Rotor position estimation in IPM motor drives based on PWM current harmonics

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Outline

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Nowadays the sensorless position control based on high frequency injected signals is largely studied.

The method is based on the injection of high frequency sine wave additional voltages, that are added to the fundamental voltages that feed the machine.

The high frequency signals can be inject in the stationary ($\alpha - \beta$) or in the rotating ($d - q$) reference frame.
Sensorless control drive scheme

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The estimated speed $\tilde{\omega}_{me}$ and position $\tilde{\theta}_{me}$ are delivered by an estimation algorithm.
The estimated position $\tilde{\theta}_{me}$ is used in the reference frame transformations.

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In the case of three phase motor drive, the fundamental voltages feeding the motor are:

\[
\begin{align*}
    u_a^*(t) &= U \cos (\theta_u) \\
    u_b^*(t) &= U \cos (\theta_u - \frac{2\pi}{3}) \\
    u_c^*(t) &= U \cos (\theta_u + \frac{2\pi}{3})
\end{align*}
\]

where \( \theta_u = \tilde{\theta}_{me} + \theta_u^r \)

Really, due to the inverter, the actual voltages applied to the motor are the result of the PWM control.
The voltages, delivered by a space vector PWM inverter, are composed by an infinite sum of sine waves. Among this harmonics there are also those around the switching frequency $f_c$. Then, there is an intrinsic high frequency injection due to the PWM modulation.
The idea proposed in this work is the exploitation of the PWM effects in the stator currents for the electrical rotor position estimation.
In the work the single edge PWM modulation with switching period $T_c = 1/f_c$ are taken into account.
It is possible to derive the expression of the voltage harmonic component at switching frequency $f_c$ of a generic phase voltage $u_c$ applying the Fourier series complex form:

$$u_c = \dot{u}^+ e^{i\omega_c t} + \dot{u}^- e^{-i\omega_c t}$$

with $\dot{u}^+$ and $\dot{u}^- \in \mathbb{C}$, $\omega_c = 2\pi f_c$

At frequency $\omega_c$ there are two voltage vectors that rotate in clockwise and anti–clockwise direction. This is peculiar of the single edge PWM. Symmetrical PWM has not harmonic vectors at frequency $\omega_c$. 
Starting from the phase $a$ voltage, it results:

$$
\dot{u}_a^+ = \frac{1}{T_c} \int_0^{T_c} u_a(t) e^{-i\omega_c t} \, dt
$$

$$
= \frac{1}{T_c} \left[ \int_0^{T_{a,\text{on}}} \frac{U_{dc}}{2} e^{-i\omega_c t} \, dt + \int_{T_{a,\text{on}}}^{T_c} -\frac{U_{dc}}{2} e^{-i\omega_c t} \, dt \right]
$$

$$
= \frac{iU_{dc}}{2\pi} \left[ e^{-i\omega_c T_{a,\text{on}}} - 1 \right]
$$

and

$$
\dot{u}_a^- = \text{conj}(\dot{u}_a^+) = -\frac{iU_{dc}}{2\pi} \left[ e^{i\omega_c T_{a,\text{on}}} - 1 \right]
$$
\[ \dot{u}_a^+ \text{ and } \dot{u}_a^- \text{ can be substituted in the initially equation:} \]

\[ u_{ac} = \frac{iU_{dc}}{2\pi} \left[ e^{-i\omega_c T_{a,on}} - 1 \right] e^{i\omega_c t} - \frac{iU_{dc}}{2\pi} \left[ e^{i\omega_c T_{a,on}} - 1 \right] e^{-i\omega_c t} \]  \hspace{1cm} (8)

Finally it results

\[ u_{ac} = \frac{U_{dc}}{\pi} \cos(\omega_c t) \sin(\omega_c T_{a,on}) - \]

\[ - \frac{U_{dc}}{\pi} \sin(\omega_c t) \cos(\omega_c T_{a,on}) + \frac{U_{dc}}{\pi} \sin(\omega_c t) \]  \hspace{1cm} (9)

with

\[ T_{a,on} = \frac{1}{U_{dc}} |u| \cos \theta_u + \frac{1}{2} \]  \hspace{1cm} (10)
Switching frequency harmonic analysis

Using the previously expression for $T_{a,\text{on}}$ and the Bessel function, it results:

$$u_{ac} = A_0 \sin(\omega_c t) + \sum_{n=1}^{+\infty} (-1)^n A_{2n-1} \cos((2n - 1)\theta_u) \cos(\omega_c t)$$

$$- \sum_{n=1}^{+\infty} A_{2n} \cos(2n\theta_u) \sin(\omega_c t)$$

with

$$A_0 = \frac{U_{dc}}{\pi} \left[ J_0 \left( \frac{2\pi|u|}{U_{dc}} \right) - \frac{1}{2} \right]$$

$$A_{2n-1} = 2 \frac{U_{dc}}{\pi} J_{2n-1} \left( \frac{2\pi|u|}{U_{dc}} \right)$$

$$A_{2n} = 2 \frac{U_{dc}}{\pi} J_{2n} \left( \frac{2\pi|u|}{U_{dc}} \right)$$

where $J_n$ denotes the Bessel function of order $n$. 

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Similarly, the phase $b$ and $c$ high frequency voltages can be computed.

\[
u_{bc} = A_0 \sin(\omega_c t) + \sum_{n=1}^{+\infty} (-1)^n A_{2n-1} \cos \left( (2n - 1) \left( \theta_u - \frac{2\pi}{3} \right) \right) \cos(\omega_c t) - \\
\sum_{n=1}^{+\infty} A_{2n} \cos \left( 2n \left( \theta_u - \frac{2\pi}{3} \right) \right) \sin(\omega_c t)
\]

\[
u_{cc} = A_0 \sin(\omega_c t) + \sum_{n=1}^{+\infty} (-1)^n A_{2n-1} \cos \left( (2n - 1) \left( \theta_u + \frac{2\pi}{3} \right) \right) \cos(\omega_c t) - \\
\sum_{n=1}^{+\infty} A_{2n} \cos \left( 2n \left( \theta_u + \frac{2\pi}{3} \right) \right) \sin(\omega_c t)
\]
High frequency harmonics of voltages $u_\alpha$ and $u_\beta$

Transforming the previously high frequency voltages in the $\alpha\beta$ reference frame, it results:

$$u'_{\alpha c} = \frac{2}{3} \sum_{n=1}^{+\infty} (-1)^n A_{2n-1} \cos(\omega_c t) \cos((2n-1)\theta_u) \left( 1 - \cos \left( (2n-1)\frac{2\pi}{3} \right) \right) +$$

$$+ \frac{2}{3} \sum_{n=1}^{+\infty} (-1)^n A_{2n} \sin(\omega_c t) \cos(2n\theta_u) \left( 1 - \cos \left( 2n\frac{2\pi}{3} \right) \right)$$

(15)

$$u'_{\beta c} = \frac{2}{\sqrt{3}} \sum_{n=1}^{+\infty} (-1)^n A_{2n-1} \cos(\omega_c t) \sin((2n-1)\theta_u) \sin \left( (2n-1)\frac{2\pi}{3} \right) +$$

$$+ \frac{2}{\sqrt{3}} \sum_{n=1}^{+\infty} (-1)^n A_{2n} \sin(\omega_c t) \sin(2n\theta_u) \sin \left( 2n\frac{2\pi}{3} \right)$$

(16)
Switching frequency harmonic analysis

High frequency harmonics of voltages $u_\alpha$ and $u_\beta$

From all the harmonics it is possible to take into account only those dependently on the cosine of $\theta_u$:

$$u_{\alpha_c} = -\frac{A_1}{2} \cos(\omega_c t + \theta_u) - \frac{A_1}{2} \cos(-\omega_c t + \theta_u)$$

$$= -\frac{A_1}{2} \cos(\omega_c t + \tilde{\theta}_{me} + \theta_u^r) - \frac{A_1}{2} \cos(-\omega_c t + \tilde{\theta}_{me} + \theta_u^r)$$ (17)

$$u_{\beta_c} = -\frac{A_1}{2} \sin(\omega_c t + \theta_u) - \frac{A_1}{2} \sin(-\omega_c t + \theta_u)$$

$$= -\frac{A_1}{2} \sin(\omega_c t + \tilde{\theta}_{me} + \theta_u^r) - \frac{A_1}{2} \sin(-\omega_c t + \tilde{\theta}_{me} + \theta_u^r)$$ (18)

The voltage vector is given by two rotating vectors at speeds $\omega_c \pm \tilde{\omega}_{me}$. 
Switching frequency harmonic analysis

High frequency harmonics of currents $i_{\alpha c}$ and $i_{\beta c}$

With these high frequency voltages, the high frequency currents, in the steady–state operation and neglecting the resistance voltage drop, result in:

$$i_{\alpha c} = -I_0^+ \sin(\omega_c t + \tilde{\theta}_{me} + \theta_r^u) -$$
$$- I_1^+ \sin(\omega_c t + \tilde{\theta}_{me} - 2\theta_{me} + \theta_r^u) -$$
$$- I_0^- \sin(\omega_c t - \tilde{\theta}_{me} - \theta_r^u) -$$
$$- I_1^- \sin(\omega_c t - \tilde{\theta}_{me} + 2\theta_{me} - \theta_r^u)$$

(19)

$$i_{\beta c} = I_0^+ \cos(\omega_c t + \tilde{\theta}_{me} + \theta_r^u) -$$
$$- I_1^+ \cos(\omega_c t + \tilde{\theta}_{me} - 2\theta_{me} + \theta_r^u) -$$
$$- I_0^- \cos(\omega_c t - \tilde{\theta}_{me} - \theta_r^u) +$$
$$+ I_1^- \cos(\omega_c t - \tilde{\theta}_{me} + 2\theta_{me} - \theta_r^u)$$

(20)
High frequency harmonics of currents $i_{\alpha c}$ and $i_{\beta c}$

with:

$$
I_0^+ = \frac{L_{\Sigma}}{L_d L_q} \frac{A_1}{2} \frac{1}{\omega_c + \tilde{\omega}_{me}} \\
I_0^- = \frac{L_{\Sigma}}{L_d L_q} \frac{A_1}{2} \frac{1}{\omega_c - \tilde{\omega}_{me}} \\
I_1^+ = \frac{L_{\Delta}}{L_d L_q} \frac{A_1}{2} \frac{1}{\omega_c + \tilde{\omega}_{me}} \\
I_1^- = \frac{L_{\Delta}}{L_d L_q} \frac{A_1}{2} \frac{1}{\omega_c - \tilde{\omega}_{me}}
$$

where $L_d$ and $L_q$ are $d$– and $q$–axis inductances at frequency $\omega_c$ respectively, $\tilde{\omega}_{me}$ is the estimated speed and

$$
L_{\Sigma} = \frac{L_q + L_d}{2} \quad L_{\Delta} = \frac{L_q - L_d}{2} \quad (21)
$$
Some simulations have been done in order to verify the validity of the high frequency currents $i_{\alpha c}$ and $i_{\beta c}$ expressions.

In order to extract the $i_\alpha$ and $i_\beta$ harmonics at $f_c$, these currents must be measured with a frequency higher than the switching frequency.

To this purpose the simulated motor currents, filtered with a band pass filter around the switching frequency, are compared with the currents given by previously equations.

The term $A_1$ is estimated by a FFT analysis.
### High frequency simulated currents

#### Current $i_{\alpha_c}$ harmonics around the switching frequency

There is a good correspondence between real and reconstructed currents. The discrepancy is due to the other harmonics that are not considered in the mathematic analysis.
High frequency simulated currents

**Current $i_\beta$ harmonics around the switching frequency**

Same considerations can be done for the current $i_\beta$. 
For the position estimation purpose, the high frequency currents can be manipulated as follows:

\[
\epsilon = i_\alpha \cdot \cos(\omega_c t - \tilde{\theta}_{me} - \theta^r_u) - i_\beta \cdot \sin(\omega_c t - \tilde{\theta}_{me} - \theta^r_u) = -I_0^+ \sin(2\omega_c t) - I_1^+ \sin(2(\tilde{\theta}_{me} + \theta^r_u - \theta_{me})) - I_1^- \sin(2\omega_c t - 2(\tilde{\theta}_{me} + \theta^r_u - \theta_{me}))
\] (22)

Filtering by a low pass filter, the terms at frequency \(2\omega_c\) are removed and it results:

\[
\epsilon_{LP} = -I_1^+ \sin(2(\tilde{\theta}_{me} + \theta^r_u - \theta_{me}))
\] (23)
Imposing \( \tilde{\theta}_m' = \tilde{\theta}_m + \theta_u \), it results:

\[
\epsilon'_L = -l_1^+ \sin(2\Delta \theta_m) \tag{24}
\]

where \( \Delta \theta_m = \tilde{\theta}_m' - \theta_m \).

- The \( \sin(2\Delta \theta_m) \) is equal to zero when also \( \Delta \theta_m \) is zero, that is when \( \tilde{\theta}_m' \) is equal to the electrical position \( \theta_m \).
- An adjustment mechanism can thus correct the estimated position to nullify the error \( \Delta \theta_m \).
- A PI regulator is used to nullify \( \epsilon'_L \) and to deliver the estimated position and speed.
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Estimated position and actual one (simulation)
Position estimation

Estimated position and actual one (simulation)

- One can note the good correspondence between the estimated position (black line) and the actual one (red line).
- Simulation confirms the possibility to estimate the rotor position starting from the high frequency stator currents due to the PWM voltage control.
- Discrepancy is due to the other PWM harmonics that are not completely rejected by the filter.
Due to the specific and particularly innovative strategy used to extract the position information, it is mandatory to validate the proposal with the experimentation.

Switching frequency has been posed equal to 6.250 kHz.

An oversampled current measure in a test bench has been implemented and all the samples have been imported in MatLab.
Experimental results

Switching frequency harmonic currents

1. Currents $i_a$, $i_b$ is measured and the third $i_c$ is derived.
2. Acquisition and saving with 400 samples per period by means of oscilloscope.
3. Post elaboration with a high pass digital filter of 4th order filter around $f_c$

IPM motor

Currents and position measuring and saving by means of oscilloscope

Post elaboration with Simulink and Matlab
The saved currents are imported in a Simulink model very similar to that used for the simulations.
High frequency stator currents $i_{\alpha c}$ (black line) and $i_{\beta c}$ (red line)

High frequency stator currents are obtained filtering the measured by a band pass filter, centered at the switching frequency.

The result is very similar to the expectation proposed by the analysis and the results obtained in the simulation.

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Estimation purpose

- The current samples are applied to the estimated scheme used in the simulation.
- The output of the estimator is compared to the actual (measured) quantities.

Estimator scheme
The estimated speed reaches the actual one after an initially transient. A reduced oscillation is present on the estimated speed: this effect may be due to a DC–BUS oscillation.
The estimated position follows the actual one.
Test confirms the possibility to extract the position.
Oscillations in the estimated speed get oscillations in the estimated position.
Rotor position estimation using PWM intrinsic voltage harmonics injection has been presented.

At first, the theoretical treatment has been discussed.

A preliminary theory confirmation has been obtained by estimating the position by means of simulations.

The position has been estimated with an acceptable precision using a post elaboration of the oversampled motor currents.

The new estimation technique appears therefore viable, provided that the current oversampling and post elaboration need are solved.

A full digital implementation of the sensorless drive coming soon.
S. Bolognani, and A. Faggion,
Thank you for the attention.