnetic perturbation \( b_n \) at the plasma edge; this directly leads to decrease the vacuum potential energies, in particular \( \delta W_{\text{vb}} \). Actually, \( \delta W_{\text{vb}} \) and \( \delta W_{\text{v∞}} \) are the principle factors causing the reduction of \( \delta W_b \) and \( \delta W_\infty \). When \( \delta W_\infty \to 0 \), the RWM is near the marginal instability, so \( \delta W_k \) become a critical factor to determine the mode stability. While \( \delta W_b \to 0 \) implies the marginal stability of external kink when a wall with the minor radius \( b \), and where the mode stability becomes more sensitive to any type of the dissipations (or excitation).
Kinetic damping on RWM in RFP plasmas

As for the group (b), (c) and (d), it shows that when the toroidal mode number becomes smaller, the fraction of current driven potential energy increase significantly in the total driven terms \( \delta W_{\text{driven}} \). The magnetic compression term becomes smaller, the vacuum energy \( \delta W_{\text{vb}} \) largely increases and so does the \( \delta W_{\text{b}} \) term. Therefore the first term in equation 5.18 becomes larger, and this implies that the n=3 and n=5 modes actually are unstable and well above the marginal state. Particularly, the kinetic energy \( \delta W_{K} \) decreases significantly as well, which makes the n=5 and n=3 mode difficult to be stabilized. Our analysis find that the increase of current driven energy \( \delta W_{\text{cur}} \) and vacuum energies \( \delta W_{\text{ve}} \) and \( \delta W_{\text{ve}} \) are due to the increase of the normal (to the flux surface) component of the magnetic perturbation \( b_n \) inside plasma (for \( \delta W_{\text{cur}} \)) and at the edge of plasma (for the vacuum energies). Equations 5.11d, 5.15 and 5.16 show that \( b_n \) is the quantity which directly determine the energy components \( \delta W_{\text{cur}}, \delta W_{\text{ve}} \) and \( \delta W_{\text{ve}} \). The increase of \( b_n \) can be understood as follows: since the smaller n number mode (dominant m=1 n=3 and m=1 n=5 modes) has its rational surface farther from the plasma than n=6 mode, correspondingly these modes have the larger parallel wave numbers \( k_{||} \) inside plasma. Making use of the approximate relation between magnetic perturbation and displacement \( [11,22], b_n \propto ik_{||}B \xi_n \), one can observe that the same perturbed displacement \( \xi_n \) will induce larger normal magnetic perturbation \( b_n \) for a mode having larger \( k_{||} \). Therefore, we conclude that smaller n mode, though having a small growth rate, is more difficult to be stabilized by kinetic dissipation and plasma rotation.

Figure 5.10: The potential energy components of the RWM, as defined in equations 5.11-5.13, 5.15 and 5.16, and normalized by the driven terms \( \delta W_{\text{driven}}=-(\delta W_{\text{pre}}+\delta W_{\text{cur}}) \), are
Kinetic damping on RWM in RFP plasmas

calculated at $\Omega = 0.0283 \omega_d$ for the n=6 mode in RFP plasmas with two $\beta_p$ values: $\beta_p = 0.06$ (group a) and 0.17 (group b); for the n=5 (group c) and n=3 (group d) modes with $\beta_p = 0.17$. The full kinetic effect is considered. $\delta W_k$ of n=5 and n=3 modes are amplified 10 times and 100 times, respectively.

As regard to the kinetic energy $\delta W_k$, the smaller toroidal wave number leads to the larger resonant transit frequency $(m-nq)\omega_p / n$ (as previous study, $l=0$ harmonic is dominant for transit resonance). Therefore, the stabilization of n=5 and n=3 modes through the transit resonance requires much larger rotation speed. This is shown in figure 5.11. The resonant condition for the transit frequencies of passing ions in the plasma center are about $\Omega \sim 0.06 \omega_d - 0.09 \omega_d$ for n=5 mode, and $\Omega \sim 0.18 \omega_d - 0.22 \omega_d$ for n=3 mode, both are much larger than $\Omega \sim 0.022 \omega_d$ of n=6 mode. It means that higher plasma rotation is required for n=5 and n=3 mode to obtain the transit resonance with the passing ions. Figure 5.12 shows the variation of $\delta W_k$ as function of the plasma rotation involving full kinetic effect for n=3 and n=5 modes. It is observed that when the plasma rotation frequency is comparable with and/or higher than the averaged resonant transit frequency, the imaginary part of the kinetic energy increases significantly. The plasma rotation frequency corresponding to the maximum value of $\delta W_k$ for n=5 mode is much higher than that of n=6 mode. Concerning the n=3 mode, until $\Omega = 0.3 \omega_A$, $\delta W_k$ has not reached its maximum. Therefore, the RWMs with smaller toroidal mode number n (n=5, n=3) can not obtain enough kinetic dissipation by the transit resonance in the low speed range as n=6 mode obtain, that is why, the kinetic energies shown in figure 5.10 for n=5 and n=3 are very small.

In conclusion, RWMs with smaller n number in RFP, having their rational surfaces farther from the plasma than n=6 mode, result in larger $k_\parallel$ inside plasma. With respect to n=6 mode, which has its rational surface nearest the plasma (magnetic axis), the modes with smaller n numbers have larger $\delta W_{vb}$ components due to the larger normal magnetic perturbation at edge. This makes the kinetic energy less significant. Moreover, larger $k_\parallel$ leads to higher resonant transit frequency, which results in a smaller $\delta W_k$ in the lower speed range where n=6 mode is stabilized.
Figure 5.11: The radial profiles of various frequencies of trapped and passing thermal particles, averaged over the velocity space and over the poloidal angle, where $\omega_\ast$ - diamagnetic frequency; $\omega_{li}$, $\omega_{le}$ - precession frequencies of trapped ions and electrons; $\omega_b$ - bounce frequency of trapped ions. The resonant transit frequencies $(m-nq)\omega_p/n$ of passing ions are plotted for $(m,n)=(1,6)$, $(1,5)$, and $(1,3)$ respectively, in RFP plasmas with $\beta_p=0.17$.

Figure 5.12: The variation of $\delta W_k$ involving full kinetic effect corresponding to the rotation scan in figure 5.9 for $n=5$ and $n=3$ modes at $\beta_p=0.17$. The kinetic energy $\delta W_k$ of $n=5$ and $n=3$ modes are normalized by $\delta W_{driven}=-(\delta W_{pre}+\delta W_{cur})$ in group (c) and (d) of figure 5.10 respectively.

5.4.2 Effect of the wall position

The results of the previous section demonstrated that $n=5$ and $n=3$ mode cannot be stabilized by kinetic effects when the resistive wall is close to the plasma ($b/a=1.12$). This is essentially due to the large $\delta W_{vb}$, with respect to which the kinetic energy becomes negligible. In this section we will show that if the resistive
wall is located farther from plasma, $\delta W_{ib}$ will be decreased. Consequently, there is a possibility of stabilizing the n=5 and n=3 modes by the kinetic effect. Figure 5.13 reports the rotation scan of the n=5 mode growth rates for different wall positions, where the full kinetic effect is included. It shows that for $\beta_p=0.17$, the mode can be stabilized at $\Omega_c/\omega_A \sim 0.072$ if the wall is located at $b/a=1.375$, and at $\Omega_c/\omega_A \sim 0.098$ if $b/a=1.35$. For higher $\beta_p=0.19$ the same wall position $b/a=1.35$ requires smaller rotation speed, $\Omega_c/\omega_A \sim 0.082$, than that of $\beta_p=0.17$.

Figure 5.13: The n=5 RWM growth rate $\gamma$ versus the plasma rotation frequency $\Omega$ including full kinetic effects of both trapped and passing particles, with various wall positions and $\beta_p$ values.

Figure 5.14 is the plot of the critical rotation frequency $\Omega_c/\omega_A$ versus the wall position $b/a$ for the representative modes n=6, 5, 3 respectively. Different $\beta_p$ values are taken for the calculation. It shows that the closer wall position to the plasma edge, the higher plasma rotation speed and/or higher beta are required for the RWM stabilization. The n=6 mode is the easiest to be stabilized; n=3 mode is the hardest one, which requires highest speed and farthest wall position; n=5 is in the middle. We find that the wall positions presented in the figure corresponding to the critical $\Omega_c$ actually are always close to the critical wall positions $b_c$ (marginal ideal kink stability), where $\delta W_b=0$. The specific data of the points in figure 5.14 are listed in table 5.1, where the corresponding $b_c$ of each case is given.
Kinetic damping on RWM in RFP plasmas

Figure 5.14: The critical velocity for stabilizing n=6 (solid), n=5 (dashed), and n=3 (dotted) modes versus wall position for various $\beta_p$ values. The full kinetic effect is considered.

Table 5.1: The data of poloidal beta value $\beta_p$, wall position b and critical rotation frequency $\Omega_c$ for different n mode corresponding to the points in figure 5.14. In addition, the critical wall position $b_c$ of each case is given.

<table>
<thead>
<tr>
<th>n=6</th>
<th>$\beta_p$</th>
<th>b/a</th>
<th>$\Omega_c/\omega_A$</th>
<th>b_c/a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.06</td>
<td>1.275</td>
<td>0.02</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>0.11</td>
<td>1.2</td>
<td>0.027</td>
<td>1.225</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>1.12</td>
<td>0.042</td>
<td>1.175</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>1.12</td>
<td>0.029</td>
<td>1.15</td>
</tr>
<tr>
<td>n=5</td>
<td>0.17</td>
<td>1.35</td>
<td>0.099</td>
<td>1.425</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>1.375</td>
<td>0.074</td>
<td>1.425</td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td>1.35</td>
<td>0.083</td>
<td>1.4</td>
</tr>
<tr>
<td>n=3</td>
<td>0.17</td>
<td>1.9</td>
<td>0.28</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>2</td>
<td>0.172</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td>1.9</td>
<td>0.266</td>
<td>2.075</td>
</tr>
</tbody>
</table>

As an example, the potential energy components of n=5 mode are calculated for different wall position and plotted in figure 5.15. Group (a) is chosen with b/a=1.12, $\beta_p$=0.17 and $\Omega_c=0.098\omega_A$ where the mode is unstable. Group (b) and (c) corresponds to the marginal stability of n=5 mode for the same wall position b/a=1.35, when $\beta_p$=0.17 and 0.19 respectively. This figure clearly shows that when the resistive wall is far from plasma, $\delta W_{ib}$ largely decreases. Consequently, it makes the $\delta W_b$ notably decreasing and the $\delta W_i$ more significant, resulting to the
stabilization of RWM by kinetic effect. The higher $\beta_p = 0.19$ can also further decrease $\delta W_{vb}$, and finally results in the smaller $\Omega_c$ critical plasma rotation frequency for stabilizing $n=5$ mode.

Figure 5.15: The potential energy components of $n=5$ mode, as defined in equations 5.11-5.13, 5.15 and 5.16 and normalized by the driven terms $\delta W_{\text{driven}} = (\delta W_{\text{pre}} + \delta W_{\text{cur}})$, are calculated at various plasma rotation velocities for different wall position and $\beta_p$ value: (a) $\beta_p = 0.17$, $b/a = 1.12$, and $\Omega = 0.098 \omega_d$ (unstable mode); (b) $\beta_p = 0.17$, $b/a = 1.35$, and $\Omega = 0.098 \omega_d$ (near marginal stability); (c) $\beta_p = 0.19$, $b/a = 1.35$, and $\Omega = 0.082 \omega_d$ (near marginal stability). The full kinetic effect is considered.

5.5 Preliminary results of the investigation of collisionality

Since the plasma collisionality cannot be neglected in current RFP experiments, in this section, we investigate how the collisionality influences the RWM kinetic stabilization. For this purpose, we choose $\beta_p = 0.03$ which is the common poloidal beta value in RFP-mod experiments. Generally, the plasma collisionality should be more important in lower beta plasmas. Since the electron-electron and electron-ion collision frequencies are much larger than ion-ion collision frequency ($\nu_{ee} \sim \nu_{ei} \sim (m_e/m_i)^{1/2} (T_e/T_i)^{3/2} v_i$), we simply assume that the effective collision frequencies in the ion and electron resonant factors have the ratio $\nu_{\text{eff}}^e/\nu_{\text{eff}}^i \approx 27$ and $\nu_{\text{eff}}^i = v_{\text{eff}}^{0i} \hat{T}^{-3/2}$ where $v_{\text{eff}}^{0i}$ is the ion-ion effective collision frequency in the plasma center, $\hat{T}$ is the plasma temperature profile normalized by the center temperature. With respect to the precession frequency of trapped particles, a low collisionality case $v_{\text{eff}}^{0i} = \overline{\omega_d}$ and a high collisionality case $v_{\text{eff}}^{0i} = 10 \overline{\omega_d}$ are chosen, where $\overline{\omega_d}$ is the
averaged precession frequency over the velocity space of the particle equilibrium distributions and over the plasma volume. Since the stabilization of RWM is mainly due to the transit ions, and in the low beta plasma the kinetic energy is less significant with respect to $\delta W_b$, the collisionality should have little effect on the kinetic stabilization. In fact, figure 5.16 presents the $n=6$ mode growth rate as a function of the normalized plasma rotation frequency $\Omega$ computed by including full kinetic effect. Three cases (no collisionality, small collisionality $v_{\text{eff}}^0 = \bar{\omega}_d$, and high collisionality $v_{\text{eff}}^0 = 10\bar{\omega}_d$) are plotted respectively. It is observed that both low and high collisionalities have very little effect on the growth rate. The case of high collisionality shows only slight destabilizing effect. This may due to the contribution of the trapped electrons. This preliminary result indicates that the collisionality is almost negligible in the low beta RFP plasmas for the kinetic dissipation. We expect that it would be even less important in the high beta case.

Figure 5.16: The rotation frequency scan including full kinetic effects and collisionality for $n=6$ modes. The mode growth rate $\gamma$ is plotted as the function of plasma rotation frequency $\Omega$ at $\beta_p = 0.03$. Three cases are chosen: no collisionality (□), low collisionality, $v_{\text{eff}}^0 = 10\bar{\omega}_d$ (○) and high collisionality $v_{\text{eff}}^0 = 10\bar{\omega}_d$ (△), where $\bar{\omega}_d$ is the averaged precession frequency of trapped particles over the velocity space of the particle equilibrium distributions and over the plasma volume.

### 5.6 Summary and discussion

The kinetic dissipation on the RWM stability in RFP plasmas is numerically investigated in this chapter, using the hybrid toroidal stability code MARS-K, in which the kinetic resonance effects are included in a non-perturbative way. The
Kinetic damping on RWM in RFP plasmas

computation of the potential energy components has been integrated into MARS-K code, enabling a better physical analysis.

In contrast with the tokamak plasmas, neither the precessional resonance, nor the bounce resonance, of trapped particles in RFP plays important roles in the stabilization of RWMs. Instead, the transit resonance of passing ions can provide the ion acoustic Landau damping, which is able to stabilize the mode for high $\beta$ plasmas. The required plasma rotation frequency is in the range of the acoustic speed, which is much slower than what previously predicted by the fluid theory involving the plasma viscosity (in Alfvén region). This is because the fluid theory cannot provide a complete description of the acoustic resonance due to the lack of the wave-particle kinetic resonance physics. In particular, it is well known that the fluid theory cannot adequately treat the parallel dynamics along the magnetic field line. This is significantly improved by the drift kinetic description.

The RWMs in RFP configurations have the characteristic of the so-called “non-resonant” modes, having their rational surfaces outside the plasma. We found that, when the resistive wall is positioned near the plasma edge, the stabilization by kinetic transit resonance only occurs for the mode with the rational surface closest to the plasma; for RFX-mod parameters, this is the n=6 INRM, which can be stabilized for $b/a=1.12$ and $\Omega_c \sim 0.028\omega_A$ for $\beta_p=0.17$; or $\Omega_c \sim 0.041\omega_A$ for $\beta_p=0.15$. The other modes with smaller toroidal wave numbers $n$ are hardly stabilized for this wall position. The physical understanding is provided by analyzing the potential energy components. It is shown that in RFP configurations, the non-resonant modes having smaller $n$ numbers correspond to larger $k_\parallel$ in the plasma. The perturbation of the modes causes a larger vacuum potential energy $\delta W_{vb}$. Therefore, the kinetic energy $\delta W_k$ becomes less significant with respect to $\delta W_b$, and the mode stability is less affected by the kinetic energy. Furthermore, the modes with smaller $n$ numbers possess larger transit frequencies due to the larger $k_\parallel$, thus the transit resonance requires higher rotation speed. The maximum of resonance-induced $\delta W_k$ actually occurs at much faster rotation than the high $n$ mode (n=6 in RFX-mod). Hence, the kinetic energy cannot bring a significant effect on the mode stability, in the slower rotation region for lower $n$ modes.

The potential energy component $\delta W_b$ can be reduced till zero by moving the resistive wall away from the plasma edge, mainly by decreasing the vacuum energy $\delta W_{vb}$. When $\delta W_b \rightarrow 0$, the ideal kink becomes marginally stable, where any
dissipation (excitation) will be significant. Therefore, moving the resistive wall farther from the plasma edge and close to the critical wall position (corresponding to $\delta W_b=0$) can make the kinetic effect much more efficient in stabilizing the RWM. In fact, we find the stabilization of the $n=5$ and $n=3$ modes in the farther wall position $b/a=1.35$ and $b/a=1.9$ respectively, which are near the corresponding marginal wall positions.

We have also carried out a preliminary investigation of the collisionality effect in low beta RFP plasmas. The collisionality plays a minor role on the kinetic stabilization of the RWM in RFP plasmas, due to the small effect of trapped particles on the mode stability. We expect that the collisionality will be even less important in the high $\beta$ plasmas.

In this work, we assume a uniform plasma rotation, which may cause a slight quantitative discrepancy when used to predict real RFP plasmas. However, due to the fact that RWMs in RFPs have their resonance surfaces outside the plasma, we do not expect a dramatic change in the conclusion. The Alfven resonance has not been discussed in the present work. For non-resonant modes, the Alfven resonance frequency is much higher than the range of the plasma rotation frequency assumed in this work, and hence beyond the scope of our current study.
In this chapter, the physics of kinetic effects on the RWM stability is studied, and a detailed comparison between RFP and tokamak configurations is made. Since RFP normally possesses a circular cross section for the plasma shape, we also consider tokamak plasmas with a circular shape, in order to exclude additional shaping effects. The toroidal, MHD-kinetic hybrid stability code MARS-K described in chapter 5, in which the drift kinetic effects are self-consistently incorporated into the MHD formulation, is adopted. The new module for computing various potential energy components has been integrated into the code, allowing in-depth analyses and a better understanding of the physical mechanisms behind computational results. A detailed comparative study of the kinetic-modified RWM stability between the two systems provides very useful insights into the physics of the rotational stabilization (passive stabilization) of the mode. In the tokamak configuration, it is found that the kinetic effect can stabilize the mode with very slow or vanishing plasma rotation, due to the mode resonance with the toroidal precession drift of thermal trapped particles. In RFP, instead, stabilization of the RWM comes mainly from the ion acoustic Landau damping (i.e. the transit resonance of passing particles). In the high beta region, the critical velocity required for the mode stabilization is predicted to be in the ion acoustic velocity range. Detailed physical analyses, based on the perturbed potential energy components, have been performed to gain understanding of the stabilizing mechanism in the two different systems.
6.1 Characteristics of fluid RWM in RFPs and tokamaks

Although RWMs share certain similar behaviour in both tokamaks and RFPs, there are a few dissimilarities resulted from the differences between the two configurations that lead to different conditions for the mode stabilization. We find that the kinetic effects work differently on the RWM instability as well in the two devices. In this section, we will describe the differences between the two configurations, and the resulting dissimilarities of the RWM characteristics in the fluid approximation.

As well known, in tokamaks, RWMs are often driven by the plasma pressure. The RWM instability appears in the range of $\beta_N$ values (normalized $\beta$) between the so-called no-wall limit $\beta_N^{\text{no-wall}}$ and the ideal wall limit $\beta_N^{\text{ideal-wall}}$ [84]. In RFPs, RWMs are the current driven modes instead. The reason is that the RFP possesses a stronger poloidal magnetic field, reaching the same order of the field strength as the toroidal field. This implies a larger plasma current in RFPs than in tokamaks, for the same value of the toroidal field. Therefore, the plasma is easier to be “kinking” due to the weaker toroidal field. In fact, ideal external kink instability is easier to be driven by the large plasma current in RFP, whenever the perfect conducting wall is detached from the plasma surfaces, even in absence of the plasma pressure. Obviously, for the current driven RWMs, the no-wall beta limit is zero, i.e. $\beta_N^{\text{no-wall}} = 0$.

Furthermore, due to the toroidal field reversion, resulted from the relaxation process, RFPs can operate in the stable regime of the “resonant” (with rational surface being inside the plasma) ideal kink modes. The RWM instabilities always have their rational surfaces outside the plasma. They can be the so-called “externally non-resonant” modes (ENRM), if the rational surfaces are located at $q < q(a) < 0$ ($q(a)$ is the safety factor at the plasma edge $r=a$), or the “internally non-resonant” modes (INRM), if the rational surfaces are located at $q > q(0) > 0$ as described in chapter 1. It is the same as chapter 5 that we choose $n>0$. It means for a given $n$ number, each poloidal harmonic with $m>0$ corresponds to INRM; the poloidal harmonics with $m<0$ represents ENRMs. In tokamaks instead, RWMs can have their rational surfaces inside the plasma, and/or outside the plasma. The difference in the location of rational surfaces (non-resonant versus resonant) results in different conditions for the mode stabilization. In section 5.1, we studied...
The rotational stabilization of RWM in RFP plasmas based on the fluid theory with classical viscous dissipation in cylindrical geometry studied. It has predicted that the required toroidal plasma rotation velocity $V_{oc}$ for stabilizing the RWM is in the range of the Alfvén velocity (around $0.2V_{A0}-1.0V_{A0}$, where $V_{A0}$ is Alfvén velocity defined by the equilibrium poloidal magnetic field). Since both INRM and ENRM, being the non-resonant modes, have their rational surfaces far from the plasma edge, $k_{||}(a)$ and/or $k_{||}(0)$ are larger ($k_{||}(a)\approx 0.2-1$ for the RFPs) than that of the RWM in tokamaks (where $k_{||}(a) \approx 0$ often holds). Therefore, the rotational stabilization of RWMs in RFPs requires higher $V_{oc}$ than in tokamaks. In fact, in tokamaks, the MHD theory predicted $V_{oc}$ is in the range of the ion sound speed (only a few percent of the Alfvén velocity) [34,39,40,74].

Finally, in RFPs the strong poloidal field makes the poloidal asymmetry weaker than that in tokamaks, leading to a less important role of the toroidal effects. From another point of view, it can be understood that the stronger poloidal field makes lower q (q is the safety factor) configurations; and the distance between the two neighboring rational surfaces, corresponding to two neighboring poloidal mode numbers (e.g. $m/n$ and $(m \pm 1)/n$), is much larger than that in tokamaks. This causes weaker toroidal mode coupling in RFPs than in tokamaks. Furthermore, compared with tokamaks, the magnetic field curvature in RFPs is dominated by the poloidal field, so the “bad curvature” region extends to the whole poloidal angle, resulting in a weak ballooning structure for the mode.

Figure 6.1 compares the typical eigenfunctions (the radial displacement $\xi_n$) of the RWM, computed by MARS-K, for the two different configurations. Figures 1(a) and (b) show the modes in RFP, with $n=6$ at $\varepsilon=a/R=0.2295$, and $n=4$ at $\varepsilon=0.4$, respectively. In the RFP plasma, for the unstable $n=6$ mode, only the $m=1$ poloidal harmonic has a large radial displacement. The other poloidal harmonics ($m=\pm 1, \pm 2, \pm 3, \pm 4\ldots$) appear with much smaller amplitude. This is the consequence of the weak toroidal coupling in the RFP configuration. Figure 6.1(b) plots the eigenfunction for a smaller aspect ratio (“fat”) RFP with $\varepsilon=0.4$, again showing the weak toroidal coupling. Figures 6.1 (a) and (b) indicate that the toroidal coupling effect in RFP is almost independent of the aspect ratio. We also note the peaking of the $m=0$ harmonic, occurring at the $m=0$ rational surface.
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

Figures 6.1(c) and (d) plot the radial displacements $\xi_n$ of the RWM in the tokamak configuration with a circular cross-section. Figure 6.1 (c) shows a case for the $n=1$ current driven mode, with $q(0)=1.13$, $q(a)=1.64$, and $\beta=0.02$. There is no rational surface inside the plasmas. Figure 6.1(d) shows a case for the $n=1$ pressure driven RWM, with $q(0)=1.14$, $q(a)=3.68$, and $\beta=0.0105$. There are two rational surfaces $q=2$ and $q=3$ located inside the plasma for this case. Both figures show a strong toroidal coupling effect, where multiple poloidal harmonics co-exist with sufficiently large amplitudes, all contributing to the $n=1$ mode growth rates. The sharp radial variation of the $m=2$ and $m=3$ harmonics, shown in figure 6.1 (d), indicates the Alfvén resonance near the $q=2$ and $q=3$ rational surfaces, respectively.

Figure 6.2 compares the 2D mode structure, in the toroidal cross-section, between the RFP [Figure 6.2(a)] and tokamak [Figure 6.2(b)] plasmas. The two cases correspond to the eigenfunctions shown in figure 6.1(a) and (c), respectively. The RWM in the RFP, with the sole $m=1$ dominant mode, has almost no ballooning character. On the contrary, the tokamak RWM has multiple poloidal harmonics growing together, and exhibiting obvious ballooning character. Only the real components of $\xi_n$ are plotted in both figures.
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

Figure 6.1: The poloidal Fourier harmonics of the normal component of the plasma displacement, plotted along the minor radius, for the fluid RWM in (a) RFP with aspect ratio \( \varepsilon=\frac{a}{R}=0.2295 \) and \( n=6 \), (b) RFP with \( \varepsilon=0.4 \) and \( n=4 \), (c) tokamak with \( q(a)=3.68 \), \( q(0)=1.14 \), and \( n=1 \) (d) tokamak with \( q(a)=1.64 \), \( q(0)=1.13 \), and \( n=1 \). A straight field line coordinate system is used. No plasma rotation is assumed.

Figure 6.2: The two-dimensional plots of the perturbed radial displacement of the fluid RWM. The maximum absolute value of the displacement is used to normalize the mode: (a) the \( n=6 \) mode in RFP with \( F=-0.06 \), \( \Theta=1.58 \), \( \beta_p=0.17 \), and \( q(0)=0.144 \), corresponding to the case of figure 6.1(a); (b) the \( n=1 \) mode in tokamak with \( q(0)=1.14 \), \( q(a)=3.68 \), \( \beta=0.01 \), and \( \beta_N=2.85 \), corresponding to the case of figure 6.1(c). Shown is the real part of the displacement on a selected toroidal cross section. No plasma rotation is assumed.

6.2 Drift kinetic effects on RWM in RFPs and tokamaks

In this section, MARS-K code is applied for the RWM study in both RFP and tokamak configuration. The MHD-kinetic hybrid, self-consistent formulation,
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

presented in section 5.2, is followed. The wave-particle interaction is included into the MHD equations via the pressure tensor term, and described by the resonant operator 5.10, for each particle species. For the convenience of following discussion, the drift kinetic potential energy \( \delta W_k \), presented in section 5.2, is rewritten as follows,

\[
\delta W_k = \frac{\nu \sqrt{\pi}}{2B_0} \sum_{\epsilon_j} \int d^{\Psi} p \left\{ \left( d^{\hat{\epsilon}_j} \right)^{5/2} e^{-\hat{\epsilon}_j^2} \sum_{\sigma} \left[ \int d\Lambda \sum_{l} \lambda^\sigma_l t_b \left( e^{-i(l+\sigma q)\phi} H_L \right) \right]^2 \right\},
\]

(6.1)

where \( \Psi \) is the equilibrium poloidal flux, \( P_{\epsilon_j} \) the ion and electron equilibrium pressure, \( \Lambda = B_0 \mu / \epsilon_k \), with \( B_0 \) being the on-axis field strength). \( H_L \) is the particle perturbed Lagrangian. The integration is carried out in both real and velocity spaces. The sum is over the poloidal Fourier and bounce harmonics \( l \), the passing and trapped particles, as well as the particle species (e, i). For trapped particles, \( \alpha=0, \nu=1/2 \), and \( t_b \) is the particle bounce period normalized by a factor \( \sqrt{M/2\epsilon_k} \); for passing particles, \( \alpha=1, \nu=1 \), and \( t_b \) denotes the normalized transit period. In this study, we consider the mode resonance with the precession frequencies of trapped particles (both ions and electrons), the ion bounce frequency, and the transit frequency of passing ions. These are the most important effects in the current study. In the following, the words “full kinetic” refer to the combined resonant effects mentioned above. The bounce and the transit frequencies of electrons are much higher than that of ions, resulting in negligible contribution to the kinetic resonance effects on RWMs. The preliminary discussion of the effect of the collisionality in RFP is studied in section 5.5. The comparison of this effect between RFP and tokamak will be made in future work.

### 6.2.1 Numerical results

In this subsection, we compare the behaviour of RWMs, including kinetic effects in the two configurations: the RFP and the tokamak with a circular cross-section. The parameters are taken to be similar to the RFX-mod [26], with \( \epsilon=a/R=0.2295 \), the wall position \( b/a=1.12 \), the electron density at the magnetic axis \( n_{e0}=2.5 \times 10^{19} / m^3 \), and the temperature ratio between the thermal ions and electrons is \( T_i/T_e=0.7 \). These parameters will be applied to all the following analyses, unless otherwise
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

stated. The density profile is assumed as \( n_e(s) = n_{e0}(1-s^2) \). For the tokamak case, the pressure profile is modelled as \( P(s) = P_0(1-s^2)^2 \). For the RFP case, we choose

\[
P = P_0(1 + a_{p1}s^2 + a_{p2}s^4 + a_{p3}s^6)
\]

For the sake of simplicity, a uniform toroidal plasma rotation frequency is considered.

The growth rates of the \( n=6 \) RWM, as a function of the normalized plasma rotation frequency, is plotted in figure 6.3 for various \( \beta \) values in the RFP plasma. Figure 6.3(a) shows the computational results involving full kinetic effects of both trapped and passing particles. Four poloidal beta values are considered (\( \beta_p = 0.06, 0.11, 0.15, 0.17 \)). While increasing \( \beta_p \), we kept the \( q(r) \) profiles nearly unchanged, with \( q(0) \approx 0.145, q(a) \approx -0.01 \), and the reversal parameter \( F \approx -0.06 \). The pinch parameter \( \Theta \) has to be changed correspondingly in the range 1.5 to 1.58. The definition of \( F, \Theta \) and \( \beta_p \) can be found in [10]. Figure 6.3(a) shows that, for the high \( \beta \) plasma, the RWM can be fully stabilizing at much slower plasma rotation than that predicted by the fluid theory [59]. Inclusion of the kinetic effects leads to a critical rotation frequency of \( \Omega \sim 0.04\omega_A \), when \( \beta_p \) reaches 0.15 ( \( \omega_A = \frac{B_0}{R_n\mu_0\rho_0} \) is the Alfvén frequency at the magnetic axis.). With further increase of \( \beta_p \) up to 0.17, the RWM can be stabilized at even slower rotation of \( \Omega \sim 0.028\omega_A \). The kinetic stabilization is mainly contributed by passing ions through the ion acoustic landau damping, as shown by figure 6.3(b).

The mode growth rates versus the normalized plasma rotation frequency \( \Omega/\omega_A \) are plotted in figure 6.3(b), for the case of \( \beta=0.17 \). Four types of kinetic contributions are compared: 1) the full kinetic effects (dot-dashed line), 2) the precession resonance of trapped particles only (dotted line), 3) the precession and the bounce resonances (long dashed line), and 4) the transit resonance of passing particles alone (short dashed line). The result of the fluid theory (without kinetic effects, solid line) is also shown in the figure for the comparison. The figure clearly shows that the transit resonance plays a principle role, stabilizing the \( n=6 \) RWM with the slowest critical rotation speed (slower than that with the full kinetic effects). The trapped particle (both ions and electrons) precession resonance alone does not fully stabilize the mode. The precession resonance combined with the bounce resonance do not stabilize the mode either. Figure 6.3(b) also shows that the critical flow velocity for the mode stabilization by full kinetic effects
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

$(\Omega/\omega_A\approx0.028)$ is larger than that by transit resonance only $(\Omega/\omega_A\approx0.02)$. This implies that the contributions from different kinetic resonances may play opposite role and slightly cancel each other.

Figure 6.3: The n=6 RWM growth rate $\gamma$ versus the plasma rotation frequency $\Omega$ in RFP plasmas: (a) the rotation frequency scan including full kinetic effects of both trapped and passing particles, with various $\beta_p$ values $\beta_p=0.06$ (solid), 0.11 (dashed), 0.15 (dotted), 0.17 (dot-dashed). The other parameters are kept almost unchanged, with $F\approx-0.06$ and $q(0)\approx0.145$; (b) the rotation frequency scan including different types of kinetic resonances, at $\beta_p=0.17$. The $\gamma$ and $\Omega$ values are normalized by the Alfvén frequency $\omega_A$ in the plasma center.

Compared to the RFP plasmas, the behaviours of RWMs in tokamaks are rather different. Figure 4(a) plots the n=1 RWM growth rates, as a function of the normalized plasma rotation frequency $\Omega/\omega_A$, for circular cross section tokamak plasmas, where the full kinetic effects are taken into account. Two different equilibria are presented: 1) the first case with $q_0=1.13$, $q_a=1.64$, and $\beta=0.02$ ($\beta_N=2.68$), shown by the solid line. In this equilibrium, there is no mode rational surface inside the plasma. We find that the RWM is current drive. The kinetic effects can not stabilize the mode with plasma rotation. 2) The second case has $q_0=1.14$, $q_a=3.68$, and $\beta=0.0105$ ($\beta_N=2.85$), shown by the dotted line. The RWM is pressure driven for this equilibrium (the mode is stable when $\beta=0$). The pressure scaling parameter $C_p = (\beta_N - \beta_{N\text{-wall}}) / (\beta_N^{\text{ideal-wall}} - \beta_{N\text{-wall}}) = 0.66$. The kinetic effects stabilize the mode without plasma rotation or with a slow rotation at $\Omega/\omega_A < ^{116}$
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

0.0048. The major contribution to the kinetic stabilization comes from the precession resonance of trapped particles.

Figure 6.4(b) shows the mode growth rates versus the rotation frequency, taking into account different kinetic effects for the pressure driven tokamak RWM (corresponding to the case shown by the dotted line in figure 6.4(a)). Here the dot-dashed line corresponds to the full kinetic effects, showing that the full stabilization can be achieved in the frequency range of $\Omega/\omega_A \approx 0.0 - 0.0048$. The dotted line corresponds to the case, where only the precessional resonance of trapped particles is considered. The dashed line presents the case, where both the precession and the bounce resonances of trapped particles are taken into account. The result of the fluid theory is also plotted (solid line). The comparison shows that the trapped particle precession resonance gives the principle contribution to the mode stabilization, at slow or vanishing plasma rotation. Again figure 6.4(b) shows that the full kinetic effect results in a narrower stable region in $\Omega/\omega_A$, than the precession resonance alone, implying a slight cancellation of the mode stabilizing effect between different kinetic resonances. As the rotation frequency $\Omega$ increases beyond the above mentioned range, the mode becomes unstable again. With further increase of the plasma rotation frequency, the transit resonance starts to play a stabilizing role by the ion acoustic damping, and mode growth rate decreases. However, we do not find a full stabilization for this equilibrium.

![Figure 6.4](image)

Figure 6.4: The $n=1$ mode growth rate versus the plasma rotation frequency for tokamak equilibria: (a) the rotation scan including the full kinetic effects of both trapped and passing particles, for two equilibria; (b) the rotation scan for the $q(a)=3.68$ case, comparing contribution from various kinetic effects, as well as the fluid theory prediction.
6.2.2 Physical understanding

Here we perform detailed analysis of both fluid and drift kinetic potential energy perturbations. With the aim of improving the physical understanding, we again consider the generalized dispersion relation [40,83]

$$\gamma \tau_w^* = -\frac{\delta W_F + \delta W_k}{\delta W_b + \delta W_k}, \quad (6.2)$$

where $\delta W_\infty = \delta W_F + \delta W_{\text{vac}}$, $\delta W_b = \delta W_F + \delta W_{\text{vb}}$, $\delta W_{\text{vb}}$ and $\delta W_{\text{vac}}$ are the vacuum energy with an ideal wall at minor radius $b$ and without the wall, respectively. $\tau_w^*$ characterizes the wall time of a resistive wall. $\delta W_F$ is the fluid energy component as given in equation 5.11, $\delta W_k$ is the potential energy from the kinetic resonance as presented in equation 6.1. The plasma inertial contribution is found to be small compared to other terms, and hence has been neglected in equation 6.2. All energy components are evaluated using the RWM eigenfunctions, obtained from the self-consistent computations. We note that, even though not fully corresponding to the self-consistent computation, equation 6.2 does approximately describe the RWM physics in the region of slow rotation velocity as we investigate here.

The kinetic energy $\delta W_k$ consists of the resonant (imaginary) part and the non-resonant (real) part, $\delta W_k = \delta W_k^{\text{re}} + i \delta W_k^{\text{im}}$. It follows from equation 6.2 that the stabilization of the RWM requires the condition

$$\delta W_\infty \delta W_k + \delta W_k^{\text{re}}(\delta W_k + \delta W_\infty) + (\delta W_k^{\text{re}})^2 + (\delta W_k^{\text{im}})^2 > 0 \quad (6.3)$$

Generally, the imaginary part always gives a stabilizing effect. The real part can be either stabilizing or destabilizing effects. Figure 6.5 compares various energy components, normalized by the total driven energy $\delta W_{\text{driven}} = -(\delta W_{\text{pre}} + \delta W_{\text{cur}})$ (the current driven plus the pressure driven energy components) of RWM, for the two configurations. Figure 6.5(a) is for the $n=6$ RWM in the RFP plasma and (b) is for the $n=1$ mode in the tokamak plasma. Each figure has several groups of columns. The first column shows the total driving energy components (current driven and pressure driven). The second column represents the stabilizing energy components, including the magnetic field line bending term $\delta W_{\text{mb}}$, the magnetic compressibility
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

term $\delta W_{mc}$ and the vacuum magnetic energy term $\delta W_{ve}$. The third column shows the vacuum energy $\delta W_{vb}$, with an ideal wall located at the minor radius $b$. This is also a stabilizing term and, together with $\delta W_F$, determines the critical wall position for the ideal kink instability. The forth column in each group shows $\delta W_\infty$ and $\delta W_b$ as defined in equation 6.2. The last two columns represent the real and imaginary parts of $\delta W_k$ respectively.

Figure 6.5 (a) shows the energy comparison for RFP plasmas. There are three groups of columns. Group (1) is for an equilibrium with $\beta_p=0$, $F=-0.063$, $\Theta=1.454$, $q(0)=0.147$, and $q(a)=-0.0104$. Neither plasma rotation nor kinetic effects are included. Due to vanishing equilibrium pressure, only the current driven energy component appears. Since the driven energy prevails in the balance between the first and second columns, the ideal kink mode is unstable, and hence the RWM is unstable. The fourth column shows a negative $\delta W_\infty$ and a positive $\delta W_b$, agreeing with the RWM instability following equation 6.2.

Group (2) is for an equilibrium with middle range $\beta_p$, $\beta_p=0.064$, and $F=-0.063$, $\Theta=1.499$, $q(0)=0.145$, $q(a)=-0.010063$. This case corresponds to the solid curve in figure 6.3(a). The flow velocity of $\Omega/\omega_A=0.06$ is assumed in the computation, at which the kinetic resonances contribute a maximal value of $\delta W_k$, along the whole velocity range considered in figure 6.3(a). The pressure driven energy in this case is only a small fraction of total driven term $\delta W_{driven}$. The current driven is still the dominant destabilizing mechanism. The computed $\delta W_\infty$ and $\delta W_b$ have rather large values. The kinetic energy component $\delta W_k$ has small real and imaginary parts, being not sufficient to stabilize the mode.

Group (3) is for a high $\beta$ RFP plasma, with $\beta_p=0.17$, and $F=-0.0620$, $\Theta=1.565$, $q(0)=0.144$, $q(a)=-0.0094$, corresponding to the dotted line in figure 6.3(a). Included are the effects of the transit resonance of passing ions on the $n=6$ mode, at the flow velocity $\Omega/\omega_A=0.0203$, which is near the marginal instability. The fraction of the pressure driven energy is significantly increased in this case, compared with the case of $\beta=0.06$. The magnetic compressional term becomes slightly larger. Both $\delta W_\infty$ and $\delta W_b$ become smaller than that in group (1) and (2). The kinetic energy components, both $\delta W_k^re$ and $\delta W_k^{im}$, play a stabilizing role and
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

lead to a very small growth rate - the mode is nearly marginally stable. With further increasing $\Omega/\omega_A$, we obtain a completely stable RWM.

Figure 6.5(b) is the counterpart plot for tokamaks. The group (1) is for an equilibrium with $q(a)=1.637$, $q(0)=1.13$, and $\beta=0.02$ ($\beta_N=2.68$), corresponding to the solid line in figure 6.4(a) at $\Omega/\omega_A=0.002$, where the kinetic effect contributes a maximal value of $\delta W_k$ along the whole range of the velocity. In this equilibrium, all rational surfaces of the $n=1$ modes are located outside the plasma. The RWM is a current driven mode - the pressure driven energy component is much smaller than the current driven component. The total stabilizing components are also smaller than the current driven component. The resulting $\delta W_{\infty}$ is rather large. Moreover, $\delta W_{vb}$ and $\delta W_b$ are much larger than that in the second group. The kinetic energy $\delta W_k$, being roughly proportional to the equilibrium pressure, is rather small and can not stabilize the mode. In tokamak plasmas, due to the strong toroidal magnetic field, the magnetic compression energy $\delta W_{mc}$ is very small and almost invisible in the figure.

Group (2) is for a tokamak plasma with $q(a)=3.68$, $q(0)=1.14$, and $\beta=0.0105$ ($\beta_N=2.85$), corresponding to the dotted curve in figure 6.4(b), which describes the effect of precessional resonance of trapped particles on the $n=1$ mode. The rotation frequency $\omega_E/\omega_A=0.007$ is chosen near the critical value for the marginal stability. The first column in this group shows a large pressure driven fraction, contributing almost half of the driving energy. In fact the RWM is a pressure driven mode for this equilibrium. The stabilizing column from the fluid terms, $\delta W_{mb}+\delta W_{v\infty}$, is larger than that of Group (1), and leads to a smaller $\delta W_{\infty}$. The vacuum energy $\delta W_{vb}$ is smaller as well, leading to a smaller $\delta W_b$. The kinetic energy $\delta W_k$ contributes sufficiently strong stabilizing effect, such that the mode is marginal stable. Further decreasing the plasma rotation $\Omega$, the mode can be completely stabilized as showed in figure 6.4(b),
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

Figure 6.5: The potential energy components of the RWM, as defined in equations 5.11-5.13, 5.15 and 5.16, and normalized by the driven terms $\delta W_{\text{driven}} = -(\delta W_{\text{pre}} + \delta W_{\text{cur}})$, are calculated at various plasma rotation velocities for (a) the $n=6$ mode in RFP plasmas with $F \approx -0.06$, $q(0) \approx 0.145$, including the mode resonance with transit frequency of passing ions only, at three chosen $\beta_p$ values: $\beta_p = 0.0, 0.06$ and $0.17$; (b) the $n=1$ mode in tokamak plasmas with $q(0)=1.13$, $q(a)=1.64$, $\beta = 0.02$, $\beta_N=2.68$ (group 1), and $q(0)=1.14$, $q(a)=3.68$, $\beta = 0.01$, $\beta_N=2.85$ (group 2). The mode resonance with precessional drifts of trapped ions and electrons are considered.

Figure 6.5 demonstrates a common feature of RWM for both configurations: the kinetic stabilization requires a large fraction of the pressure driven energy component. Furthermore, figure 6.5 (a) compares the three RFP results with increasing different $\beta_p$ values, but with almost the same $q(r)$ profiles. More detailed analysis shows that increasing the equilibrium pressure decreases the perturbed fluid displacement in the normal direction, $\xi_n$, near the plasma edge, and increases the other two components along the tangential direction; this causes smaller normal magnetic perturbation $b_n$ at plasma-vacuum interface. It turns out that the perturbed vacuum magnetic energy $\delta W_{vb}$ (proportional to $b_n^2$) becomes smaller, and so does $\delta W_b$. $\delta W_b \to 0$ implies the marginal stability of the external kink with an ideal wall. This is where the mode stability can become more sensitive to any type of dissipations (or excitations).

We find the same behaviour in tokamaks, as in RFP plasmas, if we keep the $q(r)$ invariant while changing the $\beta$ value. However, the $q$ profiles are different for the two cases shown in figure 6.5(b). The tokamak equilibrium with larger $q(a)$ implies that, for the same plasma current, the toroidal magnetic field is stronger. And the plasma is generally more resistant to the “kink” instability. Therefore, a plasma with a larger $q(a)$ needs larger pressure driven energy to become kink unstable. In fact, it is found that the second group, with $q(a)=3.68$, provides a
stronger magnetic bending and smaller $\delta W_{vb}$, than the first group with $q(a)=1.67$. The smaller $\delta W_{vb}$ for the second group is associated with the smaller magnetic perturbation at the plasma surface. For the tokamak equilibria considered here, a larger $q(a)$ leads to a higher no-wall limit $\beta_N^{\text{no-wall}}$, thus to a larger pressure driven potential energy $\delta W_{\text{pre}}$. Consequently, it leads to a small $\delta W_b$, and the mode becomes easy to be stabilized by the kinetic effects. For $1<q(a)<2$, we observe only current driven RWMs, which can not be stabilized by the kinetic effects of thermal particles.

Next, we try to understand why the precessional resonance of trapped particles, at very slow plasma rotation, can give the RWM stabilization in tokamaks, but not in RFPs. Figures 6.6 (a) and (b) plot various frequencies, entering the resonance operator $\lambda_{m,l}$ of equation 5.11, for RFP and tokamak plasmas respectively. The flux surface averaged precession drift frequency $\omega_d$, the bounce frequency $\omega_b$ of trapped particles (ions and electrons), the resonance ion transit frequency term $(m-nq)\omega_p$ ($\omega_p$ is the transit frequency), and the diamagnetic drift frequency $\omega^* (\omega^* = \omega^*_N + \omega^*_T$ includes both density gradient drift and temperature gradient drift) are plotted as a function of the magnetic flux coordinate.

In both figures 6.6(a) and 6.6(b), the dotted curves present the precession frequencies of trapped electrons $\omega_{de}$ (the precession frequency of trapped ions is defined as $\omega_{di}=-(T_i/T_e)\omega_{de}$, and plotted as dual-dot-dashed line). Note that in the tokamak configuration, the scale lengths of the magnetic curvature and gradient are in order of $O(1/R)$, which is one order of $O(\epsilon)$ smaller than that in RFP, which is in the order of $O(1/a)$. In fact, figures 6.6 demonstrates that the averaged precession frequency (over the flux surface) $\omega_d$ is near zero in the tokamak, and is around 0.005-0.01 in the RFP configuration. For very small rotation frequency $\Omega \approx 0$, the ion and electron contributions of the $\omega_d$ resonance, to the imaginary part of $\delta W_k$ ($\delta W_k^{im}$), have opposite signs, and the two contributions almost cancel each other. Therefore the precessional drifts mostly contribute to the real part of $\delta W_k$ ($\delta W_k^{re}$). As the $\Omega (>0)$ value increases, the trapped electrons start to play a more important role. The ratio $\omega^*/\omega_d$ is significant in determining the integrated value of the resonance operator $\lambda_{m,l}$. The following three points may provide the reasons why the trapped particles play different roles in tokamaks and in RFPs. First, the
fraction of the trapped particle in RFPs is smaller than that in tokamaks. This is particularly true near the low field side edge of the plasma. Secondly, the ratio $\omega_*/\omega_d$ is much larger in tokamaks, than that in RFPs, resulting in a larger value of $\delta W_k$. Finally, the tokamak plasma is characterized by a stronger toroidal coupling, where several poloidal harmonics grow together, with comparable amplitude. Each poloidal harmonic can be in resonance with particle precessions. The RWM in the RFP plasma, on the contrary, has a weak toroidal coupling, and only mode $m=1$ gives the dominant contribution, due to the precessional resonance, to the kinetic energy. Therefore, the trapped particle precessional drifts can stabilize the RWM in tokamaks for very plasma slow rotation, starting from $\Omega=0$, while the same drifts play a minor role in the stabilization of the mode in RFPs, in the frequency range $\Omega/\omega_A \sim \omega_d \approx 0.005 - 0.01$.

The particle phase space averaged precession frequency, for a tokamak plasma, is plotted in figure 6.7(a) in the toroidal cross section. It is interesting to notice the change of sign of $\omega_d$, from the low field side to the high field side. Moreover, the magnitude of $\omega_d$ stays near zero in a large region. In fact, we find that the precessional resonance gives the largest contribution to $\delta W_k$ in the yellow area, as shown by figures 6.7(b).

Figure 6.6: The radial profiles of various frequencies of trapped and passing thermal particles, averaged over the velocity space and over the poloidal angle. The diamagnetic frequency($\omega_*$), the precession frequencies of trapped ion ($\omega_{di}$) and electron ($\omega_{de}$), the bounce frequency of trapped ions ($\omega_b/n$), as well as the resonant transit frequencies $(m-nq)\omega_p/n$ of passing ions are plotted for (a) the RFP case with $m=1$, $n=6$, $F=-0.06$, $\Theta=1.58$, $q_0=1.14$, $q_0=3.68$, $\omega_0=0.06$, $\omega_b=2.85$, $m=1$, $n=2$, $\omega_b=0.01$, $\omega_b=2.85$, $m=1$, $n=2$, $\omega_b=0.01$. 

Figure 6.7: (a) The particle phase space averaged precession frequency, for a tokamak plasma, is plotted in figure 6.7(a) in the toroidal cross section. It is interesting to notice the change of sign of $\omega_d$, from the low field side to the high field side. Moreover, the magnitude of $\omega_d$ stays near zero in a large region. In fact, we find that the precessional resonance gives the largest contribution to $\delta W_k$ in the yellow area, as shown by figures 6.7(b).
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

$\beta_p=0.17$, q(0)=0.144, and (b) the tokamak case with $m=1, 2$, n=1, q(0)=1.14, q(a)=3.68, $\beta=0.0105$, $\beta_N=2.85$.

Figure 6.7: The 2D plots in the RZ-plane, of (a) the precession frequency $\omega_{de}$ of trapped electrons, averaged over the velocity space, and (b) the $|\delta W_k|$ for the $n=1$ marginally stable RWM, in the presence of the precessional resonance of trapped ions and electrons, for a tokamak plasma with $\Omega=0.007\omega_A$, q(0)=1.14, q(a)=3.68, $\beta=0.01$ and $\beta_N=2.85$. The $|\delta W_k|$ is normalized by $\delta W_{\text{driven}}$. The value of $\omega_{de}$ is normalized by $\omega_A$.

Figure 6.8: The 2D plots in the RZ-plane, of (a) $|\delta W_k|$ for the $n=6$ marginally stable RWM, including the transit resonance of passing ions alone, and (b) The value of $|\kappa \cdot \xi_\perp|$ (in arbitrary unit), for a RFP plasma with $\Omega=0.0203\omega_A$, F=-0.06, $\Theta=1.58$, $\beta_p=0.17$ and q(0)=0.144. $|\delta W_k|$ is normalized by $\delta W_{\text{driven}}$. 

124
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

The dashed line in figures 6.6(a) shows the resonant transit frequency \((m-nq)\omega_p\) for the \(m=1, n=6\) mode in the RFP plasma. In the area of minor radius \(s \approx 0.2-0.3\), corresponding to \(\Omega/\omega_A \approx 0.025-0.035\), the transit resonance starts to give a significant contribution to \(\delta W_k\), leading to the stabilization of the mode. With further increase of the flow velocity, the kinetic contribution \(\delta W_k\) becomes more important, and the mode becomes fully stable. Figure 6.8(a) shows the amplitude of the kinetic energy \(|\delta W_k|\) distribution in the toroidal cross section. The largest contribution comes from the low field side near the plasma core region (the red area). This is because the perturbed particle Lagrangian \(H_L\), which is well represented by the \(\kappa \cdot \xi_\perp\) term, has the maximum in this area, as shown by figure 6.8(b). The dominant contribution to \(\delta W_k\) comes from the \(m=1\) and the \(l=0\) harmonic in equation 6.1. We notice that the bounce frequency \(\omega_b/n\) in that area is also close to the transit one, and should in principle also give contribution to the kinetic energy. However, our computations reveal that the bounce resonance gives minor influence on the mode stability. This may be due to the small fraction of trapped particles (compared to passing particles), as well as the fact that the \(l=1\) bounce harmonic in the Lagrangian \(H_L\) is less important than the \(l=0\) harmonic.

Figure 6.9(a) plots the radial profiles for both the real and imaginary parts of the kinetic energy \(\delta W_k\), due to the transit resonance at \(\Omega/\omega_A = 0.02\), for the RFP plasma. These radial profiles are obtained as a result of the integration along the poloidal angle, for the energy distribution shown in figure 6.8(a). Figure 6.9(a) confirms that both the real and imaginary parts of \(\delta W_k\) reach their peak values in the range of \(s = 0.2-0.3\).

For the tokamak equilibrium with \(q(a)=3.68, q(0)=1.14\), and \(\beta = 0.0105\), the most important resonant transit frequencies, for the \(n=1\) RWM, are the \(m=1\) and \(m=2\) harmonics, presented by the dashed and dot-dashed curves, respectively, in figure 6.6(b). The transit resonance requires a much larger plasma rotation than the precessional resonance. For this equilibrium, we did not find the full stabilization of the RWM with the transit resonance.

Figure 6.9(b) plots of the radial distribution of both the real and imaging parts of the kinetic energy \(\delta W_k\) (integrated over the magnetic surface), contributed
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

by the precessional resonance of trapped particles. The largest kinetic contribution appears in the range of minor radius, where \( \omega_d \approx 0 \).

![Figure 6.9](image)

Figure 6.9: The radial distribution of the real and imaginary parts of \( \delta W_k \), integrated over the phase space and the poloidal angle, for (a) the n=6 marginally stable RWM in the RFP plasma, where the mode resonance with the transit frequency of passing ions alone is included, with \( \Omega=0.0203\omega_A \); (b) the n=1 marginally stable RWM in the tokamak plasma, in the presence of the mode resonance with precessional drifts of trapped ions and electrons.

6.3 Non-perturbative versus perturbative results

In this work we have adopted the self-consistent approach, where the drift kinetic effects are coupled to the MHD equations in a non-perturbative manner. Another approach is the perturbative one [85], where the fluid part is considered as the lowest order term, and the kinetic part as the next order. The drift kinetic energy is calculated using the eigenfunction of the fluid RWM or the marginally stable ideal kink mode (with an ideal wall). The non-perturbative approach allows a self-consistent modification of the RWM eigenfunction, due to the kinetic effects. Furthermore, the complex eigenvalue of the RWM is obtained self-consistently by solving the hybrid eigenmode equations, where the mode frequency dependent kinetic resonance operators are taken into account via the perturbed kinetic pressure tensor.

In the following, we give an example comparing these two approaches. We use the same tokamak equilibrium as for figure 6.1(d), with \( q(0)=1.13 \), \( q(a)=1.64 \),
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

and $\beta = 0.02$. The plasma rotation $\Omega = 0$ is assumed. The $n=1$ RWM is studied by both approaches. Since there is no mode resonant surface inside the plasma, no singular behaviour appears in the mode eigenfunction.

Figure 6.1(d) (section 6.1) showed the radial profiles of the eigenfunctions $\tilde{\xi}_n$, obtained by the fluid theory. The $m=1$ harmonic is dominant, with the $m=2$ harmonic having relatively smaller amplitude. Shown in figure 6.10 is the eigenfunction for $\tilde{\xi}_n$, obtained by the self-consistent approach. Clearly the amplitude of the $m=2$ harmonic is largely increased due to the kinetic effects, and in fact becomes one of the dominant modes.

Consequently the energy components, normalized by $\delta W_{\text{driven}}$ and calculated from the two different eigenfunctions, are also different as shown in figure 6.11. Two groups of the energy components are plotted. The first one comes from the perturbative approach, where the driving terms (current driven $\delta W_{\text{cur}}$ and pressure driven $\delta W_{\text{pre}}$) and the stabilizing terms (the field line bending energy $\delta W_{\text{mb}}$, the magnetic compression energy $\delta W_{\text{mc}}$, and the no-wall vacuum energy $\delta W_{\text{vb}}$) are computed using the eigenfunction of the fluid RWM. The second group of energy is computed using the RWM eigenfunction obtained from the self-consistent approach. In the second group, the current driven term contributes a larger fraction, due to the kinetic modification of the mode eigenfunction. The increasing amplitude of the $m=2$ harmonic, near the plasma edge, also increases the two vacuum energy components ($\delta W_{\text{vb}}$ and $\delta W_{\text{vb}}$), resulting in a larger mode growth rate. The kinetic energy from the two approaches, though different, is still much smaller than the fluid energy $\delta W_{\infty}$. Therefore, for this equilibrium, the kinetic effects can not stabilize the RWM following either approach. The variation of the mode eigenfunction, as well as the energy components, does not result in a qualitative difference in the mode eigenvalues (perturbative approach: $(\gamma+i\omega)/\omega_\alpha = 2.51 \cdot 10^{-3} - i3.55 \cdot 10^{-4}$; self-consistent approach: $2.74 \cdot 10^{-3} - i2.98 \cdot 10^{-4}$). for the case considered here. Because $\delta W_k << \delta W_{\infty}$, it is understandable that the perturbative approach should not give large discrepancy in the mode eigenvalue, compared to the self-consistent approach. However, in other cases, where the RWM is close to marginal stability, or the plasma rotation is included, the two approaches can lead to rather large (even qualitative) discrepancies.
Kinetic effects on RWM stability–comparison between RFPs and tokamaks

Figure 6.10: The radial profiles of the poloidal Fourier harmonics of perturbed radial displacement, for the self-consistently computed n=1 RWM including the full kinetic effect, for the tokamak equilibrium with q(a). A straight field line coordinate system is used. No plasma rotation is assumed.

Figure 6.11: The potential energy components, as defined in equations 5.11-5.13, 5.15 and 5.16 and normalized by the driven terms $\delta W_{\text{driven}} = (\delta W_{\text{pre}} + \delta W_{\text{cur}})$, plotted for the n=1 RWMs, for the q(a)=1.64 tokamak equilibrium. Group (1) is the fluid RWM case. Group (2) corresponds to the RWM including the full kinetic. No plasma rotation is assumed.

6.4 Summary and discussions

We have studied the drift kinetic effects on the RWM for both RFP and tokamak plasmas, using the toroidal MHD-kinetic hybrid code MARS-K. In MARS-K, the kinetic resonant effects are incorporated into MHD formulation via the perturbed kinetic pressure tensor. The unknown mode growth rate and the mode eigenfunction are self-consistently obtained by solving the hybrid eigenmode
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

equations. A new module introduced in chapter 5 for computing various potential energy components, based on the kinetic-modified RWM eigenfunction, has been developed and integrated into MARS-K. The module helps to make comprehensive energy analyses and to obtain in-depth physics understanding.

Our studies reveal that the kinetic dissipations on the RWM instability, in RFPs and tokamks, are due to different kinetic mechanisms, resulting in different stabilizing conditions for the mode in the two systems. The differences in the RWM behavior are directly linked to the characteristics of the two different magnetic configurations. The RFP configuration is characterized by the low q profile, with the poloidal magnetic field \( B_\chi \) being in the same order as the toroidal field \( B_\phi \), and with a toroidal field reversal in the plasma edge. The tokamak plasma has a stronger toroidal field \( B_\phi \) being the order of \( 1/\varepsilon \) greater than \( B_\chi \), and with \( B_\phi \propto 1/R \). We briefly summarize the comparison results in the following table 6.1.

Table 6.1: Comparison between the RFP and the tokamak plasmas on the RWM stability, with the inclusion of kinetic resonances

<table>
<thead>
<tr>
<th>RFP</th>
<th>Tokamak</th>
</tr>
</thead>
<tbody>
<tr>
<td>RWMs are current driven modes.</td>
<td>RWMs are often pressure driven modes.</td>
</tr>
<tr>
<td>All RWMs are “non-resonant” modes.</td>
<td>RWMs can be either “resonant” or “non-resonant” modes.</td>
</tr>
<tr>
<td>Weak toroidal coupling, only one poloidal harmonic dominant</td>
<td>Strong toroidal coupling, multiple poloidal harmonics grow together with comparable amplitudes</td>
</tr>
<tr>
<td>Almost no ballooning character in the mode structure</td>
<td>Usually clear ballooning character in the mode structure</td>
</tr>
<tr>
<td>Some internal non-resonant modes can be stabilized by the plasma rotation speed in the ion acoustic range for high ( \beta ) plasmas.</td>
<td>Pressure driven RWM can be stabilized at very slow plasma rotation (( \Omega &lt; 0.5\omega_A )), or even without rotation.</td>
</tr>
<tr>
<td>Kinetic stabilization is mainly due to the mode resonance with transit frequency of passing particles (ion</td>
<td>Kinetic stabilization at slow velocity is mainly due to the mode resonance with precessional drifts of trapped particles.</td>
</tr>
</tbody>
</table>
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

<table>
<thead>
<tr>
<th>acoustic Landau damping.</th>
<th>The precessional resonance of trapped particles can not stabilize the RWM.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current driven modes can not be stabilized by the precessional resonance.</td>
</tr>
</tbody>
</table>

A comparison of various potential energy components in both configurations demonstrated certain common futures, namely that the kinetic stabilization requires a large fraction of the pressure driven energy component. Decreasing the vacuum energy $\delta W_{vb}$ and $\delta W_{\infty}$, which are related to the normal component of the magnetic field perturbation at the plasma surface, is actually a critical ingredient in bringing the kinetic energy component to play an important role in the mode stabilization. This is because the vacuum energy components $\delta W_{vb}$ and $\delta W_{\infty}$ are the principal factor to reduce $\delta W_b$ and $\delta W_\infty$, to the level comparable to $\delta W_k$. We have found that increasing the plasma $\beta$ leads to the decrease of the normal component of the displacement, as well as the normal perturbed magnetic field at plasma edge, which in turn reduce the perturbed vacuum energy. In tokamaks, at a fixed total current, a larger $q(a)$ value implies a stronger toroidal field, in which the plasma is more resistant to the kink instability, thanks to the larger magnetic bending energy. This, in general, will lead to a higher $\beta_{n,\text{no-wall}}$, and probably smaller $\delta W_{vb}$ and $\delta W_{\infty}$. The combined effect with larger $\delta W_k$ (at higher pressure) eventually leads to an easier stabilization of the RWM by kinetic effects, in tokamak plasmas.

In this study, we use the wall position at $b/a=1.12$ for all of the calculations. Therefore, the effects of the wall position on the kinetic stabilization have not been discussed. Actually, this effect is also an important ingredient for the RWM stability because it is directly related to the vacuum energy component $\delta W_{vb}$. this study is presented in chapter 5 in RFP plasmas.

A uniform flow velocity has been assumed in this study. Compared to the shear flow, this assumption may lead to certain quantitative discrepancies. In tokamak plasmas, if the mode rational surfaces are located inside the plasma, the stability may be more sensitive to the profile of the flow velocity. We leave this issue for the future work. Another assumption is the circular cross section for the tokamak plasmas, in order to make a comparison with the circular RFP plasma.
Kinetic effects on RWM stability—comparison between RFPs and tokamaks

We believe that the physics for the shaped tokamak plasmas is similar to the pressure driven case that we have studied in the present work.

In order to see how the kinetic effect can modify the fluid RWM eigenfunctions, one simple tokamak equilibrium has been investigated, without rational surfaces inside the plasma. The results show a large modification of the RWM eigenfunction by the kinetic effects, compared to the fluid eigenfunction. However, the equilibrium used in this example is very unstable with respect to the RWM, such that both $\delta W_\infty$ and $\delta W_b$ are much larger than $\delta W_k$. Therefore, the perturbative approach only gives a small quantitative discrepancy in the mode eigenvalue, compared to the non-perturbative approach. In many other situations that we investigated, in particular, when the plasma is near the marginal stability, or in the presence of plasma rotation, the two approaches can lead to rather large, sometimes even qualitative, discrepancies. Further investigations will be pursued and reported elsewhere.
In this thesis, the extensive theoretical studies are dedicated to the subject of the RWM stability physics and both passive and active stabilization of RWMs. The knowledge provided by the studies is much relevant not only to the RWM control (passive and active stabilization) in the RFP plasmas but also to the future advanced tokamak devices. In order to carry out these studies, two numerical codes have been adopted and upgraded in the RFX server: CMR-F (cylindrical MHD) code and MARS-K (MHD-kinetic hybrid toroidal) codes. The major results obtained in these three years and the possible further works are listed as the follows:

**Comparison between cylindrical model and experimental observation**

We numerically studied the RWMs stability properties in RFP plasmas by cylindrical MHD model. In order to validate the model and the corresponding CMR code, we concentrate the study on the careful comparison with the experimental observation in RFX-mod. In the numerical calculation, the equilibrium parameters are selected carefully in the pre-computed matrix to well fit the experimental measured values of $F$, $\Theta$, and $\beta_p$. The obtained growth rates by CMR code show a good agreement with the experimental results. Moreover, it shows that the cylindrical model is also consistent with experimental observed RWM instability spectra. It is found that when the field reversal becomes shallower (less negative $F$), INRMs become more unstable while ENRMs are more stable; as the field reversal becomes deeper (more negative $F$), on the contrary, ENRMs becomes more unstable while INRMs are more stable.

The sensitivity of the mode growth rates to the equilibrium parameters $F$, $\Theta$ and $\beta_p$, as well as the pressure profiles have been investigated in detail. It is shown that a careful matching the equilibrium parameters with experiments is necessary in the comparison. In particular, the parameter $F$ and $\Theta$ can significantly influence the mode growth rates. This fact should be noticed while applying a theoretical
Conclusions and further work

model to experiments and/or comparing the theoretical results with the experiments in order to draw a meaningful conclusion.

Physical understanding of RWM instability spectrum in RFP

For the instability spectrum, we explain why the variations of the growth rates with F are in the opposite directions between the ENRMs and INRMs in RFP plasmas, based on the CMR code, although these behaviours have been observed experimentally since years [3,4]; the physical mechanism behind the observation is not yet well understood. By calculating the potential energy components, it is found that when the RFP plasma changes its equilibrium by following the fixed F-Θ curve, the normalized plasma potential energy $\delta W_p / \xi_a^2$ of a given mode is unvaried, which implies that RFP plasma intends to keep the potential energy $\delta W_p$ being a constant value by following a fixed F-Θ relation. This phenomena seems to imply that for whatever type of the field reversal of the discharge, RFP plasmas automatically adjusted its current profiles and the Θ value, intending to keep its potential energy unchanging and remaining as small as possible to the perturbation. Therefore the variation of the RWM growth rates versus F is determined only by the change of the vacuum energy, mainly by $\delta W_v$ which plays the stabilizing role to RWMs. Due to the opposite helical winding between INRM and ENRM in RFPs, the vacuum potential energies ($\delta W_v$, $\delta W_{vb}$) for these two modes reflect the change of F oppositely; thus the changes of the growth rates versus F for the two types of the modes go to different directions. One should also notice that the vacuum potential energy is a decreasing function with the wave number k. The growth rate of RWMs is determined by the balance of the potential energy components.

Physical understanding of the feedback control of RWM in RFP

We introduced the feedback system into the cylindrical model described in chapter 2 in RFP plasma. The eigenmode equation with appropriate boundary condition which is generalized to include the PID feedback controller is provided for the study; and the dispersion relation for RWM with feedback control in terms of the potential energy components are derived under the assumption of $\bar{\omega} << \omega_n$.

Since RWMs in RFPs appears as non-resonant modes, it leads to the solution of modes inside the plasma (eigenfunction of RWMs, $\xi_r (r)$ or $b_r (r)$) being uniquely determined by the equilibrium parameters (except a normalization factor).
Conclusions and further work

The quantity \((\xi'_{f}/\xi_{f})_{a}\) is independent from the feedback application. This conclusion is valid also for the case having plasma rotation and the viscose dissipation if \(v_{0} < v_{oc}\). The relative plasma potential energy \(\delta W_{p}/\xi_{a}^{2}\) is not influenced by the feedback, which shows to be potentially unstable to the external kink even when the feedback stabilization of RWM exists. The role of the feedback control is to modify the magnetic perturbation outside the wall, thus to modify (reduce) the eddy current induced by the plasma perturbation on the resistive wall. As for \(v_{0} > v_{oc}\), the RWM is stabilized by the plasma rotation and dissipation, and the mode rotates with high speed. Both the rotating magnetic perturbation and the possible signal from the feedback coils are totally shielded by the resistive wall because \(\omega \gg \tau_{b}^{-1}\); And the feedback system play no role in this situation.

Based on these conclusions, when the growth rate without feedback, \(\gamma_{0}\) is known, the linear time evolution of the RWM under the feedback control (linear theory) can be easily studied by the calculation only in the vacuum region. In fact, we don’t need to calculate the plasma responses for each time step. The effects of the wall proximity and the location of the sensor have been discussed. The time dependent solution for different feedback scenario of PID controller has also investigated in details. The influence of the response time scale of the feedback system on the stabilizing the RWMs is also studied.

Kinetic damping on RWM in RFP plasmas

The kinetic dissipation on the RWM stability in RFP plasmas is numerically investigated in this chapter, using the hybrid toroidal stability code MARS-K, in which the kinetic resonance effects are included in a non-perturbative way. In order to carry out our study MARS-K is implemented into RFX-mod server. Later, we parallelized the kinetic calculation in MARS-K, which improves the code performance significantly. In order to gain insight physical understanding of kinetic effect on RWMs in tokamak and RFP, the analysis based on the quadratic potential energy components have been performed. The corresponding module is developed and integrated into MARS-K, enabling a better physical analysis.

It is found that the transit resonance of passing ions can plays important roles in the stabilization of RWMs. The transit resonance provides the ion acoustic landau damping, which is able to stabilize the mode in high \(\beta\) RFP plasmas.
Conclusions and further work

Trapped particles do not play a significant role in the kinetic stabilization. The required plasma rotation frequency is much slower than what previously predicted by the fluid theory involving the plasma viscosity (in Alfvén region). The mostly unstable mode, having its rational surface closest to the plasma, can be stabilized for the wall near the plasma with the rotation frequency in the ion acoustic range; for RFX-mod parameters, this is the n=6 INRM. For other RWMs with smaller toroidal wave numbers n, the stabilization conditions depend on the wall position and plasma $\beta_p$. We find the stabilization of the n=5 and n=3 modes in the farther wall position b/a=1.35 and b/a=1.9 respectively, which are near the corresponding marginal wall positions. The physical understanding is provided by analyzing the potential energy components. Moreover, our numerical results indicate that kinetic stabilization always occurs when the wall locates near the critical position $b_c$ where $\delta W_b \approx 0$. In fact, both factors, increasing $\beta_p$ and farther wall position b, can lead to the decrease of $\delta W_b$ and hence a more significant role from the kinetic resonant energy $\delta W_k$. We have also carried out a preliminary investigation of the collisionality effect in low beta RFP plasmas and found that the collisionality plays a minor role on the kinetic stabilization of the RWM in RFP plasmas.

Kinetic effects on RWM stability–comparison between RFPs and tokamaks

We have studied the drift kinetic effects on the RWM for both RFP and tokamak plasmas, using the toroidal MHD-kinetic hybrid code MARS-K and the new developed potential energy components analyzing module.

Our studies reveal that the kinetic dissipations on the RWM instability, in RFPs and tokamaks, are due to different kinetic mechanisms, resulting in different stabilizing conditions for the mode in the two systems. The differences in the RWM behavior are directly linked to the characteristics of the two different magnetic configurations. The RFP configuration is characterized by the low q profile, with the poloidal magnetic field ($B_\chi$) being in the same order as the toroidal field ($B_\phi$), and with a toroidal field reversal in the plasma edge. The tokamak plasma has a stronger toroidal field $B_\phi$, being the order of $1/\varepsilon$ greater than $B_\chi$, and with $B_\phi \propto 1/R$. We briefly summarize the comparison results in the table 6.1.

A comparison of various potential energy components in both configurations demonstrated certain common futures, namely that the kinetic
Conclusions and further work

stabilization requires a large fraction of the pressure driven energy component. Decreasing the vacuum energy $\delta W_{vb}$ and $\delta W_{vxc}$, which are related to the normal component of the magnetic field perturbation at the plasma surface, is actually a critical ingredient in bringing the kinetic energy component to play an important role in the mode stabilization. This is because the vacuum energy components $\delta W_{vb}$ and $\delta W_{vxc}$ are the principal factor to reduce $\delta W_b$ and $\delta W_{s_c}$, to the level comparable to $\delta W_k$. We have found that increasing the plasma $\beta$ leads to the decrease of the normal component of the displacement, as well as the normal perturbed magnetic field at plasma edge, which in turn reduce the perturbed vacuum energy. Therefore, both $\beta$ value and the wall position can modify $\delta W_{vb}$ in RFP and tokamak plasmas. In addition, in tokamaks, the safety factor $q(a)$ also can influence RWM instability. At a fixed total current, a larger $q(a)$ value implies a stronger toroidal field, in which the plasma is more resistant to the kink instability, thanks to the larger magnetic bending energy. This, in general, will lead to a higher $\beta_N^{no-wall}$, and probably smaller $\delta W_{vb}$ and $\delta W_{vxc}$. The combined effect with larger $\delta W_k$ (at higher pressure) eventually leads to an easier stabilization of the RWM by kinetic effects, in tokamak plasmas.

Further work

Based on MARS-K code, we will further develop the code and study the physics of energetic particle effects on MHD instabilities (e.g. RWM, Toroidal Alfvén Eigenmode) in RFP and tokamak plasmas. Moreover, we would like to include more physics of kinetic effect into MARS-K e.g. finite banana orbit width of trapped particles which could not be neglected in some circumstances. This subject is important to the future RFP with neutral beam injection and the advanced tokamak scenarios e.g. ITER.

We will carry out the analytical study by involving kinetic effects into the cylindrical MHD model and create a hybrid code CMR-K code. The new model in cylindrical geometry will be easier for us to do the physical analysis. It will be possible to make comparison of this model with MARS-K code and experiments, in order to obtain more physical understanding of kinetic effect on RWM in both RFP and tokamak plasmas.

As a further application, the behavior of tearing mode might be studied by MARS-K in the presence of feedback system and/or resonant magnetic perturbation, and compared with the observation from the RFX-mod experiments.
Conclusions and further work
The goal of this appendix is to derive the perturbed kinetic pressure and kinetic energy. We denote perturbed quantities by a superscript “(1)” and introduce the coordinates \((s, \chi, \phi)\). In MARS-K, the equilibrium magnetic field is written

\[
B = \nabla \phi \times \nabla \Psi' + F' \nabla \phi = \Psi' \nabla \phi \times \nabla s + \frac{JF}{R^2} \nabla s \times \nabla \chi
\]  

(A.1)

where \(J\) is the Jacobian, \(\Psi'\) is the poloidal magnetic flux, \(\Psi' = d\Psi / ds\), \(F\) is related to the net poloidal current flowing in the plasma and the toroidal field coils \([11]\).

The total perturbed pressure as a tensor is defined as

\[
p^{(1)} = p^{(1)} + p^{(1)}_\perp \mathbf{b} \mathbf{b} + p^{(1)}_\perp \left(1 - \mathbf{b} \mathbf{b}\right)
\]  

(A.2)

The perturbed kinetic pressure terms are

\[
p^{(1)}_\parallel e^{-i\omega t} e^{i\phi} = \sum_{e,i} \int d\Gamma Mv^2_{\parallel} f^{(1)}_L,
\]  

(A.3)

\[
p^{(1)}_\perp e^{-i\omega t} e^{i\phi} = \sum_{e,i} \int d\Gamma \frac{1}{2} Mv^2_{\perp} f^{(1)}_L = \sum_{e,i} \int d\Gamma M \mu B f^{(1)}_L,
\]  

(A.4)

where \(d\Gamma = \frac{2\pi}{M^2} \sum_{\sigma} d\tau d\mu B / \|v\|\), \(\mu = \frac{mv^2}{2B}\), \(B\) is the strength of the magnetic field.

The perturbed distribution function satisfies \([80]\)

\[
\frac{df^{(1)}_L}{dt} = \frac{\partial F}{\partial E} \frac{\partial H^{(1)}_L}{\partial t} - \frac{\partial F}{\partial P_\phi} \frac{\partial H^{(1)}_L}{\partial \phi} - v_{eff} f^{(1)}_L,
\]  

(A.5)

By assuming the perturbations having the form

\[
X^{(1)} = \tilde{X}^{(1)}(s, \chi) e^{-i\omega t} e^{i\phi}
\]  

(A.6)
Appendix

The perturbed Lagrangian can be given [80, 81]

\[ H^{(1)} = -\varepsilon_{i} H_{L} e^{-i\omega_{T} t} \]  
(A.7)

\[ H_{L} = \frac{1}{\varepsilon_{i}} \left[ M_{i} \eta \cdot \xi_{\perp} + \mu \left( Q_{i} + \nabla B \cdot \xi_{\perp} \right) \right] \]  
(A.8)

\( \nu_{eff} \) is the effective collisionality coefficient, \( Q_{i} \) is the parallel magnetic perturbation, \( \kappa \) is the magnetic curvature. In the limit \( \delta_{b} \to 0 \), where \( \delta_{b} \) is the banana orbit width, the toroidal canonical momentum [80, 86] is

\[ P_{\phi} = -Z e \Psi \]  
(A.9)

This sign convention for \( \Psi \) \((0 < \Psi < \Psi_{a})\) agrees with that in MARS-F. Equation (A.5) can be written as

\[ \frac{df_{i}^{(1)}}{dt} = -i(\omega - n\omega_{r}) \frac{\partial F}{\partial \varepsilon} H^{(1)} - \nu_{eff} f_{i}^{(1)} \]  
(A.10)

\[ \omega_{r} = -\frac{\partial F / \partial \phi_{r}}{\partial F / \partial \varepsilon} = \frac{1}{Z e} \frac{\partial F / \partial \Psi}{\partial F / \partial \varepsilon} \]  
(A.11)

Assume Maxwellian distribution for thermal particles

\[ F = N \left( \frac{M}{2\pi T} \right)^{3/2} e^{-\varepsilon_{i}/T} = N \left( \frac{M}{2\pi T} \right)^{3/2} e^{-\varepsilon_{i}/T + Z e \Phi/T} \]  
(A.12)

The equation \( \omega \) can be cast into

\[ \omega = \omega_{r} + (\hat{\varepsilon}_{i} - 3/2) \omega_{T} + \omega_{k} \]  
(A.13)

where

\[ \omega_{r} = -\frac{1}{Z e} \frac{dT}{d\Psi}, \quad \omega_{r} = -\frac{1}{Z e} \frac{T}{N d\Phi}, \quad \hat{\varepsilon}_{i} = \frac{\varepsilon_{i}}{T} \]

Assume only toroidal equilibrium flow \( V_{0} = R^{2} \Omega (\Psi) \nabla \phi \), thus

\[ E = -\nabla \Phi = -V_{0} \times B = \Omega \nabla \Psi \], thus \( \omega_{k} = -\frac{d\Phi}{d\Psi} = \Omega \)

The solution of equation (A.10) is

140
Appendix

\[ f_L^{(1)} = -i \left[ \omega - n\omega^* \right] f^0 e^{-\nu_0 t} \int_{-\infty}^{t} H^{(1)}(\tau) e^{i\omega^* \tau} d\tau \]  

(A.14)

where \( f^0 = \frac{\partial F}{\partial x}. \)

Let’s write \( H_L \) symbolically as

\[ H_L = C(s, \chi, \Lambda) \sum_m X_m e^{imx} \]  

(A.15)

where \( \Lambda = \frac{\mu}{\epsilon_k} B_0 \) being the pinch angle is only involved in \( C(s, \chi, \Lambda). \)

The perturbed distribution function (A.14) becomes

\[ f_L^{(1)} = i \left[ \omega - n\omega^* \right] f^0 e_{\epsilon_k} e^{-\nu_0 t} \int_{-\infty}^{t} C(\tau) \sum_m X_m e^{i\omega t + \nu_0 e_{\epsilon_k} \tau + im\chi(\tau) + im\phi(\tau)} d\tau \]  

(A.16)

\( \chi \) and \( \phi \) can be decomposed into secular part and periodic part

\[ \chi(\tau) = <\tilde{\chi} \tau + \hat{\chi}(\tau) \]  

(A.17)

\[ \phi(\tau) = <\tilde{\phi} \tau + \hat{\phi}(\tau) \]  

(A.18)

Where \( < \cdot \cdot > \) denotes the bounce average \( \int_{-\tau_b}^{\tau_b} d\tau / \tau_b \) of trapped/passing particles.

Thus

\[ f_L^{(1)} = i \left[ \omega - n\omega^* \right] f^0 e_{\epsilon_k} \sum_m X_m e^{i\omega t + \nu_0 e_{\epsilon_k} \tau + im\chi(\tau) + im\phi(\tau)} \int_{-\infty}^{t} C(\tau) e^{im\tilde{\chi}(\tau) + im\tilde{\phi}(\tau)} e^{-i\omega t + \nu_0 e_{\epsilon_k} \tau + im\chi(\tau) + im\phi(\tau)} d\tau \]  

(A.19)

\[ = i \left[ \omega - n\omega^* \right] f^0 e_{\epsilon_k} \sum_m X_m e^{i\omega t + \nu_0 e_{\epsilon_k} \tau} \int_{-\infty}^{t} \tilde{H}(\tau) e^{i\omega t + \nu_0 e_{\epsilon_k} \tau + im\chi(\tau) + im\phi(\tau)} d\tau \]  

(A.20)

where \( \tilde{H}(\tau) \) is a periodic function of \( \tau \) (with a bouncing period), which can be expanded in Fourier series of bouncing orbit

\[ \tilde{H}(\tau) = C(\tau) e^{im\tilde{\chi}(\tau) + im\tilde{\phi}(\tau)} = \sum_{l=-L_0}^{L_0} H_{m\ell} e^{i\omega_{\ell}\tau} \]  

(A.21)

with
Appendix

\[ H_{ml} = \frac{1}{\tau_b} \oint C(\tau) e^{i m \Omega(\tau) + i n \Phi(\tau) - i \Omega_{b} \tau} d\tau \]  
(A.22)

For an unstable perturbation (Re(\(-i\omega\)) > 0), by carrying out the time integration of equation (A.20), we obtain

\[ f_{k}^{(i)} = i \left[ \omega - n \omega^* \right] f_{k}^{n} e_{k} \sum_{m,l} X_{m}^{n} H_{ml} e^{-i \omega_{l} \tau} \int_{-\infty}^{\infty} e^{-i \omega_{l} \tau + i m \phi + i n \phi + i \Omega_{b} \tau} d\tau \]  
(A.23)

\[ = i \left[ \omega - n \omega^* \right] f_{k}^{n} e_{k} \sum_{m,l} X_{m}^{n} H_{ml} e^{-i \omega_{l} \tau} \frac{\exp\left[-i \omega_{l} \tau + i m \phi + i n \phi + i \Omega_{b} \tau\right]}{i(m - \hat{\chi} + n < \phi > + \omega_{b} - i v_{eff})} \]  
(A.24)

\[ = -f_{k}^{n} e_{k} \sum_{m,l} X_{m}^{n} H_{ml} \frac{n\left(\omega_{N} + \left(\hat{\chi} - 3 / 2\right) \omega_{T} + \omega_{E}\right) - \omega}{m - \hat{\chi} + n < \phi > + \omega_{b} - i v_{eff}} e^{-i \omega_{l} \tau + i m \phi + i n \phi + i \Omega_{b} \tau} \]  
(A.25)

In the zero orbit width limit, \(< \hat{\chi}>\) and \(< \phi>\) can be written as

\[ < \hat{\chi}> = \frac{\oint \hat{\chi} d\tau}{\oint d\tau} = \frac{\omega_{b}}{2\pi} \oint \hat{\phi} d\tau = \alpha \omega_{b} \]  
(A.26)

\[ < \phi> = \frac{\oint \phi d\tau}{\oint d\tau} = \frac{\omega_{b}}{2\pi} \left[ \int_{\tau_{0}}^{\tau + \tau_{b}} \frac{d(\phi - q \hat{\chi})}{d\tau} d\tau + \int_{\tau_{0}}^{\tau + \tau_{b}} \frac{d(q \hat{\chi})}{d\tau} d\tau \right] \]  
(A.27)

\[ < \phi> = \frac{\omega_{b}}{2\pi} \int_{\tau_{0}}^{\tau + \tau_{b}} \frac{d(\phi - q \hat{\chi})}{d\tau} d\tau + \alpha \omega_{b} \]  
(A.28)

where \(\alpha = 0\) is for trapped particles, \(\alpha = 1\) is for passing particles.

Note that for passing particles, \(\omega_{b}\) can be positive or negative depending on \(\sigma = \text{sign}(v_{f})\). For trapped particles, \(\omega_{b}\) is always positive. For \(\delta_{b} / r \to 0\), the first term on the right-hand side of equation (A.27) reduces to the bounce-averaged magnetic precession frequency \(\omega_{b} = \omega_{E} + \omega_{b}\) [87], therefore

\[ \lambda_{ml} = \frac{n\left(\omega_{N} + \left(\hat{\chi} - 3 / 2\right) \omega_{T} + \omega_{E}\right) - \omega}{n \omega_{b} + \left[ \alpha(m + nq) + l \right] \omega_{b} - i v_{eff} - \omega} \]  
(A.29)

\[ = \frac{n\left(\omega_{N} + \left(\hat{\chi} - 3 / 2\right) \omega_{T} + \omega_{E}\right) - \omega}{n \omega_{b} + \left[ \alpha(m + nq) + l \right] \omega_{b} + n \omega_{b} - i v_{eff} - \omega} \]  
(A.30)
Appendix

\( f_L^{(1)} = -f_k^0 e_{\kappa} e^{-i\omega_{\kappa} \tau} \sum_{m,j} X_m H_{ml} \lambda_{ml} e^{-i \theta(t) + im <x+1>/h} \) \hspace{1cm} (A.31)

Based on equations (A.3), (A.4) and (A.31) and the following definitions,

\[ d\Gamma = \sqrt{\frac{2\pi T^{3/2}}{B_0 M^{3/2}}} \sum_{\sigma} d\varepsilon_{\kappa} d\Lambda \sqrt{\frac{\varepsilon_{\kappa} B}{1 - \Lambda / h}}, \] \hspace{1cm} (A.32)

\[ \varepsilon_{\kappa} = e_{\kappa}/T \] \hspace{1cm} (A.33)

\[ f_{\varepsilon}^0 = -\frac{N}{T \left(2\pi T\right)^{3/2}} e^{-\varepsilon_{\kappa}/T} \] \hspace{1cm} (A.34)

where \( h = B_0 / B, \varepsilon_{\kappa} = \varepsilon - Ze\Phi, \) the kinetic pressure terms \( p_{||}^{(1)} \) and \( p_{\perp}^{(1)} \) become

\[ p_{||}^{(1)} = \frac{1}{\sqrt{\pi}} \sum_{e_{\kappa} m_{\xi}} \sum_{j_{\kappa} m_{\eta}} \frac{P_{e_{\kappa}j_{\kappa}}}{B_0} \int d\varepsilon_{\kappa} d\Lambda \varepsilon_{\kappa} \sqrt{\frac{\varepsilon_{\kappa} B}{1 - \Lambda / h}} e^{-i \omega_{\kappa} \tau} \lambda_{ml} X_{ml} B \sqrt{1 - \Lambda / h} e^{-im <x+1>/h} \] \hspace{1cm} (A.35)

\[ p_{\perp}^{(1)} = \frac{1}{\sqrt{\pi}} \sum_{e_{\kappa} m_{\xi}} \sum_{j_{\kappa} m_{\eta}} \frac{P_{e_{\kappa}j_{\kappa}}}{B_0} \int d\varepsilon_{\kappa} d\Lambda \varepsilon_{\kappa} \sqrt{\frac{\varepsilon_{\kappa} B}{1 - \Lambda / h}} \lambda_{ml} X_{ml} \frac{BA}{2h \sqrt{1 - \Lambda / h}} e^{-i \omega_{\kappa} \tau} \] \hspace{1cm} (A.36)

where \( P_{\|} \) and \( P_{\perp} \) are the equilibrium pressure of ions and electrons respectively.

The kinetic energy is derived from the kinetic pressure equations (A.2)-(A.4), (A.29) and (A.31).

\[ p^{\text{kinetic}} = \bar{p}_{||} \hat{b} \hat{b} + \bar{p}_{\perp} \left(1 - \hat{b} \hat{b}\right) \] \hspace{1cm} (A.37)

\[ \bar{p}_{||} = p_{||}^{(1)} e^{-i\omega_{\kappa} \tau} \] \hspace{1cm} (A.38)

\[ \bar{p}_{\perp} = p_{\perp}^{(1)} e^{-i\omega_{\kappa} \tau} \] \hspace{1cm} (A.39)

Starting from the definition of kinetic energy

\[ \delta W_k = -\frac{1}{2} \int -\nabla \cdot p^{\text{kinetic}} \cdot \varepsilon_{\kappa}^* d^3x \] \hspace{1cm} (A.40)
Appendix

\[ \frac{1}{2} \left[ \nabla \tilde{p}_\perp + B \left( \frac{\tilde{p}_\perp - \tilde{p}_\parallel}{B} \cdot \mathbf{b} \right) + \left( \tilde{p}_\parallel - \tilde{p}_\perp \right) \kappa \right] \cdot \xi_\perp^* d^3x \]  
(A.41)

\[ = -\frac{1}{2} \left[ \tilde{p}_\perp \nabla \cdot \xi_\perp^* - \left( \tilde{p}_\parallel - \tilde{p}_\perp \right) \kappa \cdot \xi_\perp^* \right] d^3x + \frac{1}{2} \int \tilde{p}_\perp \xi_\perp^* \cdot ds \]  
(A.42)

We neglected the surface integration term in equation (A.42) since vanishing equilibrium pressure is assumed at the plasma boundary and \( \tilde{p}_\perp \propto P_{ei} \) according to equation (A.36).

With \( d^3x = J ds d\phi \), the kinetic energy can be written as

\[ \delta W_K = \pi \int J ds d\phi \left[ \tilde{p}_\perp \frac{1}{B} \left( Q_i^* + \nabla B \cdot \xi_\perp^* \right) + \tilde{p}_\parallel \kappa \cdot \xi_\perp^* \right] \]  
(A.43)

Substitute equations (A.3) and (A.4) into equation (A.43)

\[ \delta W_K = \pi \int J ds d\phi \left[ \frac{1}{B} \left( Q_i^* + \nabla B \cdot \xi_\perp^* \right) \sum_{e,d} \int d\Gamma \mu B_f^i \right. \]  
(A.44)

\[ + \left. \kappa \cdot \xi_\perp^* \sum_{e,d} \int d\Gamma M v^2 f^i \right] \]  
(A.45)

With equations (A.8) and (A.31), the expression of kinetic energy becomes

\[ \delta W_K = \pi \sum_{e,d} \int J ds d\phi \left[ \int d\Gamma \left( -f_t^{0\nu} \right) e_k \sum_{m,d} X_{m,H_{ml}} \lambda_m e^{-\imath \nu t + \imath \epsilon \nu t} \epsilon_k H_L^* \right] \]  
(A.46)

where \( H_L^* \) is the complex conjugate of \( H_L \).

With equations (A.32) and (A.33), \( \delta W_K \) arrives at

\[ \delta W_K = \pi \sum_{e,d} \int J ds d\phi \left[ \int \frac{\sqrt{2\pi} \pi^{3/2}}{B_0 M^{3/2}} \sum_{\sigma} d\tilde{\epsilon}_k \sigma \Lambda \frac{\tilde{\epsilon}_k^2}{\sqrt{1 - \Lambda^2 / h}} e_k^2 \left( -f_t^{0\nu} \right) \sum_{m,d} X_{m,H_{ml}} \lambda_m e^{-\imath \nu t + \imath \epsilon \nu t} \epsilon_k H_L^* \right] \]  
(A.47)

Substitute equation (A.34) into (A.47)

\[ \delta W_K = \frac{\sqrt{2\pi}}{2} \sum_{e,d} \int J ds d\phi \left[ \frac{1}{B_0} d\tilde{\epsilon}_k \sum_{\sigma} \int \sigma \Lambda \frac{\tilde{\epsilon}_k^2}{\sqrt{1 - \Lambda^2 / h}} e_k^2 \left( NT e^{-\imath \nu t} \right) \sum_{m,d} X_{m,H_{ml}} \lambda_m e^{-\imath \nu t + \imath \epsilon \nu t} \epsilon_k H_L^* \right] \]
Appendix

\[ \frac{\pi}{2B_0} \sum_{ij} \left[ P_{ij} \left( d \tilde{E}, \tilde{E}_k \right) \right] \sum_n \left[ \int d \Lambda \frac{J_B}{\sqrt{1 - \Lambda / h}} \left( \sum_{m,l} X_{nl} H_{nl} \lambda_{nm} e^{-i\phi(t)} \sum_{m,j} H^*_m X^*_m e^{-i\phi(t)} \right) \right] \]  \hspace{1cm} (A.48)

By using \( H^*_m(t) = \sum_{m,l} H_{ml}^* X^*_m e^{-im\tilde{E}t + in\phi(t) - im\phi(t)} \), it is obtained

\[ \delta W_k = \frac{\pi}{2B_0} \sum_{ij} \left[ P_{ij} \left( d \tilde{E}, \tilde{E}_k \right) \right] \sum_n \left[ \int d \Lambda \frac{J_B}{\sqrt{1 - \Lambda / h}} \left( \sum_{m,j} X_{nl} H_{nl} \lambda_{nm} e^{-i\phi(t)} \sum_{m,j} H^*_m X^*_m e^{-i\phi(t)} \right) \right] \]  \hspace{1cm} (A.49)

Let \( l = k - am \), thus

\[ \lambda_k = \frac{n}{nq + k} \left[ \omega_n - \frac{3}{2} \omega + \omega_E \right] - \omega \]  \hspace{1cm} (A.50)

Equation (A.50) has the form

\[ \delta W_k = \frac{\pi}{2B_0} \sum_{ij} \left[ P_{ij} \left( d \tilde{E}, \tilde{E}_k \right) \right] \sum_n \left[ \int d \Lambda \frac{J_B}{\sqrt{1 - \Lambda / h}} \left( \sum_{l,j} \lambda_l \left[ \sum_{m,j} X_{nl} H_{nl} \lambda_{nm} e^{-i\phi(t)} \sum_{m,j} H^*_m X^*_m e^{-i\phi(t)} \right] \right) \right] \]  \hspace{1cm} (A.51)

Then the order of integrations can be changed, based on the relation

\[ \int_{x}^{x^h} d \chi \left[ f_{x^h}^{x} d \Lambda \right] = \int_{x}^{x^h} d \Lambda \left[ f_{x^h}^{x} d \chi \right] \]  \hspace{1cm} (A.52)

where \( x^h \) and \( x_l \) are two returning points for trapped particles. \( x^h \) and \( x_l \) have 2\( \pi \) phase difference for passing particles.

\[ \delta W_k = \frac{\pi}{2B_0} \sum_{ij} \left[ P_{ij} \left( d \tilde{E}, \tilde{E}_k \right) \right] \sum_n \left[ \int d \Lambda \frac{J_B}{\sqrt{1 - \Lambda / h}} \left( \sum_{l,j} \lambda_l \left[ \sum_{m,j} X_{nl} H_{nl} \lambda_{nm} e^{-i\phi(t)} \sum_{m,j} H^*_m X^*_m e^{-i\phi(t)} \right] \right) \right] \]  \hspace{1cm} (A.53)

\[ \delta W_k = \frac{\pi}{2B_0} \sum_{ij} \left[ P_{ij} \left( d \tilde{E}, \tilde{E}_k \right) \right] \sum_n \left[ \int d \Lambda \frac{J_B}{\sqrt{1 - \Lambda / h}} \left( \sum_{l,j} \lambda_l \left[ \sum_{m,j} X_{nl} H_{nl} \lambda_{nm} e^{-i\phi(t)} \sum_{m,j} H^*_m X^*_m e^{-i\phi(t)} \right] \right) \right] \]  \hspace{1cm} (A.54)

\[ \delta W_k = \frac{\pi}{2B_0} \sum_{ij} \left[ P_{ij} \left( d \tilde{E}, \tilde{E}_k \right) \right] \sum_n \left[ \int d \Lambda \frac{J_B}{\sqrt{1 - \Lambda / h}} \left( \sum_{l,j} \lambda_l \left[ \sum_{m,j} X_{nl} H_{nl} \lambda_{nm} e^{-i\phi(t)} \sum_{m,j} H^*_m X^*_m e^{-i\phi(t)} \right] \right) \right] \]  \hspace{1cm} (A.55)
Appendix

Since \( \langle \dot{\chi} \rangle = \alpha \omega_b \), \( \delta W_k \) takes the form

\[
\delta W_k = \frac{\sqrt{\pi}}{2B_0} \sum_{c \sigma} \int d\mathcal{C} \mathcal{P} e \int d\mathcal{C} e \sum_{\mathcal{C}} \left[ \int_{-\infty}^{\infty} d\mathcal{A} \left( \sum_{k \lambda} \epsilon_{k \lambda} \frac{J_B}{\sqrt{1 - \Lambda}} \right) \right]
\]

(A.56)

By defining

\[
\tau_b = \frac{M}{2 \epsilon_h} \tau_b, \quad \hat{\tau}_b = \frac{J_B / \Psi}{\sigma / \sqrt{1 - \Lambda}} d\chi = \frac{J_B}{\sigma / \sqrt{1 - \Lambda}} d\chi \]

it is found that

\[
\delta W_k = \frac{\sqrt{\pi}}{2B_0} \sum_{c \sigma} \int d\mathcal{C} \mathcal{P} e \int d\mathcal{C} e \sum_{\mathcal{C}} \left[ \int_{-\infty}^{\infty} d\mathcal{A} \left( \sum_{k \lambda} \epsilon_{k \lambda} \frac{J_B}{\sqrt{1 - \Lambda}} \right) \right]
\]

(A.57)

Since \( \omega_b \Lambda = 2 \pi \tau_b \), the kinetic energy can be further written as

\[
\delta W_k = \frac{\sqrt{\pi}}{2B_0} \sum_{c \sigma} \int d\mathcal{C} \mathcal{P} e \int d\mathcal{C} e \sum_{\mathcal{C}} \left[ \int_{-\infty}^{\infty} d\mathcal{A} \left( \sum_{k \lambda} \epsilon_{k \lambda} \frac{J_B}{\sqrt{1 - \Lambda}} \right) \right]
\]

(A.58)

\[
= \frac{\sqrt{\pi}}{2B_0} \sum_{c \sigma} \int d\mathcal{C} \mathcal{P} e \int d\mathcal{C} e \sum_{\mathcal{C}} \left[ \int_{-\infty}^{\infty} d\mathcal{A} \right]
\]

(A.59)

where \( \nu = 1/2 \) for trapped particles, \( \nu = 1 \) for passing particles.

With the following relation

\[
\left| \left< H \mathcal{P} \mathcal{C} e^{-\mathcal{C} \mathcal{A}} d\mathcal{A} \right> \right| = \frac{1}{\tau_b} \int H \mathcal{P} \mathcal{C} e^{-\mathcal{C} \mathcal{A}} d\mathcal{A}
\]

(A.61)

\[
H_L(t) = \sum_{m,k} H_{mk} X^* e^{i\epsilon_{k-\Lambda} t - \mathcal{C} \mathcal{A} X^*}
\]

(A.62)
Appendix

\[
\left| \left\langle H e^{i \hat{\gamma}(t)} e^{-i \hat{\Delta} \tau} d \tau \right\rangle \right|^2 = \frac{1}{\tau_b} \sum_m X_m \int_0^{\tau_b} \sum_l H_{m,l} e^{il \omega \tau - i (l- \omega) \tau + i \gamma(\tau)} d \tau
\]
(A.63)

\[
= \frac{1}{\tau_b} \sum_m X_m \int_0^{\tau_b} \sum_l H_{m,l} e^{il \omega \tau - i (l- \omega) \tau} d \tau
\]
(A.64)

\[
= \left| \sum_m X_m H_{m,j-\omega} \right|^2
\]
(A.65)

finally, the kinetic energy can be written as

\[
\delta W_k = \frac{\sqrt{\pi}}{2B_0} \sum_{\alpha \beta} \int d^4 \Phi \left\{ \int d^4 \Phi \hat{\gamma}^{1/2} e^{-i \hat{\gamma}} \sum_{\alpha} \left[ \int d^4 \Phi \sum_{\beta} \lambda_{\alpha \beta} \left( H e^{-i \Delta \tau + i \gamma(\tau)} d \tau \right)^2 \right] \right\}
\]
(A.66)
Bibliography


Bibliography


Bibliography


Bibliography


Publications

Publications related to this thesis


2) **Wang Z. R.** and Guo S. C., “Physical understanding of the instability spectrum and the feedback control of resistive wall modes in Reversed Field Pinch”, 2011 Nuclear Fusion, 51 053004s, Dublin Ireland 21-25 June 2010, ECA Vol. 34A P4.171 (related to chapter 3 and chapter 4)


4) **Wang Z. R.**, Guo S. C., and Liu Y. Q., “Kinetic damping of resistive wall mode in reversed field pinch”, submitted to Nuclear Fusion (has been approved by internal referee in Consorzio RFX). (related to chapter 5)


Other publications


Acknowledgements

During the three years of my PhD study, it was a pleasure for me to work with all the wonderful people in our lab, Consorzio RFX.

First of all, I would like to express my immense gratitude to Dr. Shichong Guo, being a great advisor, whose expertise, broad understanding of plasma physics, and patience, greatly influenced me in my life. She patiently provided the vision, encouragement and advise necessary for me to proceed through the doctoral program and complete my dissertation which would not have been possible without her expert guidance. She also has given me many opportunities to participate and present our works in many conferences and to work in collaboration with many outstanding researchers. In particular, I want to thank for her selfless help whenever I met any problem.

I would like to thank my supervisor Prof. Antonio Buffa for making possible my PhD study in Consorzio RFX. I wish to thank him for his kindly help when I met any problem in work and life during these three years, and taking time to review my thesis and giving me many constructive suggestions.

I’m especially grateful to Dr. Yueqiang Liu of Culham Centre for Fusion Energy. His experience in theoretical physics and MARS-K code has allowed us to obtain important and significant results of kinetic effect on RWM. It is my pleasure to work with such an expert at plasma physics and also a friendly person.

I want to thank Prof. Piero Martin and Ms. Fiorella Colautti for their efforts on arranging our PhD program and kindly help me on solving many administrative matters. I would like to thank Prof. Escande for taking time to review my thesis and helping me in improving the expression of my thesis.

I’m very much grateful to Dr. Lidia Piron for the translation of the summary of my thesis into Italian. I’m thankful to all the members of FT group and the group leader Dr. Susanna Cappello. I’m so glad to work with you. I would like to thank to Dr. Tommaso Bolzonella and Dr. Matteo Baruzzo for kindly
Acknowledgements

providing the experimental data and fruitful discussions on the work of comparison between theory and experimental observation of RWM. I’m incredibly fortunate to have such friendly PhD colleagues: Stefano Munaretto, Barbara Momo and Silvia Spagnolo. I want to thank all my long-term and best friends who always support me. You all have been very patient with me.

Finally, the greater acknowledgment is dedicated to my parents, Gang Wang and Ling Yu. Their love provided my inspiration and was my driving force. I owe them everything and wish I could show them just how much I love and appreciate them.